Cognitive Neuroscience
Numbers

Cinzia Chiandetti, PhD
A.A. 2018–2019
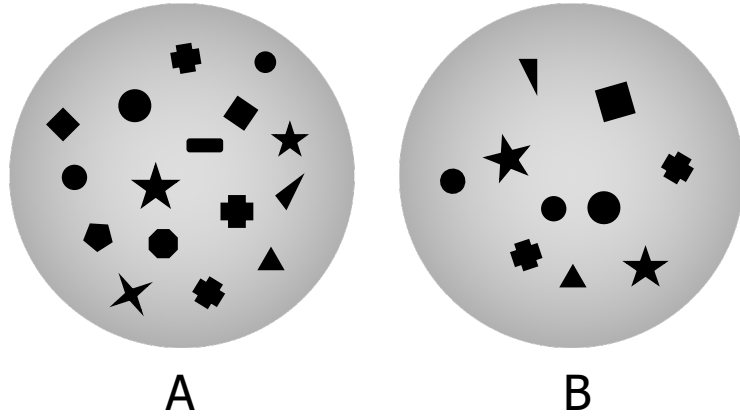
1CFU

Core knowledge

Numeri

A   B

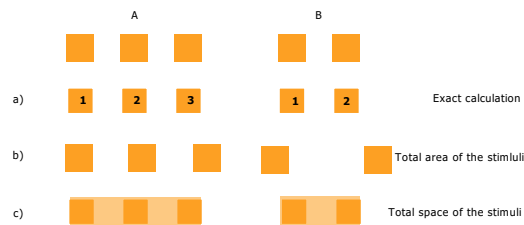
Numbers

- The discrimination of relative numerosity
  - is the ability to establish that 2 ensambles of elements are not equal and to approximately discriminate the relative numerosity
  - is a protonumerical capability (Davis & Perusse, 1988)

Numbers

- Usually, it is challenging to establish the specific type of info used (by the animal) during the test

- even when discriminating between the two ensambles, the choice could be based on non-numerical variables that usually co-variate with numerosity (extranumerical variables)
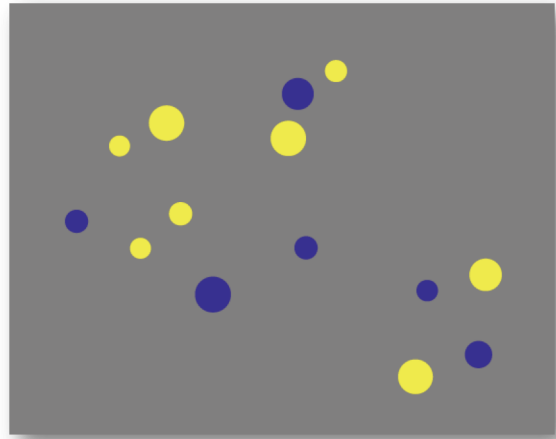
Numbers

MAIN NUMBER-CORRELATED CONTINUOUS VARIABLES		
SENSORY MODALITY FOR PRESENTATION OF THE STIMOLI		
VISUAL	ACOUSTIC	OLFACTORY
TOTAL SURFACE AREA (l'insieme numericamente più ampio occupa generalmente una maggior area superficiale complessiva)	**TOTAL TIME** (Insiemi tonali più numerosi richiedono un tempo maggiore di presentazione)	**QUANTITY OF CHEMICAL INFO** (l'insieme più numeroso tende a portare con sé un più alto grado di informazione olfattiva)
DENSITY (a parità di superficie il gruppo più numeroso è caratterizzato da una maggiore densità)		
OCCUPIED SPACE (il gruppo più numeroso tende ad occupare uno spazio totale maggiore)	**FREQUENCY OF THE ELEMENTS** (Tenendo sotto controllo il tempo totale di presentazione, aumenta la frequenza degli elementi che compongono l'insieme più numeroso)	**QUANTITY OF PHEROMONES** (il gruppo più numeroso solitamente rilascia una quantità maggiore di sostanze biochimiche)
LUMINOSITY (la quantità di luce riflessa tende a variare a seconda del numero di elementi presenti sulla scena)		
INTER-ELEMENTS DISTANCE (più è elevata la numerosità di un insieme, più la distanza che intercorre tra i suoi elementi ha la probabilità di essere minore)		

Numbers

- Which color has the larger number of items?

  - There are more yellow dots

- How many yellow dots?

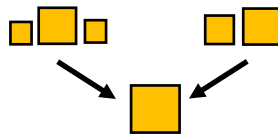 - There are 8 yellow dots

Numbers

- How to control for extra-numerical variables?
  1. matching the area
  2. modifying the spatial position
  3. changing the nature of the stimuli (transfer)
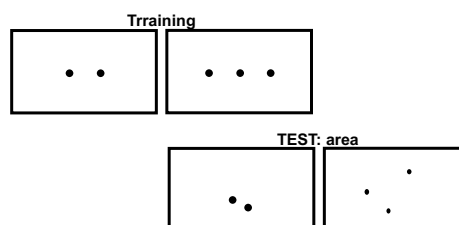 4. with a sequential presentation

Numbers

- How to control for extra-numerical variables?
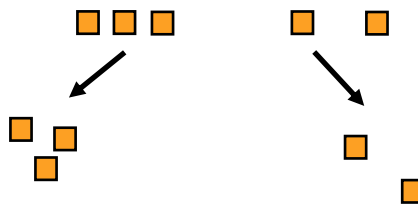   1. matching the area

Numbers

- How to control for extra-numerical variables?
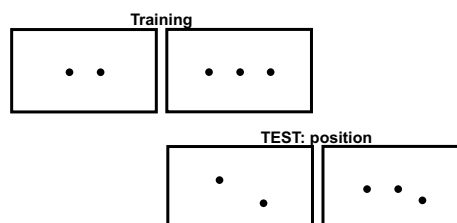   1. matching the area, example

Numbers

- How to control for extra-numerical variables?
  - 1. matching the area
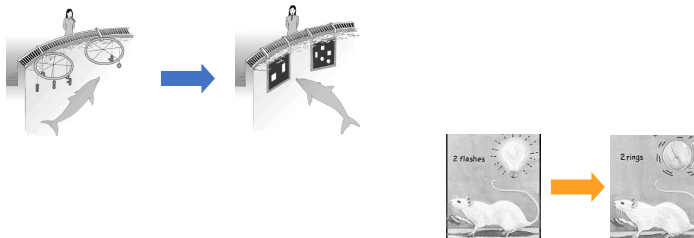 - 2. modifying the spatial position

Numbers

- How to control for extra-numerical variables?
  - 1. matching the area
  - 2. modifying the spatial position, example

Numbers

- How to control for extra-numerical variables?
    1. matching the area
    2. modifying the spatial position
    3. changing the nature of the stimuli (transfer), examples
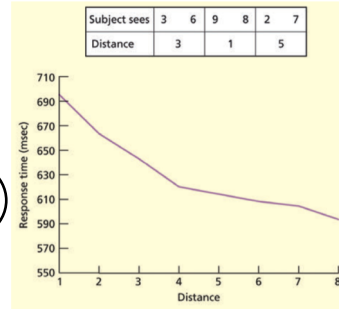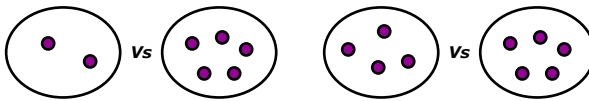

Numbers

- How to control for extra-numerical variables?
    1. matching the area
    2. modifying the spatial position
    3. changing the nature of the stimuli (transfer)
    4. with a sequential presentation

Numbers

- Two effects rule the relative numerosity judgements

PRINCIPLE of the **NUMERICAL DISTANCE**

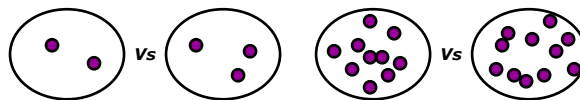the higher the distance, the better the disrimination between the two magnitudes
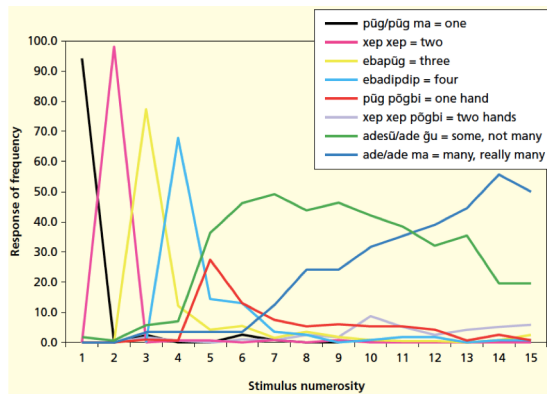

Numbers

- Two effects rule the relative numerosity judgements

PRINCIPLE of the **NUMERICAL MAGNITUDE**

When the distance between two sets is the same, the larger the number of elements between the two sets that need to be compared to give an answer, the worse the performance

Numbers

Legend:
- pūg/pūg ma = one
- xep xep = two
- ebapūg = three
- ebadipdip = four
- pūg pōgbi = one hand
- xep xep pōgbi = two hands
- adesū/ade ğu = some, not many
- ade/ade ma = many, really many

- Tribal populations (likethe Amazonian Munduruku) with a limited vocabulary for numbers and no formal mathematical training
  - Asked to name numbers of sets from 1 to 15 dots (in order to study how they count and what names they have in their vocabulary)

Numbers

- **Estimation of magnitude**: subjects were shown two clouds of dots and had to judge which of the two sets is larger
- **Approximate computation**: they were shown short video clips illustrating simple operations (20 seeds + 30) and the subject must say whether the total was more or less than another set (for example, of 40 seeds)
- **Computing with exact numbers:** they were asked to give the result of a precise mathematical operation (for example, 6 seeds minus 4 seeds)
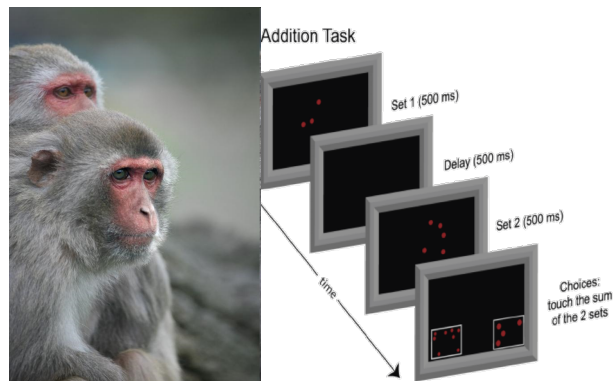
Numbers
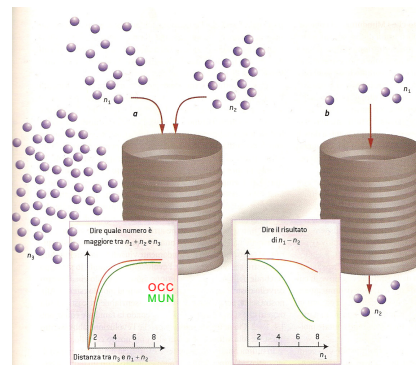
Pica et al., 2004

Numbers

- If they have to add 5 to 7 seeds, they choose an answer like "approx. a dozen" (meaning 11, 12 or 13) -> correct
- If they have to answer about distant numbers (i.e., to sum 8 to 20) they cannot answer in no way

- There is a distinction between symbolic and non-symbolic representations:
  - The symbolic representations allow both the exact and approximate quantification
  - The non-symbolic representations allow an approximate quantification only
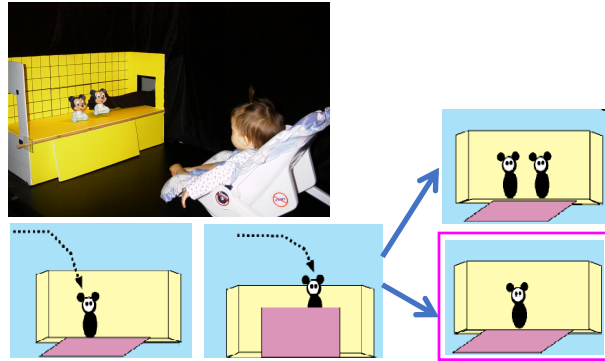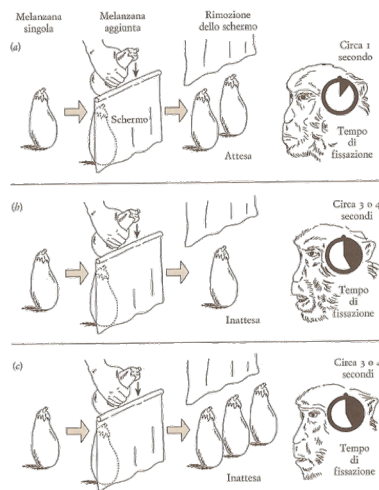
Numbers

Numbers

- Macaques *vs.* Students

  - Students provide the 94% of correct answers

    average time for an answer: 1 sec
  - Macaques provide the 76% correct answers

    average time for an answer: 1 sec
  - Both groups suffered when the two alternatives were very similar (e.g. correct summation = 11; choice between 11 *and* 12)
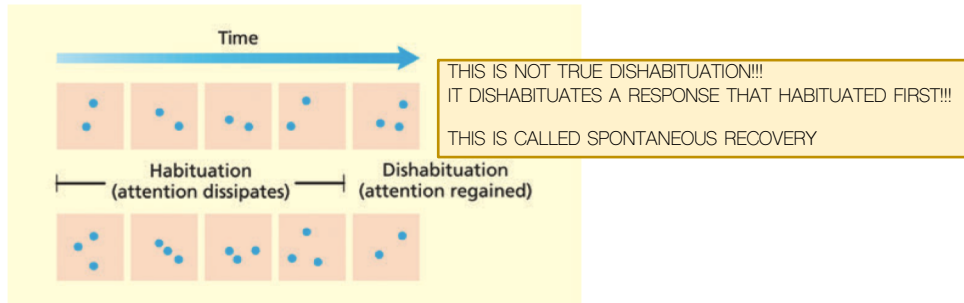
Numbers

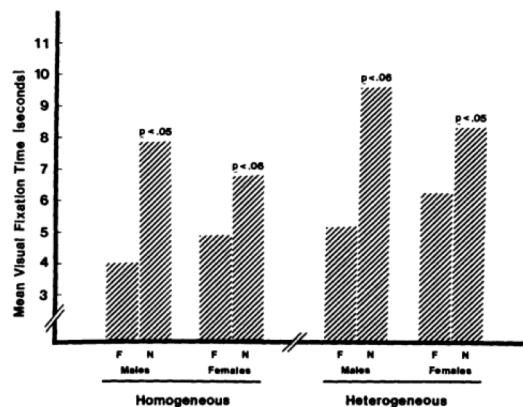- Wynn, 1992

Numbers

Numbers

• Antell and Keating (1983) tested 1 day old newborns

Numbers

• One alternative explanation for the results observed by Antell and Keating is the possibility that attention was regained by any new stimulus instead that the novel numerosity

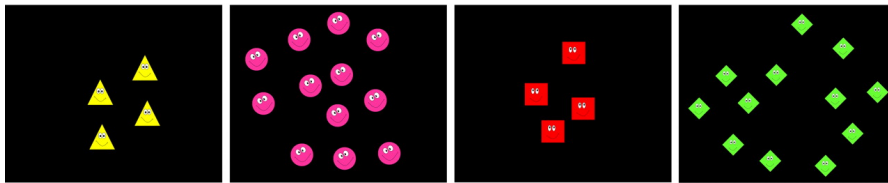• Strauss e Curtis (1981)

Numbers – 50h newborns

Familiarization (2 min)

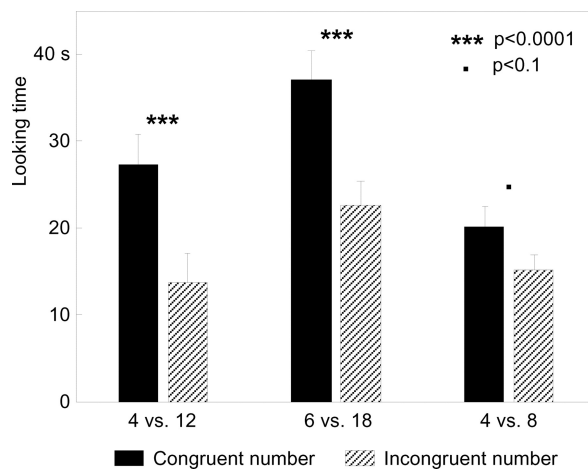... " tu-tu-tu-tu-tu-tu-tu-tu-tu-tu-tu-tu " ... " ra-ra-ra-ra-ra-ra-ra-ra-ra-ra-ra-ra- " ...

or

... " tuuuuu-tuuuuu-tuuuuu-tuuuuu " ... " raaaaa-raaaaa-raaaaa-raaaaa " ...

Test (4 trials)

Izard et al., 2009

Numbers – 50h newborns

Izard et al., 2009

Numbers

- In conclusion: The names of numbers are
  - neither essential to manage gross arithmetical concepts (quantities, more/less)
  - not important to run approximate estimations

- The names of numbers are important
  - to compute exact calculations with more than 4-5 elements
  - to number the elements in an automatic way

Numbers

- A sub-set of our core numerical abilities is shared with different species
  - human and non-human animals share a "core set" of abilities to compare non-verbally approximate numerosities
  - and to compute raw, non-verbal, arithmetic
- Indeed, when you prevent human participants to count explicitly, they can resort to use only an approximate system of magnitudes:
  - our non-verbal representation of numerosities is comparable to that of other species
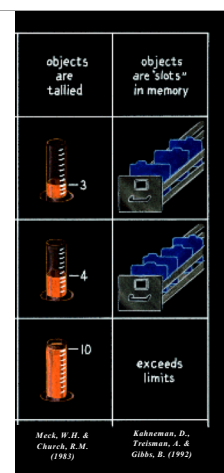
Numbers

- Two systems describe the core numercial cognition:
  - For SMALL NUMEROSTIES there is the OBJECT-FILE SYSTEM: each element is a symbol, there is a one-to-one equivalence for object-number (there is a mental image of each object)
  - Calculations are computed in parallel and are stored in a temporary storage system
  - It has limits:
    - It allows precise computatinos but has a limit of 3-4 units especially when groups differ of 1 unit only


objects are "slots" in memory

exceeds limits

Kahneman, D., Treisman, A. & Gibbs, B. (1992)

Numbers

- For LARGE NUMEROSITIES there is the ACCUMULATOR MODEL:
  - It allows a "*fuzzy*" mathematics: two sets are discernable whenever the distance between the two increases
  - An organism can estimate rapidly relevant quantities for survival
  - Inputs are accumulated and only at the end are quantified
  - It means that different numerosities correspond to different levels within the accumulator and if the difference is small, two sets can be named with the same label

objects are tallied

objects are "slots" in memory

exceeds limits

Meck, W.H. & Church, R.M. (1983)

Kahneman, D., Treisman, A. & Gibbs, B. (1992)

Numbers

- ANS: Aproximate Number System

- It suffers from general imprecision, which is due to its logarithmic nature. That is, larger numbers are represented closer together than smaller numbers

- number discrimination and apprehension of quantities are more prone to error as a function of the magnitude and decreased ratio between sets

larger numbers

smaller numbers

0 1 2 3 5 6 7 8 9 10 11   100
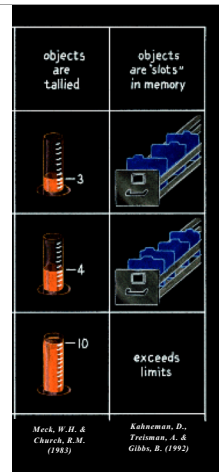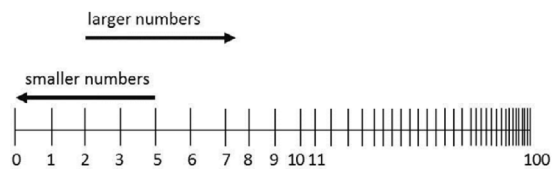
Numbers

- At the neurophysiological level, how are represented different numerosities?
- We can use single-cell recording in nonhuman primates to answer this question
  - With the assumption that
    - in different species these neurons support raw basic numerical abilities
    - In humans they can be linked (maybe modified) by symbolic representations (Nieder and Dehaene, 2009)

- Different populations of neurons have been found

Numbers

Monotonic Coding of Numerosity in Macaque Lateral Intraparietal Area

Jamie D. Roitman[1☆*], Elizabeth M. Brannon[2,3], Michael L. Platt[1,3]

1 Department of Neurobiology, Duke University, Durham, North Carolina, United States of America, 2 Department of Psychology and Neuroscience, Duke University, Durham, North Carolina, United States of America, 3 Center for Cognitive Neuroscience, Duke University, Durham, North Carolina, United States of America

- One class responds to higher quantities (Roitman et al., 2007)
  - *[…] a population of neurons in the lateral intraparietal area of monkeys encodes the total number of elements within their classical receptive fields in a graded fashion*
  - *the modulation of neuronal activity by visual quantity developed rapidly, within 100 ms of stimulus onset, and was independent of attention, reward expectations, or stimulus attributes such as size, density, or color.*
  - *The responses of these neurons resemble the outputs of ''accumulator neurons'' postulated in computational models of number processing.*
  - *Numerical accumulator neurons may provide inputs to neurons encoding specific cardinal values, such as ''4,'' that have been described in previous work […]*

Numbers

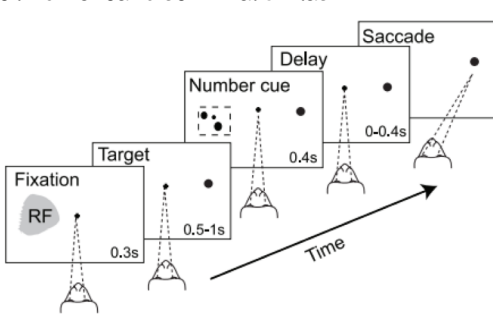**Monotonic Coding of Numerosity in Macaque Lateral Intraparietal Area**

Jamie D. Roitman[1☆*], Elizabeth M. Brannon[2,3], Michael L. Platt[1,3]
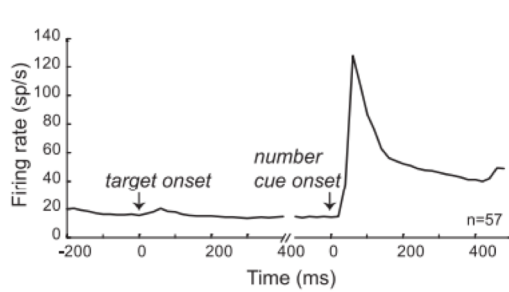
1 Department of Neurobiology, Duke University, Durham, North Carolina, United States of America, 2 Department of Psychology and Neuroscience, Duke University, Durham, North Carolina, United States of America, 3 Center for Cognitive Neuroscience, Duke University, Durham, North Carolina, United States of America
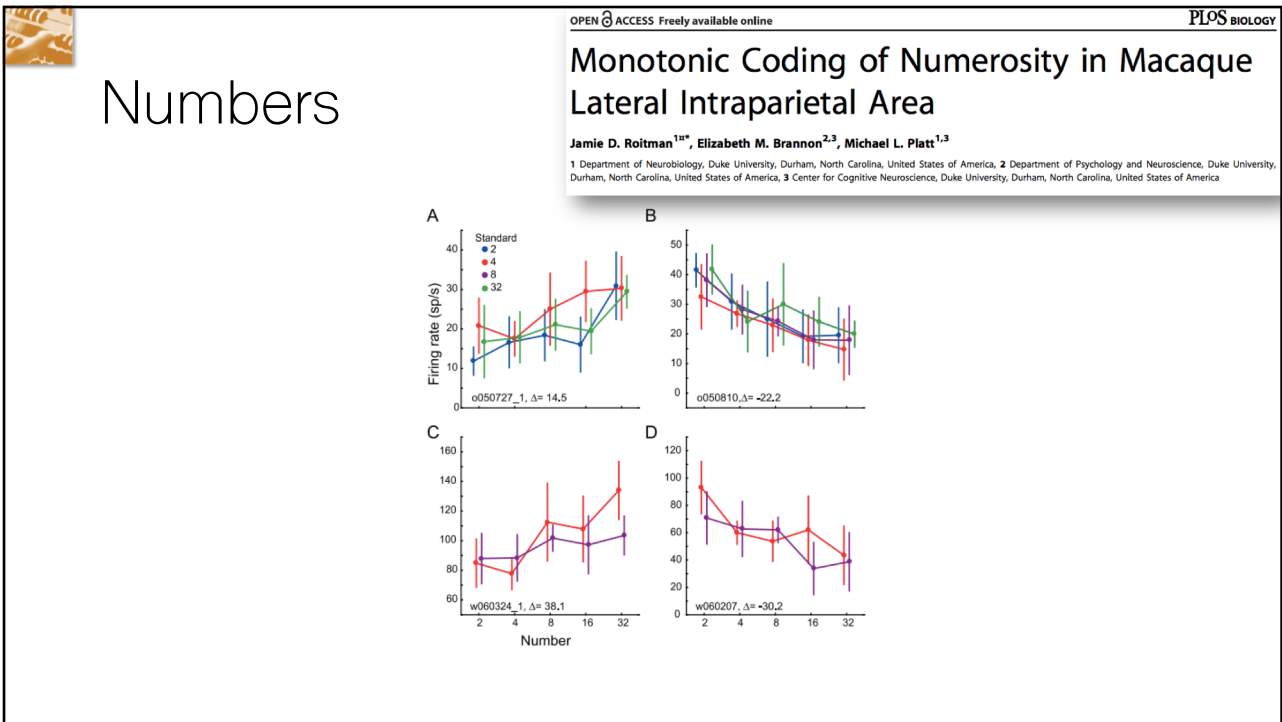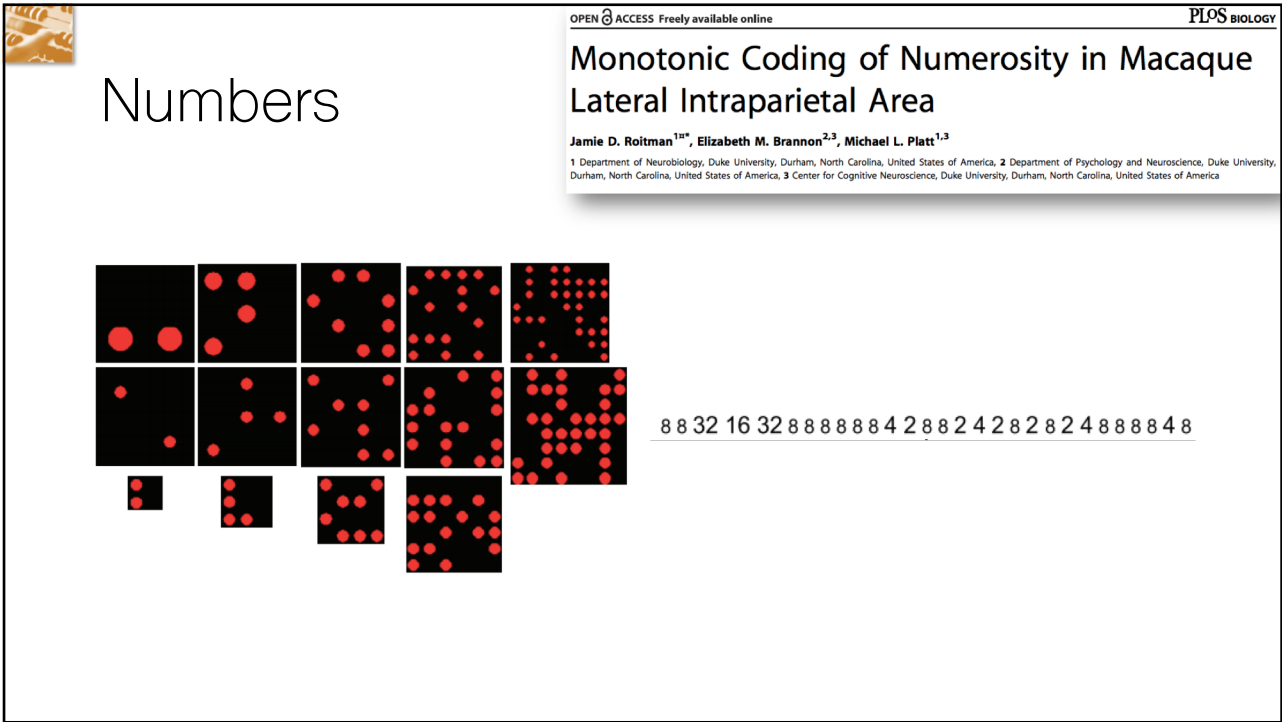
implicit numerical discrimination task

Numbers

8 8 32 16 32 8 8 8 8 8 8 4 2 8 8 2 4 2 8 2 8 2 4 8 8 8 8 4 8
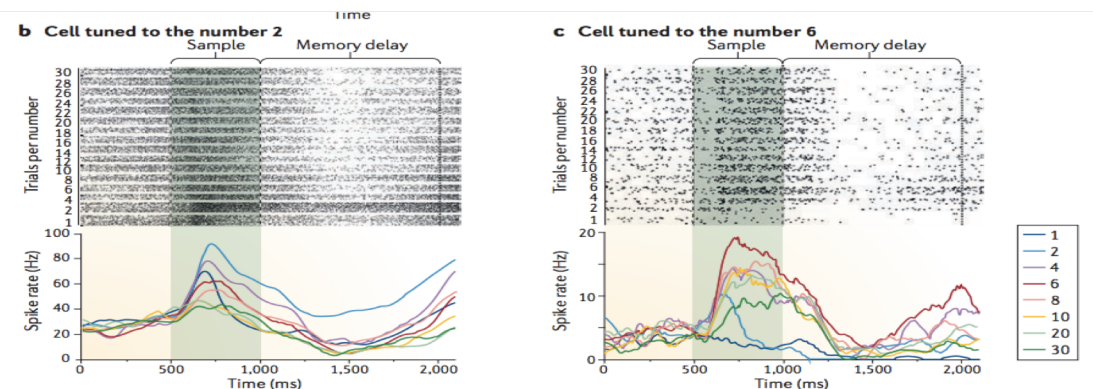
Numbers

Numbers

- The most interesting class of cells is tuned to a specific quantity
  - These cells have the highest activity in response to a stimulus of a certain number (e.g., 5 dots)
  - Some are activated by different modalities (e.g., 5 visual stimuli, 5 bursts of noise)
  - They respond with the optimum to '5' and to a lesser extent to '4' and '6' and even lesser to '7' and '3' and so forth

  - It is not unusual to record from cells whose activity is tuned to '10' but respond still vigorously to '9' or '11' elements showing the magnitude effect
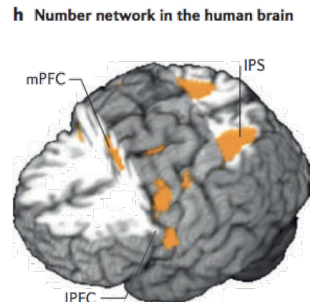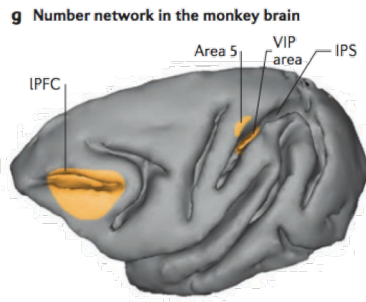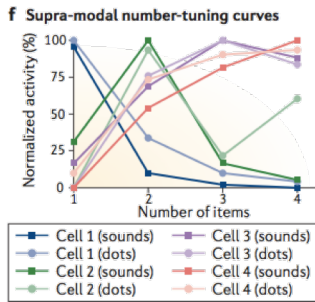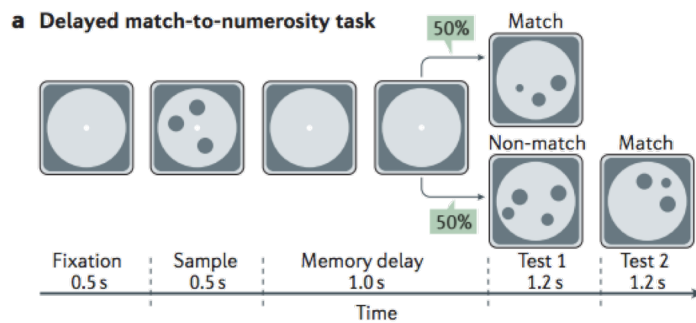
Numbers

Numbers

They are silent in front of other words (e.g., car, flower, etc.)
They are recorded from the IPS and the PFC in the macaque's brain

Numbers

Numbers

d **Sequential visual number task**

e **Sequential auditory number task**
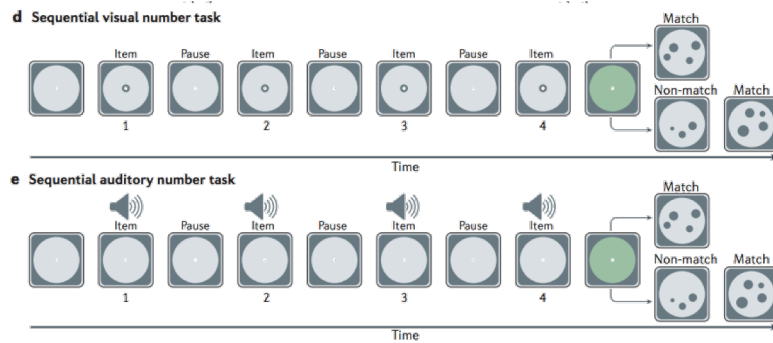
Numbers

- It is possible to train animals to associate a certain number of dots with an arbitrary symbol of the same quantity

  - Frontal neurons are selectively activated by both symbolic and nonsymbolic aspect
  - Meaning that they show a discharge frequency when the animal sees 3 dots or sees symbol '3'

Numbers

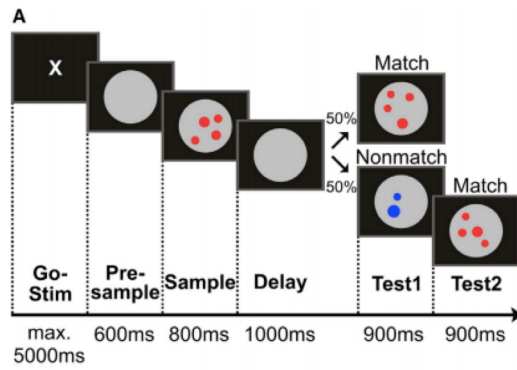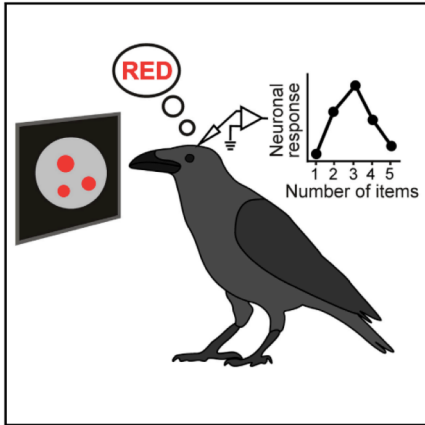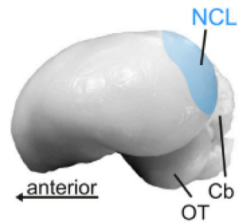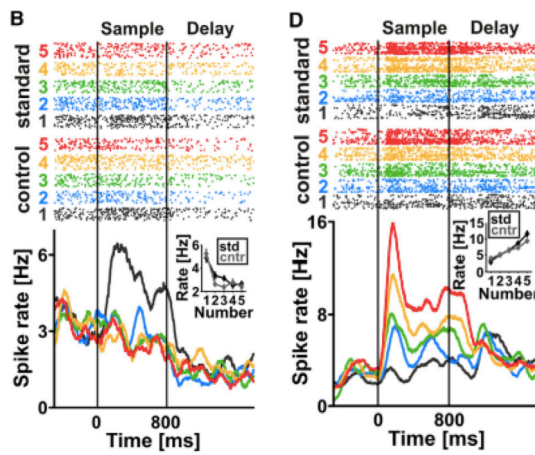Wagener et al., 2018

Numbers

Wagener et al., 2018

Numbers

- The first neurophysiological evidence in a bird brain of spontaneous number-related activity
- Despite macroscopic difference in brain architecture between monkeys and birds, there is comparable neuronal activity for numbers
  - Evolution found a different strategy to organize brains so that they can respond to the specific number of items in a particular set

  - However, these were adult individuals (should be tested in the young)
- What is this representation for, is still debated:
  - Maybe it evolved for social reasons -> are less evident in solitary species?

Numbers – the ZERO

- Since each collection of real elements has NO ELEMENTS it was thought that the zero was NOT PRESENT in the core set of representations

- Some facts supported this view:
  - hystorically, there is a late use of zero as a symbol
  - ontogenetically, the child learns very late to use the zero (ask a child who knows already the sequence of numbers: "zero, one, two, three, four…" which is the smallest number; he/she would say "one!"
  - **The conclusion was that it has to be learned, it is not in the basic kit**

Numbers

- The question for children is a ill-posed one
- Indeed, Sulkowski and Hauser showed that macaques solve these operations: 0+1 = 1 and 1-1 = 0
- Biro and Matsusawa showed the same in chimps
- Alex the parrot could use zero as a symbol
- ...and think to its ecological meaning

Numbers

- Thanks to the ANS, children and other species can reason about the empty collection: indeed, we are faster in saying that
  - 8 <18 than 8 < 12
  - 0 < 12 than 0 < 4

  - The conclusion is rather that there is nothing special in allocating the empty collection on the ladder of quantities in the first position...

Numbers

- However, Okuyama et al., showed that the zero has a special state; indeed, there are "two neurons for a zero":
  a) Neurons with exclusive response (increased freq for zero only)
  b) Neurons with continuous response (max freq for an empty collection)
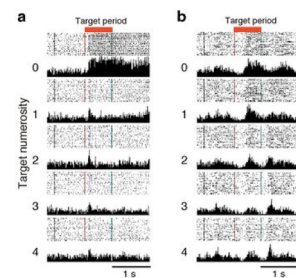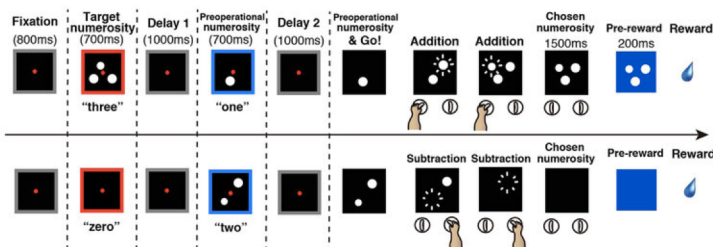
Numbers

- In a sense, between 'nothing' and 'something' there could be an asymmetric cognitive representation
  - Wynn showed that infants are surprised when an object that was placed behind an opaque screen has disappeared

  - Conversely, infants are not surprised if an object magically appears
    - Maybe it is a copy, a new object or the same…hard to disambiguate

Numbers
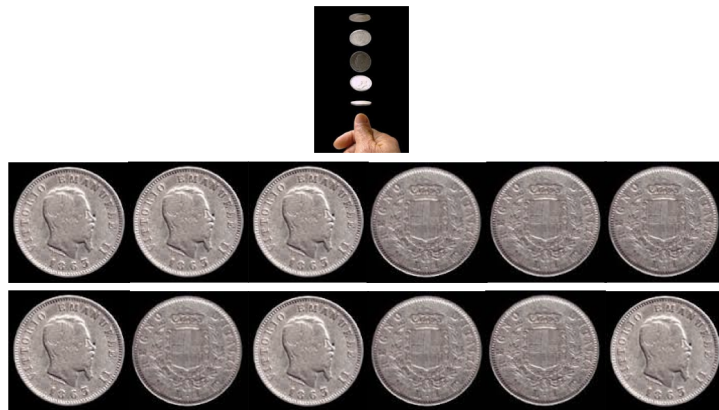
Numbers

- 12-mo olds:
  - If a blue ball comes out from the container (given the ensamble of 3 yellow and 1 blue ball) they are surprised
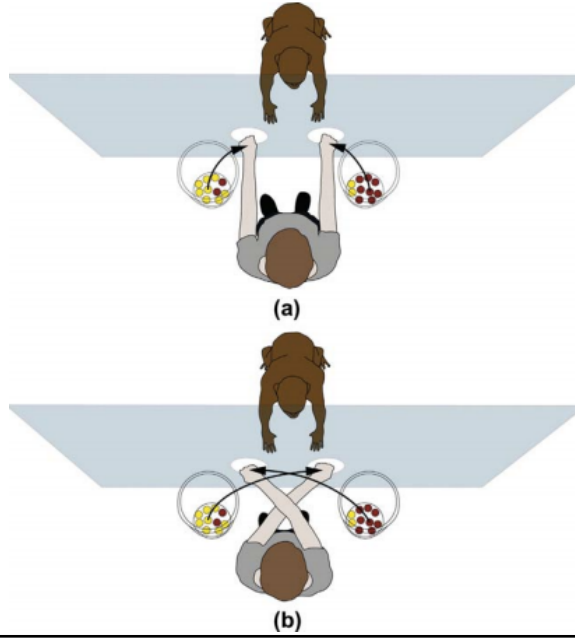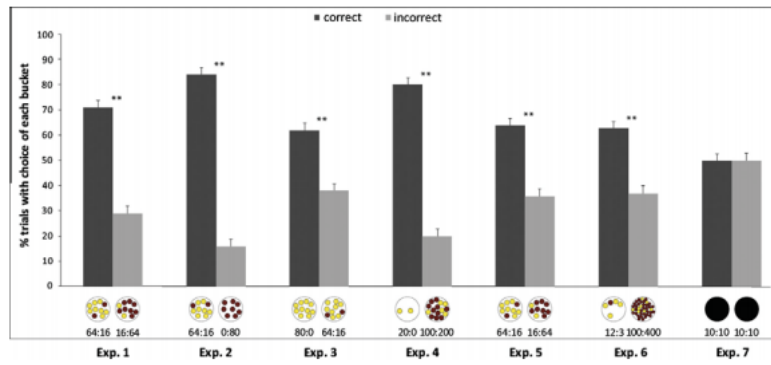
Numbers

Numbers

Numbers

- Mayan populations (Kaqchikel and K'iche)
  - With no formal instruction in mathematics
- In a series of test of probabilistic reasoning, adults responded as good as Mayan children (who go to school) and Italian adults

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Numbers
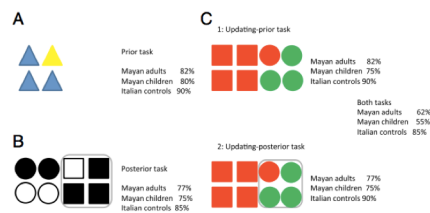
Fig. 2. Performance of Maya and Italian participants in tasks asking for a prior and/or posterior probability evaluation. In the prior task (A), participants had to bet on the color of a randomly drawn chip. In the posterior task (B), participants had to bet on the color of a randomly drawn chip whose shape was indicated by the experimenter. In the updating task (C), participants had to make both a prior and a posterior bet. The grey line indicates the possibilities compatible with the evidence, namely, the shape of the drawn chip: in B, the experimenter has drawn a square chip; in C, she has drawn a round chip. For each task, the figure reports the percentage of participants making the optimal bet.

Numbers

A

100%		0%

Mayan adults 100%
Mayan children 100%
Italian adults 100%
Italian children 100%

B

| 25% | 100% | | 75% | 100% |

Mayan adults 82%
Mayan children 90%
Italian adults 100%
Italian children 90%

Mayan adults 80%
Mayan children 80%
Italian adults 90%
Italian children 90%

C

| 75% | 25% | | 75% | 25% |

More red dots

Mayan adults 92%
Mayan children 90%
Italian adults 95%
Italian children 85%

Mayan adults 87%
Mayan children 85%
Italian adults 90%
Italian children 85%

Less red dots

Numbers

The preverbal ability to represent and manipulate quantities may constitute the foundation for the symbolic number system used for learning formal arithmetic

As young children develop language and a language-based symbolic number system (i.e., counting words and digits), it is believed there is a mapping of the counting words and visual symbols onto the innate number system

The foundational Approximate Number System (ANS) operates over multiple modalities, forming representations of the number of objects, sounds, or events in a scene
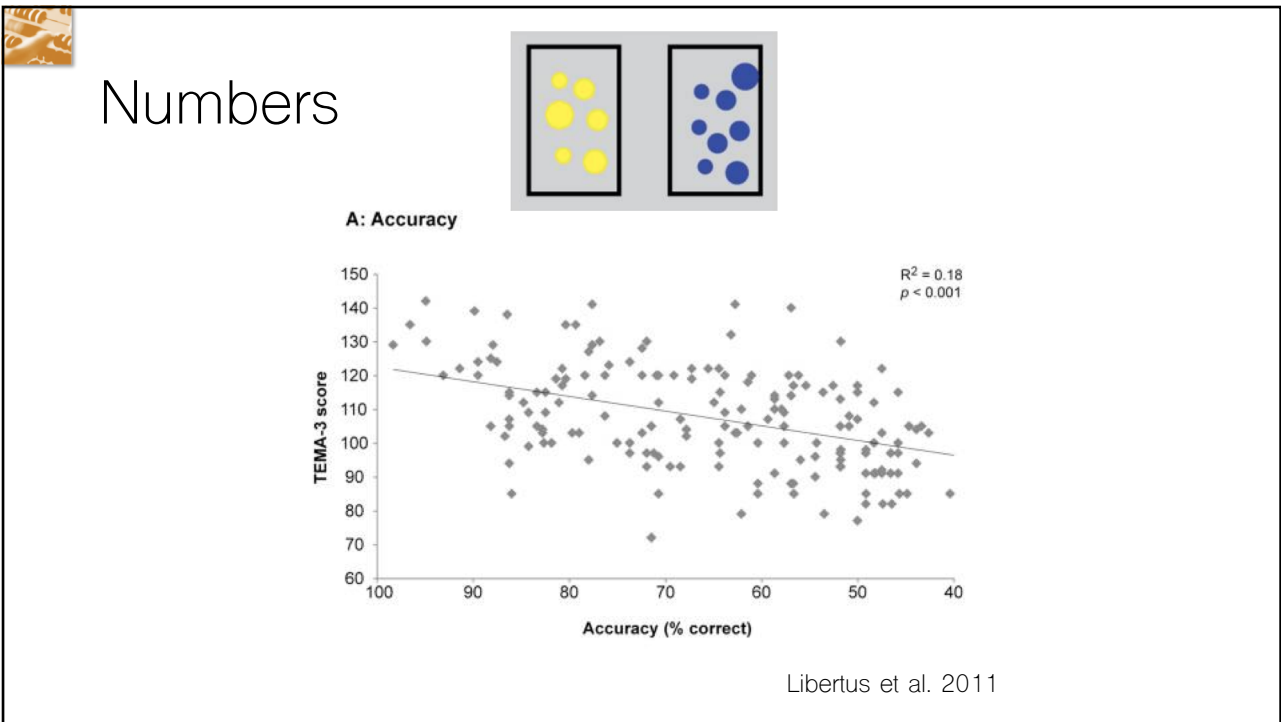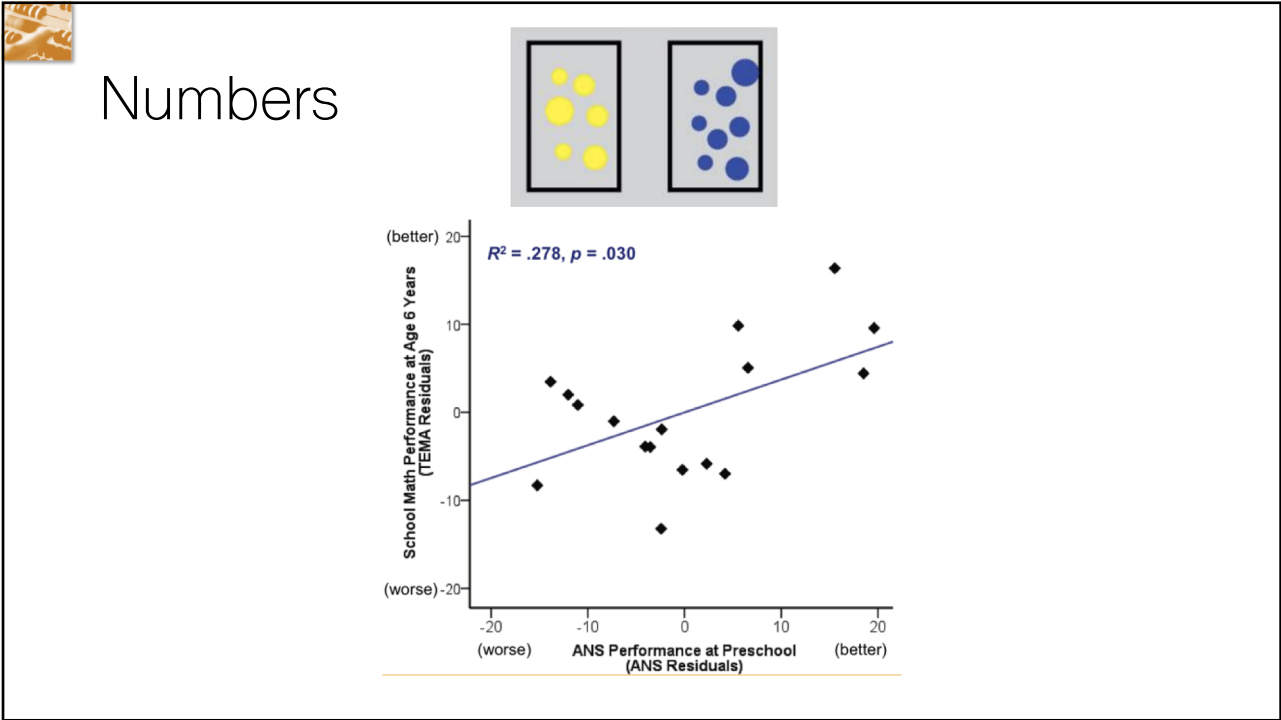
Numbers

- Individual differences in non-verbal number acuity correlate with maths achievement (Halberda et al., 2008)
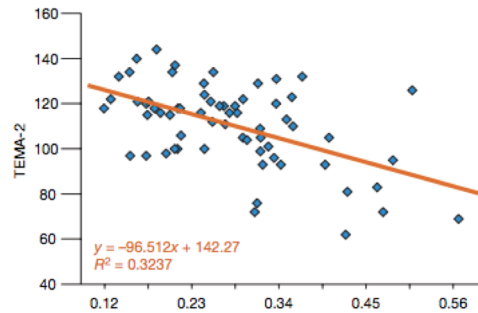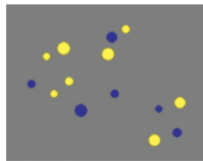
Numbers

- The resolution of the ANS is specified by a Weber fraction:
  - it measures the smallest numerical change to a stimulus that can be reliably detected
  - it is equal to the difference between the two numbers divided by the smaller number; for example, 7:8 → (8 – 7)/7 .14

- When asked to indicate the more numerous of two simultaneously presented arrays containing 20 – 80 dots, French adults' Weber fraction is .12 and Amazonian adults' Weber fraction is .17; thus on average these adults could discriminate ratios differing by about 7:8 (Pica et al., 2004)

Numbers
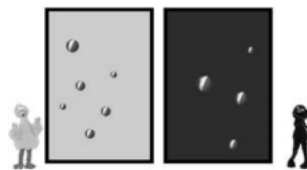

Numbers

Libertus et al. 2011

Numbers

- results obtained in a test that evaluates the number sense show a bi-directional relationship with formal arithmetic abilities:

  - children with higher acuity in the ANS tend to obtain higher scores in formal mathematic tests but not in other domains (logic, vocabulary, etc.) suggesting a specific relation

  - symmetrically, formal education in mathematics sharpens precision in the ANS

Numbers

- the resolution of the ANS, as specified by a Weber fraction, increases with age such that adults can discriminate numerosities that infants cannot.
- Studying the Weber fraction of the ANS in 3-, 4-, 5-, and 6-year-old children and in adults, it has been shown that the resolution of this system continues to increase throughout childhood, with adult-like levels of acuity attained surprisingly late in development.


"Who has more balls?"

Numbers
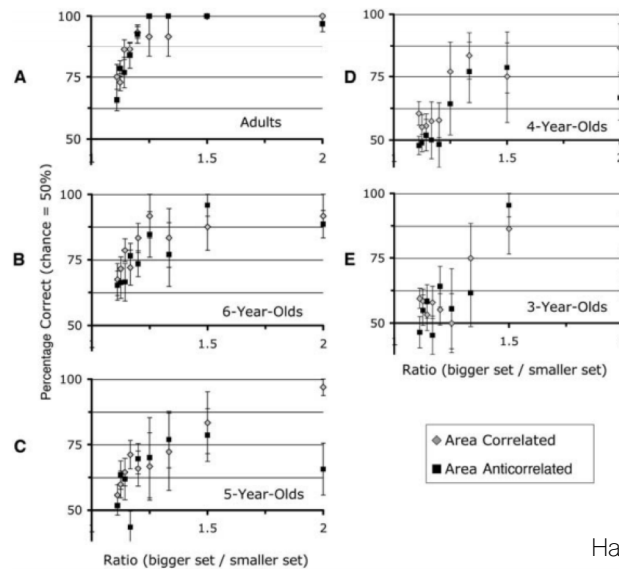
Halberda and Feigenson, 2008

Numbers

- Many children have significant mathematical learning disabilities (MLD, or dyscalculia) despite adequate schooling

- **Developmental Dyscalculia** (DD) is a learning disorder affecting the normal acquisition of arithmetic skills in an otherwise-normal child and that affects roughly 3-6% of the population

Numbers

Mazzocco et al.,2011
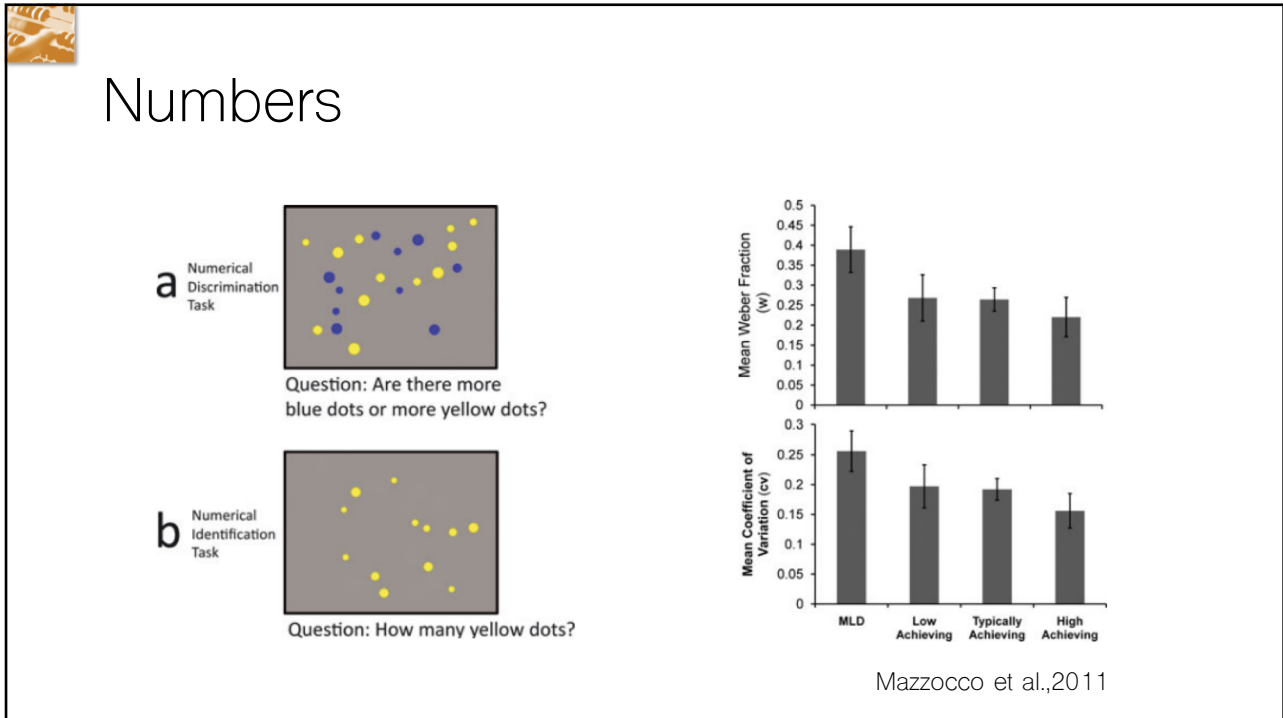
Numbers

- The number sense hypothesis proposes that the deficit in DD is located in the *approximate number system* (ANS) responsible for representing large and approximate numbers via a logarithmic analog mental number line (Feigenson et al., 2004; de Hevia et al., 2006; Dehaene, 2011)

- In contrast, the numerosity-coding hypothesis states that DD is caused by a deficit in the processing of smaller and exact sets of numbers (Butterworth, 2010)
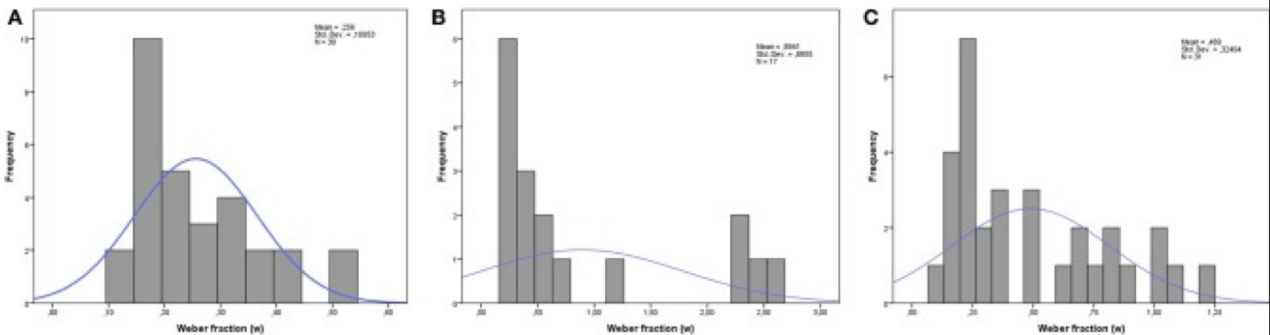
Numbers

- fourth graders with DD
- age-matched controls
- a group of ability-matched second graders
- were compared on a range of magnitude processing tasks pertaining to space, time, and number

- Children with DD displayed difficulties across all magnitude dimensions compared to age-matched peers and showed impaired ANS acuity compared to the younger, ability-matched control group, while exhibiting intact symbolic number processing
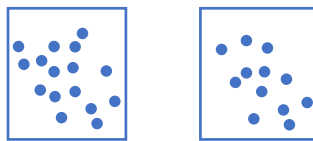
Numbers

Numbers

- conclusions
- (1) children with DD suffer from a general magnitude-processing deficit
- (2) a shared magnitude system likely exists
- (3) a symbolic number-processing deficit in DD tends to be preceded by an ANS deficit

Numbers

Choose the ensemble containing more dots

- 10-years old with dyscalculia perform as if they were 5-years old
- The degree of numerical acuity is predictive of symbolic manipulation of numbers
- The ANS seems to be the core set of fundamental numerical concepts

Numbers

In numerical cognition, there is an association between pre-verbal abilities and formal arithmetical capabilities

- The verbal system for exact calculation can be added to pre-verbal abilities
- The set of core numerical knowledge is at the basis of the formal conceptual mathematical thinking

Numbers

- There is an extreme continuity in the processes that subtend the numerical reasoning in primates (human and nonhuman)

- In our species, two forms of representation are used jointly and allow a precise representation that is not restricted to specific quantities

- Other animals may have a precise representation for small quantities or a raw representation for large quantities

Core knowledge

The criteria hypothesized by Spelke (2000) seem to be satisfied as for the system of knowledge that support our approximate calculation on large quantities and exact computations on small numbers:

– Given at birth

– Independent from experience and formal culture/acculturation

– Largely shared between species

– At the basis of learning processes