Solutions to the Test of Mathematics

January 28th, 2020

Name:.....................................**Surname:**...............................

Matriculation number:........................

1. How many numbers are there of 3 figures all different from 0 with sum of the figures equal to 7 (example: 151)?

   Solution: $\binom{6}{2} = 15$

2. Find the domain of the function

$$f(x) = log\left(x - \sqrt{1 - x^2}\right).$$

   Solution: $A = \left]\frac{1}{\sqrt{2}}, 1\right]$

3. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} -\frac{1}{x} & \text{if } x \leq -1 \\ \frac{1}{x+1} + 1 & \text{if } x > -1 \end{cases}.$$

   Determine the inverse function $x = f^{-1}(y)$.

   Solution:

$$x = f^{-1}(y) = \begin{cases} -\frac{1}{y} & \text{if } 0 < y \leq 1 \\ \frac{1}{y-1} - 1 & \text{if } y > 1 \end{cases}.$$

4. Determine the following limit:

$$\lim_{x \to 1} \frac{sin^2\left(\sqrt{x} - 1\right)}{(1 - \sqrt{x})^2}.$$

   Solution:

$$\lim_{x \to 1} \frac{sin^2\left(\sqrt{x} - 1\right)}{(1 - \sqrt{x})^2} = 1.$$

5. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = \frac{e^{1+\sqrt{x}}}{x}.$$

   Solution: Domain: $A =]0, +\infty[$, $\lim_{x \to 0^+} f(x) = +\infty$, $\lim_{x \to +\infty} f(x) = +\infty$, $f(x) > 0 \ \forall x \in A$.

$$f'(x) = e^{1+\sqrt{x}} \frac{\sqrt{x} - 2}{2x^2},$$

   $f'(x) = 0 \Leftrightarrow x = 4$ (point of absolute minimum), $f(4) = \frac{e^3}{4}$.

6. Determine the following indefinite integral:

$$\int \frac{1}{cos^2 x \sqrt{tan\, x - 1}} \, dx.$$

   Solution:

$$\int \frac{1}{cos^2 x \sqrt{tan\, x - 1}} \, dx = 2\sqrt{tan\, x - 1} + c.$$