Condensed Matter Physics I
I written test
academic year 2017/2018
November 8, 2017

(*Time: 2 hours*)

NOTE: Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise 1: Sommerfeld model and occupation of electronic energy levels

1. Find the probability that an electronic energy level 0.1 eV below the Fermi energy in copper is occupied at T = 300 K. For copper, E_F=7.04 eV.

Exercise 2: Sommerfeld model for 2D electron gas

Consider a free electron gas in 2D in the Sommerfeld model, with density n.

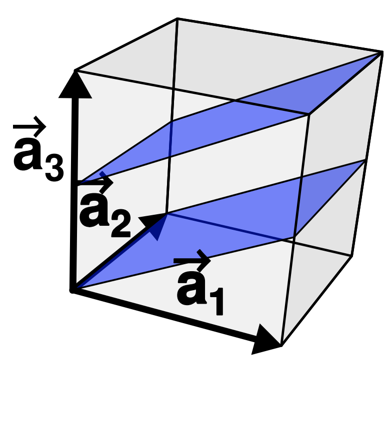
1. Find the Fermi energy in terms of the electron density n.
2. Show that the average energy for each electron is $E_F/2$.
3. Show that the chemical potential μ can be written as:

$$\mu = k_B T\, ln(e^{E_F/k_BT} - 1) \qquad \text{and in particular} \quad \mu \approx \begin{cases} E_F & \text{if} \quad k_BT << E_F \\ kT\, ln(E_F/k_BT) & \text{if} \quad k_BT >> E_F \end{cases}$$

Exercise 3: Crystalline structures: family of lattice planes

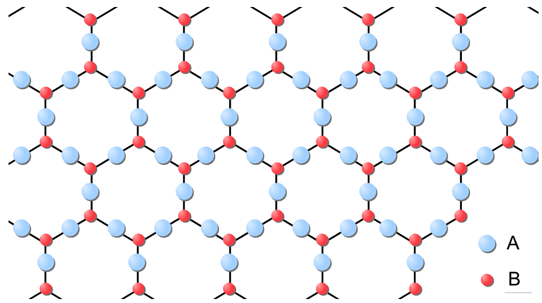
Shown in the figure is a conventional cubic cell with two parallel lattice planes.

1. Give the Miller indices of the family of lattice planes with reference to the cubic cell.
2. Calculate the interplanar distance.
3. Given the lattice planes drawn in the figure, which cubic Bravais lattice (SC, BCC, FCC?) we are considering in particular?



Exercise 4: Crystalline structures: Bravais lattices and primitive unit cells in 2D

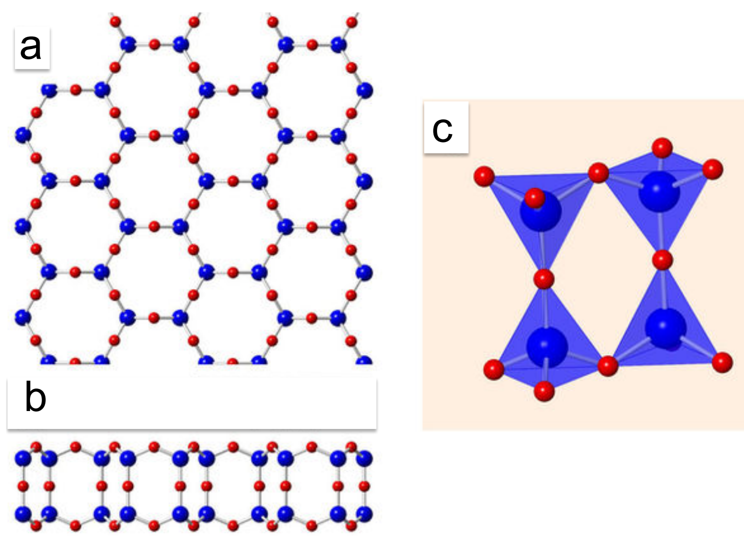
Consider the 2D lattice shown in panel (a) of the figure below.



1. Draw a set of primitive vectors and the corresponding unit cell.
2. How many atoms of each type are in the primitive unit cell? Mark them in the unit cell previously drawn.

Exercise 5: Crystalline structures: Bravais lattices and primitive unit cells in 3D

In the following figure, panel (a) is similar to the figure of Exercise 4, but now it represents the top-view projection of a bilayer silica (Si blue, O red); the side-view is shown in panel (b) and tetrahedral units forming the 3D structure of the bilayer are shown in panel (c). (figure adapted from T. Bjorkman et al., Scientific Reports 3, 3482 (2013))



1. Give the formula unit of the compound, Si_mO_n.
2. Describe the 3D structure of the bilayer silica: specify which is the unit cell, draw it, specify how many atoms it contains.