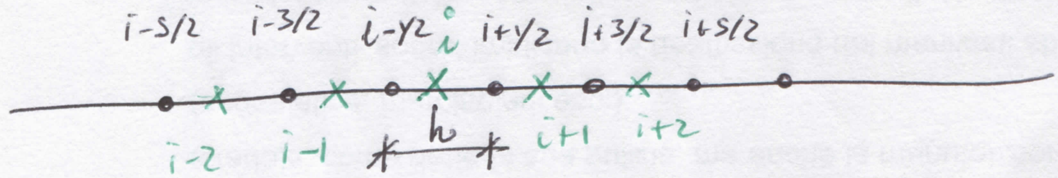
Interpolation and filtering

• Interpolation

$$\beta \hat{f}_{i-2} + \alpha \hat{f}_{i-1} + \hat{f}_i + \alpha \hat{f}_{i+1} + \beta \hat{f}_{i+2}$$

$$= \frac{c}{2}\left(f_{i+5/2} + f_{i-5/2}\right) + \frac{b}{2}\left(f_{i+3/2} + f_{i-3/2}\right)$$

$$+ \frac{a}{2}\left(f_{i+1/2} + f_{i-1/2}\right)$$



Taylor-series expansion yields the coefficients of different schemes.

The resolution of the scheme can be featured by a "transfer function" $T(\omega)$:

$$\hat{f}_i = T(\omega)\, e^{ikx_i}$$

For instance, for an explicit, second-order interpolation:

$$\hat{f}_i = \frac{f_{i+1/2} + f_{i-1/2}}{2}$$

$$\Rightarrow T(\omega)\, e^{ikx_i} = e^{ikx_i}\, \frac{e^{ikh/2} + e^{-ikh/2}}{2}$$

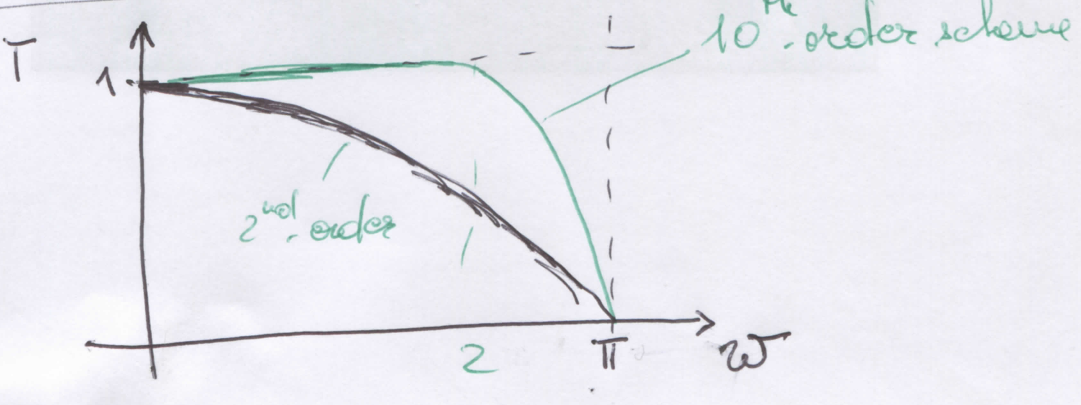
$$= e^{ikx_i} \cos(\omega/2)$$

$$\Rightarrow \boxed{T(\omega) = \cos(\omega/2)}$$

A 10^{th}-order compact scheme has coefficients:

$$\alpha = \frac{10}{21} \quad \beta = \frac{5}{126} \quad a = \frac{5}{3} \quad b = \frac{5}{14} \quad c = \frac{1}{126}$$

$$\boxed{T(\omega) = \frac{a\cos(\omega/2) + b\cos(3\omega/2) + c\cos(5\omega/2)}{1 + 2\alpha\cos\omega + 2\beta\cos(2\omega)}}$$

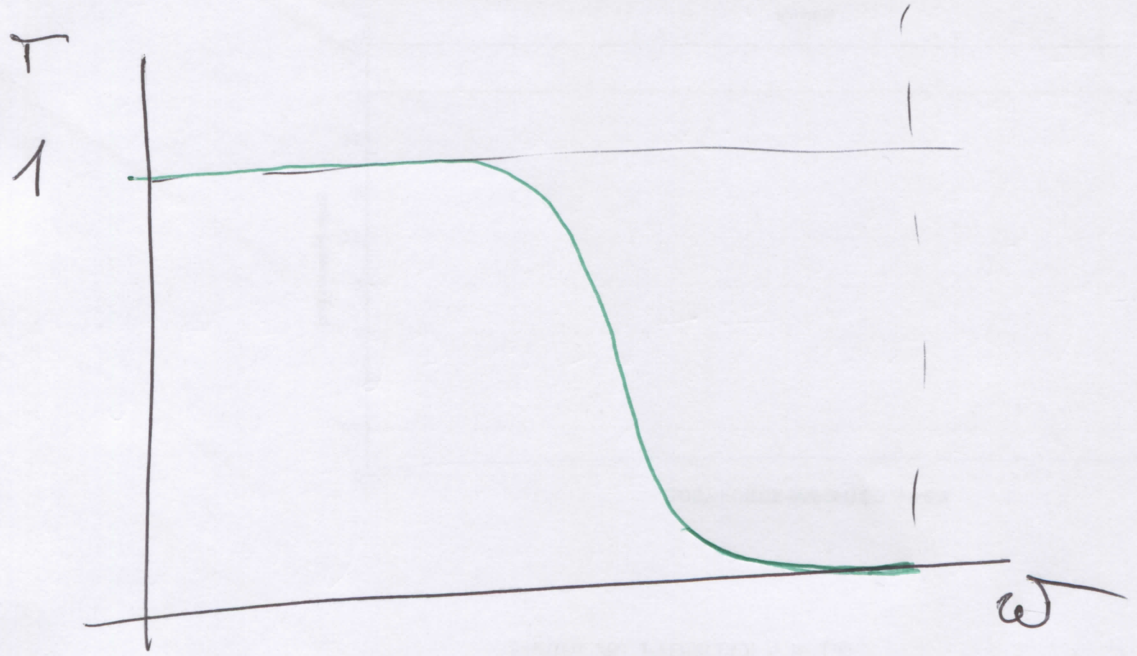


ω	$T(\omega)$	
	2^{nd} order	10^{th}-order
0	1.000	1.000
$\pi/6$	0.966	1.000
$\pi/3$	0.866	1.000
$\pi/2$	0.707	1.000
$2/3\,\pi$	0.500	0.992
$5/6\,\pi$	0.259	0.868
π	0.000	0.800

A 6^{th}-order filter satisfying all the filtering constraints is proposed by Lele (1991).

$\alpha = 0 \quad \beta = \frac{3}{10} \quad a = \frac{1}{2} \quad b = \frac{3}{4} \quad c = \frac{3}{10}$

$d = \frac{1}{20}$



ω	T(ω)
0	1.000
π/6	1.000
π/3	0.964
π/2	0.500
2/3 π	0.036
5/6 π	0.000
π	0.000

Boundary filters can be derived as well
(see Lele, 1991, p 51)