

# Solution to the Test of Mathematics

January 14th, 2020

1. How many numbers are there of 5 figures (from 10000 to 99999) beginning with 1 and sum of the figures equal to 4 (example: 10102)?

**Solution:**  $4 + 4 \cdot 3 + 4 = 20$ .

2. Find the domain of the function

$$f(x) = \log \left( \frac{x - \sqrt{x} - 2}{x - 5} \right).$$

**Solution:**  $[0, 4[\cup]5, +\infty[$ .

3. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} e^{-x} & \text{if } x \leq 0 \\ -\frac{1}{x} + 1 & \text{if } x > 0 \end{cases}.$$

Determine the inverse function  $x = f^{-1}(y)$ .

**Solution:**

$$x = f^{-1}(y) = \begin{cases} -\log y & \text{if } y \geq 1 \\ \frac{1}{1-y} & \text{if } y < 1 \end{cases}.$$

4. Determine the following limit:

$$\lim_{x \rightarrow 0} \frac{\arctan x^2}{1 - \sqrt{x^2 + 1}}.$$

**Solution:**

$$\lim_{x \rightarrow 0} \frac{\arctan x^2}{1 - \sqrt{x^2 + 1}} = -2.$$

5. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = e^{\frac{\sqrt{x}}{\sqrt{x-2}}}.$$

**Solution:** Domain:  $A = [0, +\infty[\setminus\{4\}$ .  $f(0) = 1$ ,  $\lim_{x \rightarrow +\infty} f(x) = e$ ,  $\lim_{x \rightarrow 4^-} f(x) = 0^+$ ,  $\lim_{x \rightarrow 4^+} f(x) = +\infty$ .

$$f'(x) = -e^{\frac{\sqrt{x}}{\sqrt{x-2}}} \frac{1}{\sqrt{x}(\sqrt{x-2})^2}.$$

6. Determine the following indefinite integral:

$$\int \frac{\arctan \sqrt{x}}{\sqrt{x}(1+x)} dx.$$

**Solution:**

$$\int \frac{\arctan \sqrt{x}}{\sqrt{x}(1+x)} dx = \arctan^2 \sqrt{x} + c.$$