Assignment 2

Group 5

Abstract

Estimating the SEs of coefficients in basic univariate regression; testing simple hypotheses on coefficients; probability distributions with R

Keywords: regression, OLS, R.

Consider the three following vectors of returns:

rf risk-free

rm market portfolio

rx asset X

for periods $1 \dots 20$:

	rf	rm	rx
1	0.64	5.51	8.81
2	0.53	5.79	9.77
3	0.77	5.88	9.83
4	0.87	5.05	8.35
5	0.53	4.98	8.51
6	0.75	4.69	8.02
7	0.82	5.10	8.46
8	0.44	3.47	5.55
9	0.67	6.40	10.99
10	0.92	7.21	12.09
11	0.79	6.24	10.46
12	0.72	4.45	7.43
13	0.48	5.48	8.99
14	0.58	6.82	11.13
15	0.74	5.49	8.77
16	0.64	4.34	7.21
17	0.89	5.70	9.28
18	0.85	4.79	7.59
19	0.75	7.19	12.09
20	0.99	6.15	10.06

and the regression model from Assignment 1:

$$erx_t = \alpha + \beta erm_t + u_t$$

Building on the skills you acquired from Assignment 1, i.e.

- inputting the three vectors of returns:
- calculating *excess* returns for market (erm) and stock X (erx)
- estimating the above model through `lm`

your group is required to :

1. estimate the covariance of the estimator $\hat{\beta}_{OLS}$ using matrix notation
2. estimate the same model using the function `lm()` and inspect it through `summary()`; show that $SE(\hat{\beta}_{OLS})$ are the same
3. test the following hypotheses:
   (a) H_0: $\alpha = 0$, H_A: $\alpha \neq 0$
   (b) H_0: $\alpha = 0$, H_A: $\alpha > 0$

Please provide the results under form of a (more) readable (than last time) document in free format, with a reasonable amount of comments where appropriate.

The deadline for this assignment is Tuesday, November 5th.

1. Results (not for distribution)

Make excess returns

```
> erm <- rm - rf
> erx <- rx - rf
```

1.1. Estimate covariance through matrix algebra

Estimate $Cov(\hat{\beta}_{OLS}) = s^2(X'X)^{-1}$ through matrix algebra:

1. estimate $\hat{\beta}_{OLS}$

```
> X <- cbind(1, erm)
> beta.hat <- solve(crossprod(X), crossprod(X, erx))
```

2. fetch model residuals $\hat{u}$

```
> u.hat <- erx - X %*% beta.hat
```

3. calculate $s^2 = \hat{u}'\hat{u}$

```
> s2 <- crossprod(u.hat)/(dim(X)[[1]]-dim(X)[[2]])
```

4. estimate the covariance matrix $Cov(\hat{\beta}_{OLS})$

```
> cov.bhat <- as.numeric(s2) * solve(crossprod(X))
> print(cov.bhat)

                          erm
     0.07051049 -0.014132728
erm -0.01413273  0.002933318
```

5. calculate $SE(\hat{\beta}_{OLS})$

```
> cov.bhat <- as.numeric(s2) * solve(crossprod(X))
> se.bhat <- sqrt(diag(cov.bhat))
> print(se.bhat)

                  erm
0.26553811 0.05416012
```

1.2. Estimate model, extract covariance and compare

Estimate model:

```
> mod <- lm(erx ~ erm)
```

extract standard errors from `summary`:

```
> print(summary(mod)$coefficients[, 2])
```

```
(Intercept)         erm
 0.26553811  0.05416012
```

(alternatively, just `print(summary(mod))` and look at them).

1.3. Two-sided test

Test H_0: $\alpha = 0$, H_A: $\alpha \neq 0$:

1. Calculate test statistic

   ```
   > tstat <- (beta.hat/se.bhat)[1,1]
   ```

2. calculate (two-sided) critical value

   ```
   > crit1 <- qt(0.025, df=dim(X)[[1]]-dim(X)[[2]], lower.tail=FALSE)
   > ## alt.:
   > ## crit1 <- qt(1 - 0.025, df=dim(X)[[1]]-dim(X)[[2]])
   ```

3. compare (two-sided): reject H_0 if `TRUE`

   ```
   > abs(tstat) >= abs(crit1)
   ```

   ```
   FALSE
   ```

1.4. One-sided test

Test H_0: $\alpha = 0$, H_A: $\alpha \geq 0$

1. Calculate test statistic : same as above

2. calculate (one-sided) critical value

   ```
   > crit2 <- qt(0.05, df=dim(X)[[1]]-dim(X)[[2]], lower.tail=FALSE)
   ```

3. compare (one-sided): reject H_0 if `TRUE`

   ```
   > tstat >= crit2
   ```

   ```
   FALSE
   ```

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