Condensed Matter Physics I
II written test
academic year 2009/2010
December 15, 2009

(Time: 3 hours)

Exercise 1: *Density of states for 2D periodic crystals: critical points*

For a 2D lattice, calculate, plot and discuss the behaviour of the density of states for a band described for sufficiently small k by the following expressions:

1. around a minimum:
$$E(\mathbf{k}) = E_0 + (\hbar^2/2)(k_x^2/m_x + k_y^2/m_y)$$
2. around a maximum:
$$E(\mathbf{k}) = E_0 - (\hbar^2/2)(k_x^2/m_x + k_y^2/m_y)$$
3. around a saddle point:
$$E(\mathbf{k}) = E_0 + (\hbar^2/2)(k_x^2/m_x - k_y^2/m_y).$$
(*In the latter case, you should be able to find a logarithmic singularity:* $g(E) \sim constant \cdot ln(4a^2/|E - E_0|)$*, with "a" related to an arbitrary small cutoff for k where the quadratic expansion of the band holds; hint: remember Ex. 8.2 of Ashcroft-Mermin textbook. In this case you may need to solve an integral like this:*
$$\int \frac{dx}{\sqrt{x^2+a}} = ln(x + \sqrt{x^2+a}).)$$

Exercise 2: *Tight-binding model and Bloch electron velocity*

Consider a lattice with a band given by:

$$E(k) = E_0 + 4\gamma[\sin^2\left(\frac{k_x a}{2}\right) + \sin^2\left(\frac{k_y a}{2}\right) + \sin^2\left(\frac{k_z a}{2}\right)].$$

1. Which Bravais lattice and which kind(s) of atomic orbitals does this band originate from? *(the answer will be considered valid only if fully justified by calculations)*
2. Write the expression of the Bloch velocity for this band.
3. Show that the velocity for $\mathbf{k}$ lying on a face of the Brillouin zone is parallel on such face *(also in this case the answer will be considered valid only if fully justified by calculations; it's enough for one face)*
4. Generalize the expression of the energy band given above, for the same lattice, in order to include also next-nearest neighbor interactions (indicating with t the corresponding hopping integral).

Exercise 3: *Weak potential*

Consider Schrödinger's single particle equation for an electron moving on a plane under in a potential

$$U(x,y) = -2U\left[cos\left(\frac{2\pi}{a}x\right)cos\left(\frac{2\pi}{a}y\right)\right].$$

1. Write the location of minima and maxima of $U(x,y)$. Give a sketch in the (x,y) plane of the Bravais lattice formed by all the minima (maxima).
2. Which is the underlying Bravais lattice? Give a possible choice of basis vectors.
3. Give the corresponding reciprocal lattice vectors and sketch the First Brillouin Zone(FBZ).
4. Which are the non vanishing $U_{\mathbf{G}}$ and their value? (*Hint: write trigonometric functions in terms of exponentials*).
5. Assume U "small" and calculate with first order perturbation theory the effect on the energy bands at $\mathbf{G_1}/2$, where $\mathbf{G_1}$ (one of the star of) the shortest reciprocal lattice vectors.