Exercise for the examination

Description of the exercise

- Let's consider an experimental setup composed by two layers of pixels detectors. Each detector has an area of 10x10 cm^2, and it is composed by a matrix of pixels detectors with a pitch of 200 µm
- The two detectors are separated by 20 cm and are used to detect cosmic rays. As a first approximation they are in vacuum.
- Cosmic rays are generated as straight lines (“tracks”), generated according to the angular distributions

$$\Phi(\vartheta) \propto \cos^2(\vartheta)$$

$$\Phi(\varphi) \propto \textbf{isotropic}$$

- The (x,y) coordinates are generated with a flat distribution
- When the tack hits the detector, a “Point” is reconstructed in each detector. The (x,y) coordinates of the point are discretized according to the pitch of the pixel. The information of “generated (x,y)” and the discretized position have to be stored in the object “Point”.
- For each event 1 track is generated.

Description of the exercise

- For each event, a “Track” object has to be defined: each track is defined using two “Point” objects and it is characterized by the two “reconstructed angles”, α and β. (Note that in general, α and β are different from ϑ and φ)
- Generate 10^6 events and reconstruct the distribution of α.
  - Compare the obtained distribution with the expected one ($\Phi(\vartheta) \propto \cos^2(\vartheta)$)
  - If you define a track as “two joined points” how does the “tracking efficiency” vary as a function of ϑ?
  - How does your results change if you have a pitch of 100 μm or 400 μm?
- Possible complications (not mandatory):
  - Introduce an “inefficiency” of your detector (e.g. 100% efficiency means that all your pixels works fine, 95% means that 5% of the total number of pixels are off)
  - Include multiple scattering assuming that each detector has a width of 300 μm
- The simulation can be done either in “pure” C++ or using ROOT (better to “visualize” the results of your simulation)

Multiple scattering

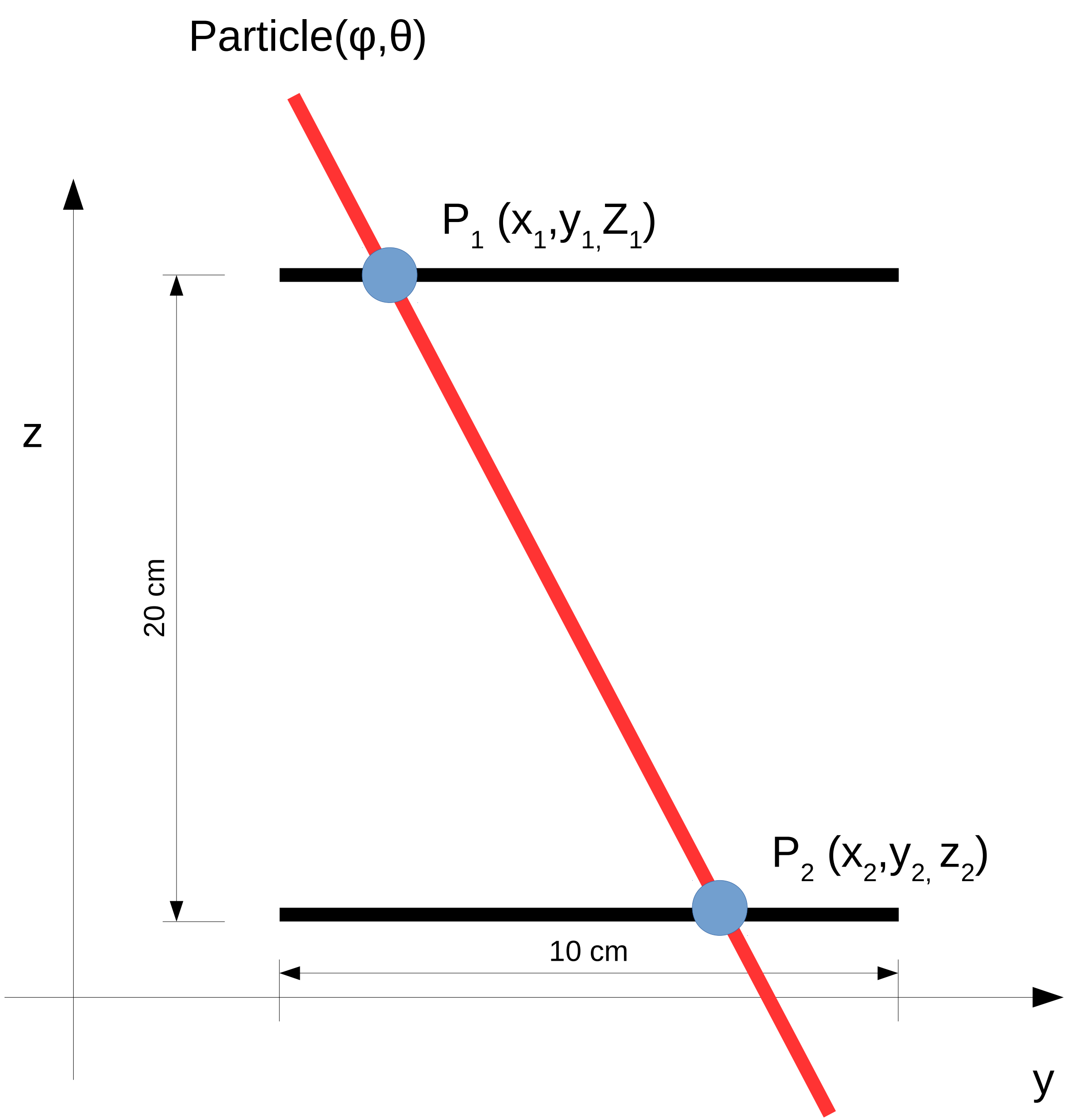
- The Coulomb scattering distribution is well represented by the theory of Moliere. It is roughly Gaussian for small deflection angles, but at larger angles it behaves like Rutherford scattering, having larger tails than a Gaussian distribution.
- For many applications it is possible to use a Gaussian approximation (which was found to describe the central 98% of the projected angular distribution [1]), with a width given by

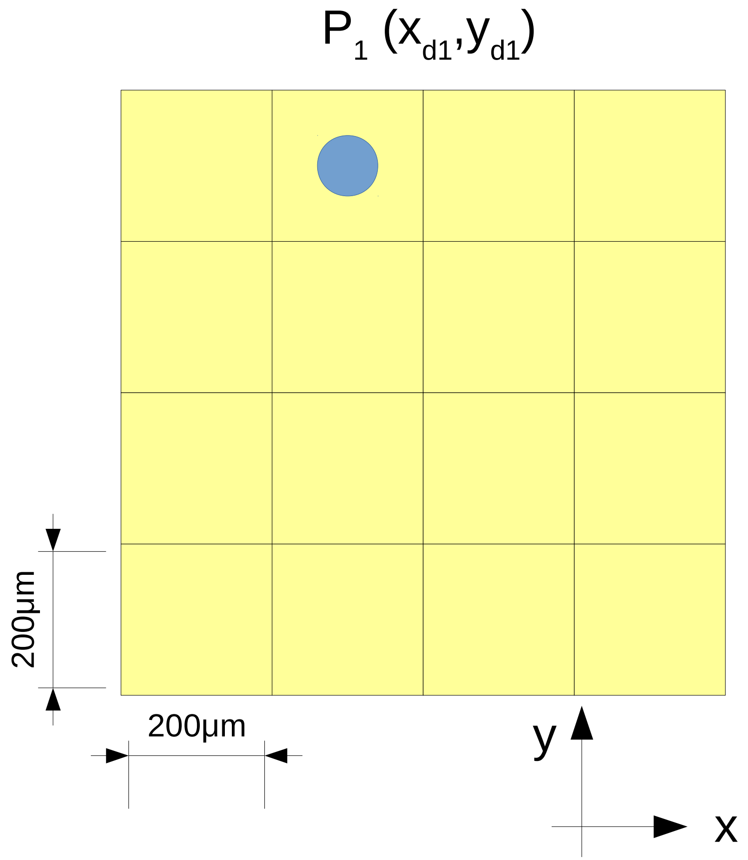
$$\theta_0 = \frac{13.6\,MeV}{\beta c p} z \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln \left(\frac{x}{X_0} \right) \right]$$

  - Here p, βc, and z are the momentum, velocity, and charge number of the incident particle, and x/X_0 is the thickness of the scattering medium in radiation lengths.
- In your case, assume a muon, with 1 GeV/*c* momentum crossing 300μm of silicon (X_0 = 21,82 g cm^{-2}, ρ = 2,329 g cm^{-3})

[1] http://pdg.lbl.gov/2005/reviews/passagerpp.pdf

Schematic view of the apparatus





Angles in 3D

$$\text{2D: } \theta = arctan(x, y)$$

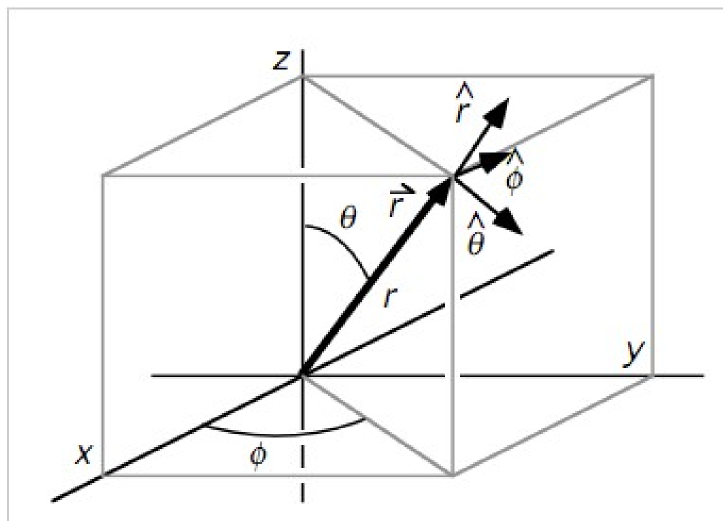
$$\text{3D: } r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = arctan(\sqrt{x^2 + y^2},\ z)$$

$$\phi = arctan(y, x)$$

Angle calculation in 2D and 3D

The spherical coordinate system is illustrated below.



Spherical coordinates