

INTEREST RATES

A.Y. 2019/2020

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AGENDA



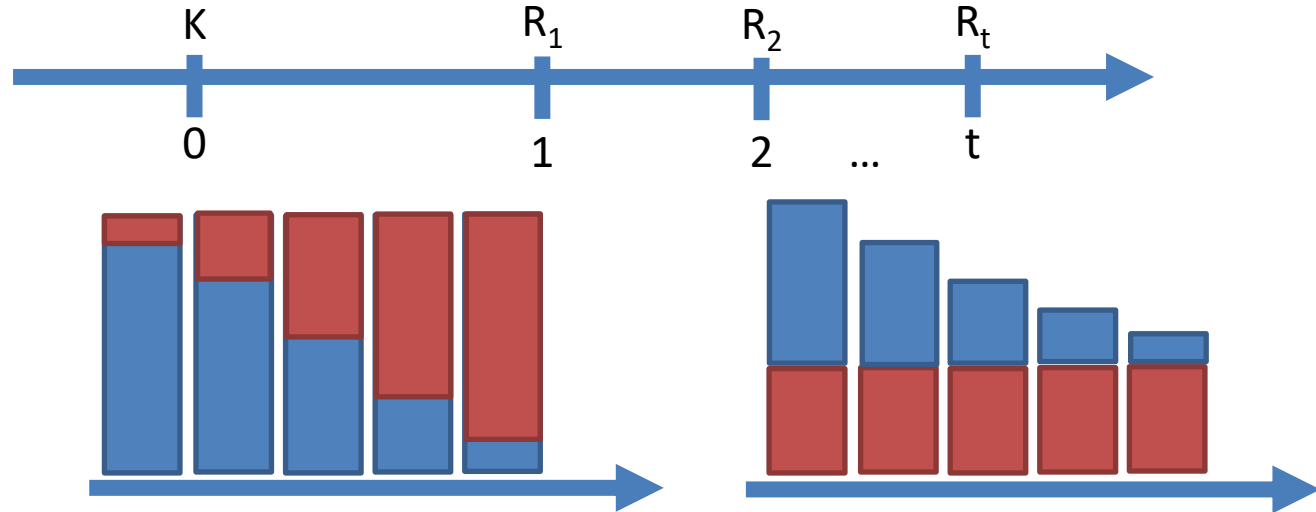
- Why do we need IR and how do we measure them?
- What are real IR and why are they important?
- How do we use IR to measure returns and risks?
- Can we predict interest rates?

MEASURES OF IR

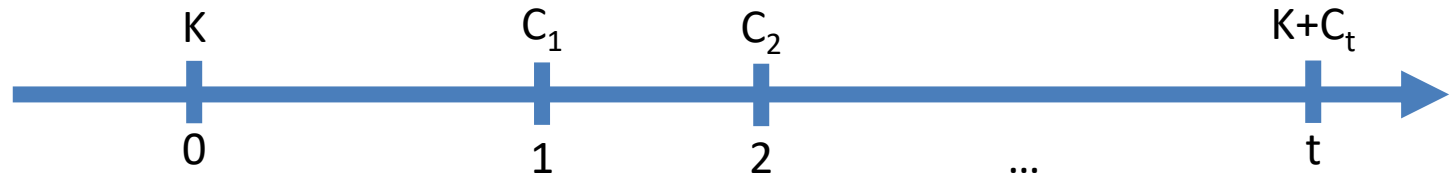
Balloon/simple loan:



Fully amortised loan:



Coupon bonds:



Discount bond:



MEASURES OF IR

How to compare different bond instruments quickly and easily?

Yield to maturity (or internal rate of return, or effective interest rate):

- the IR that balances the PV of future cash-flows with its current value
- For simple loans, YTM equals the nominal interest rate
- For ZC bonds:

$$i_{YTM} = \sqrt[n]{\frac{NV}{CV}} - 1$$

- For others (and in general) calculation is more complex (*goal-seek, Excel*):

$$CV_{FP} = \sum_{t=1}^n \frac{FP}{(1 + i_{YTM})^t}$$

$$CV_{CB} = \sum_{t=1}^n \frac{C}{(1 + i_{YTM})^t} + \frac{NV}{(1 + i_{YTM})^n}$$

- Note that the greater the YTM, the smaller the current value

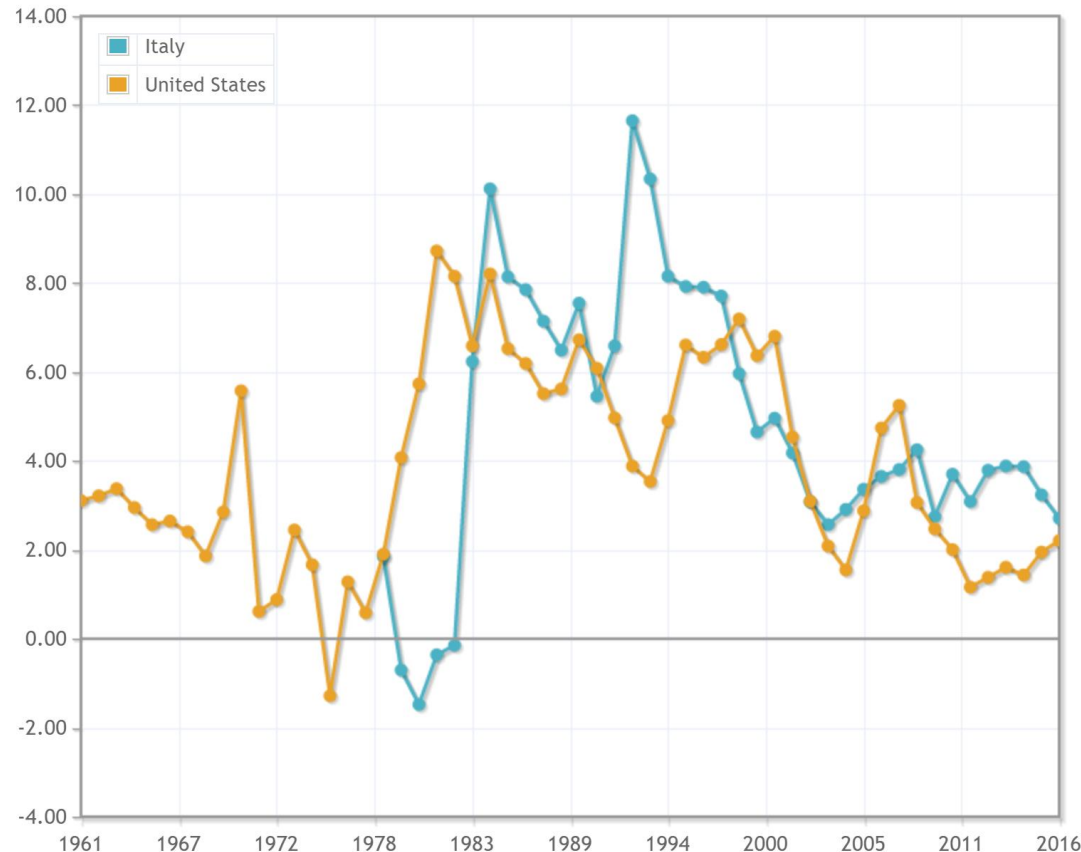
MEASURES OF IR

Is the YTM the perfect tool? (Spoiler: no)

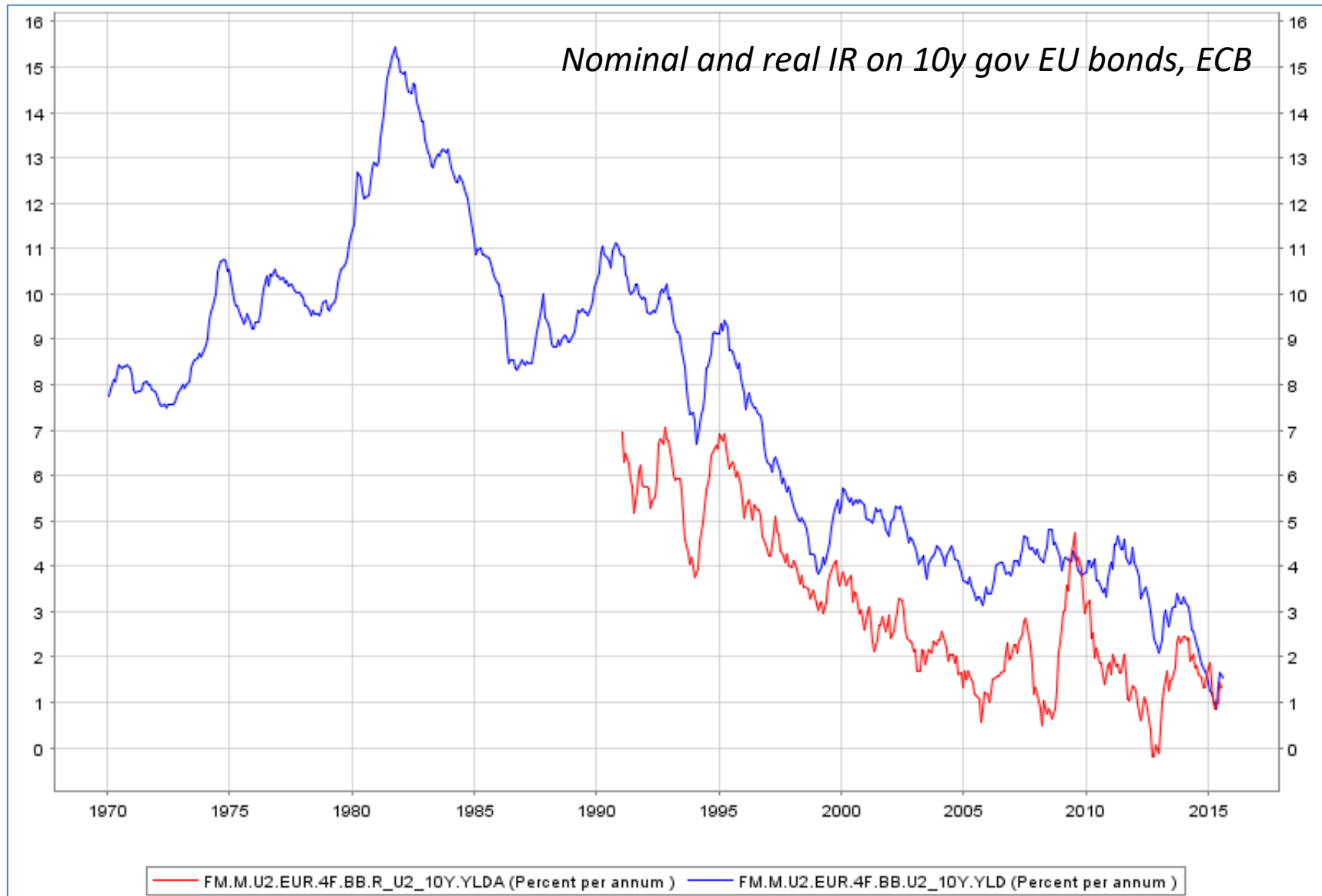
- You need to keep the instrument until maturity
 - Coupons are reinvested at the same IR
 - It is a nominal measure...
 - Ex-ante real IR are adjusted by expected inflation
- $$i = i_r + \pi^e + i_r \cdot \pi^e$$
- Ex-post real IR measures performance... when it's over
 - But also taxes should be considered
(net effective real IR)

Real interest rate (%)

Real interest rate is the lending interest rate adjusted for inflation as measured by the GDP deflator. The terms and conditions attached to lending rates differ by country, however, limiting their comparability. [More info >](#)



MEASURES OF IR



NEGATIVE IR?

- Pay to lend?
 - Central banks: ECB -0.2% on deposits in 9/2014 (but also DEN, SWE, CH)
 - Governments: DE from -0.4% to 0 for 1m-8y bonds (but also NED, SWE, DEN, CH, AUT), with FIN and DE issuing bonds with negative IR from inception on 2/2015
 - Corporations: Nestlé for its 4y € bonds in 2/2015...
- Should be good if you are a borrower?
 - Maybe, unless people keep money at home
 - Maybe, unless this shrinks profitability of commercial banks
 - Maybe, until this triggers a currency war
- Does it make any sense?
 - Real IR do... not always though
 - Storing money, building wealth reserves, accessing settlement services: all cost
 - A number of bonds give access to CB lending, increasing their demand
 - Taxation applies on nominal interest rates



IR AND RETURNS

- Rate of return: payments to the owner of a security plus the change in its value as a fraction of its purchase price
- IR and RoR are related but usually differ because of capital gains:

$$RoR = \frac{C + P_{t+1} - P_t}{P_t} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t} = i_c + g$$

- If holding period equals time to maturity, return equals yield to maturity only for ZCs: reinvestment risk
- The bigger the time to maturity, the bigger the effect on capital gains due to changes in IR: longer term bonds are more volatile (interest-rate risk)
- Increasing IR produces capital losses, decreasing IR produces gains
- Despite capital gains and losses are unrealised, they represent an opportunity cost
- If holding period is longer than time to maturity, this is another source for reinvestment risk (uncertainty over future IR)



IR AND RETURNS

So, how can we compare bonds with different maturities, coupons and prices?

A simple way is to use the duration (effective maturity)

- It's the weighted average lifetime of a debt instruments' cashflows
- For ZCs equal to the time to maturity
- Other instruments are seen as a portfolio of ZCs, weighted by their proportion over the portfolio (a useful additive property)

$$DUR = \frac{\sum_{t=1}^n \frac{CF_t}{(1+i)^t} \cdot t}{\sum_{t=1}^n \frac{CF_t}{(1+i)^t}}$$

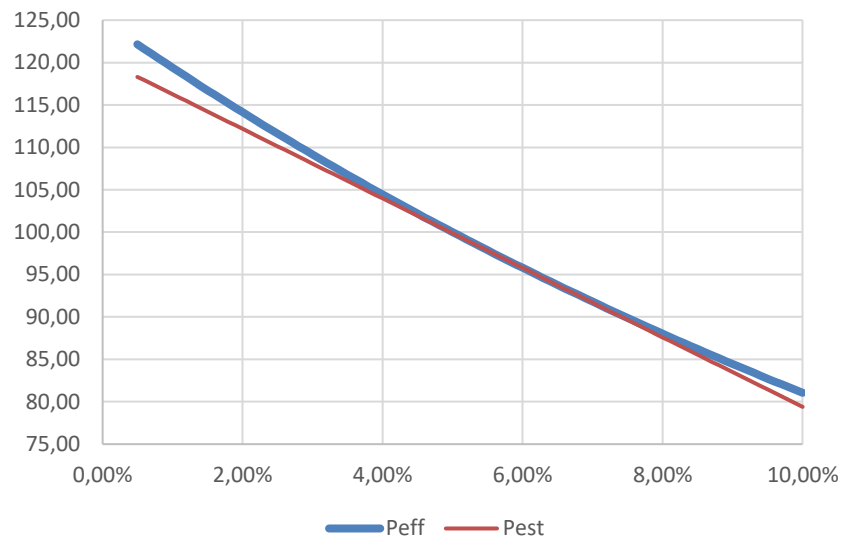
- Longer terms and smaller coupons mean bigger duration
- Increases in interest rates decrease duration
- For small changes in IR, duration is a good proxy of interest rate risk

$$\% \Delta P = \frac{(P_{t+1} - P_t)}{P_t} = -DUR \cdot \frac{\Delta i}{(1+i)}$$

IR AND RETURNS

But duration too is not perfect

- Linear proxy of a convex price/return relationship



Example: $M=5$, yearly coupon

$IR=6\%$: $P=95,79$ and $DM=-4,28$

If $\Delta i=1\%$, $P_{eff}=91,80$ e $P_{est}=91,69$

- Convexity

$$CON = \frac{1}{P \cdot (1+i)^2} \cdot \sum_{t=1}^N \left[\frac{CF_t}{(1+i)^t} \cdot (t^2 + t) \right]$$

DEMAND AND SUPPLY FRAMEWORK

Why do interest rates change?

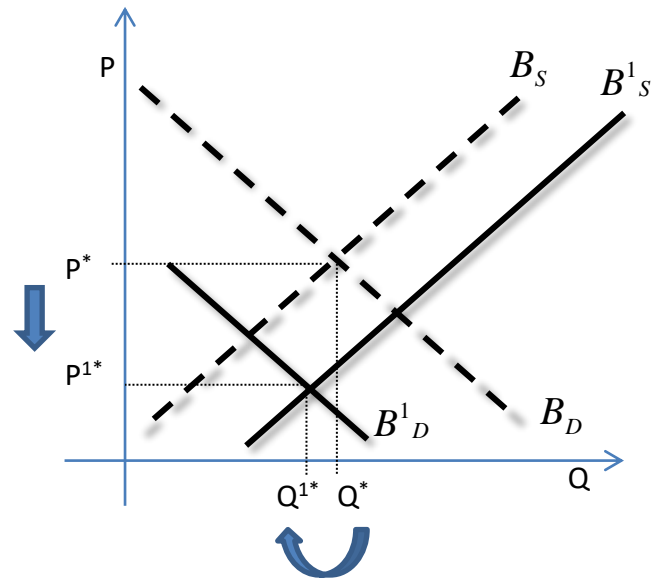
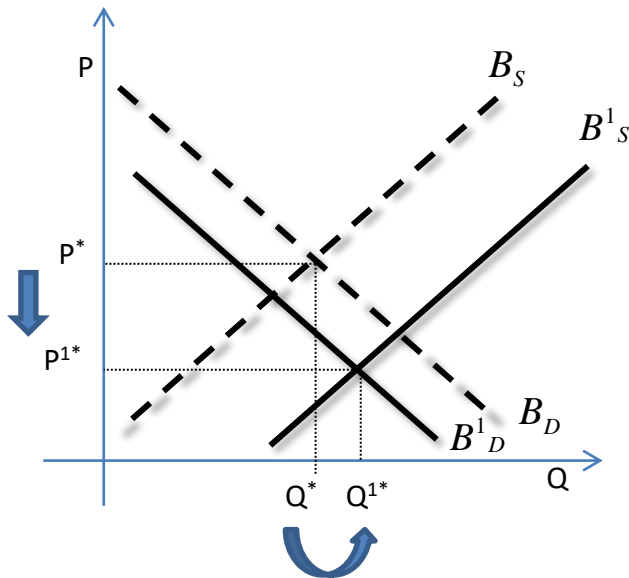


- Bonds' demand:
 - (+) Wealth owned by an individual
 - (+) Expected return relative to other assets
 - (–) Expected future interest rates
 - (–) Expected future inflation
 - (–) Risk (uncertain return) relative to other assets
 - (+) Liquidity relative to other assets
- Bonds' supply:
 - (+) Profitability of investments (more earnings)
 - (+) Expected inflation (heaper borrowing)
 - (+) Government deficits (more public debt)

DEMAND AND SUPPLY FRAMEWORK

Changes in IR due to inflation:

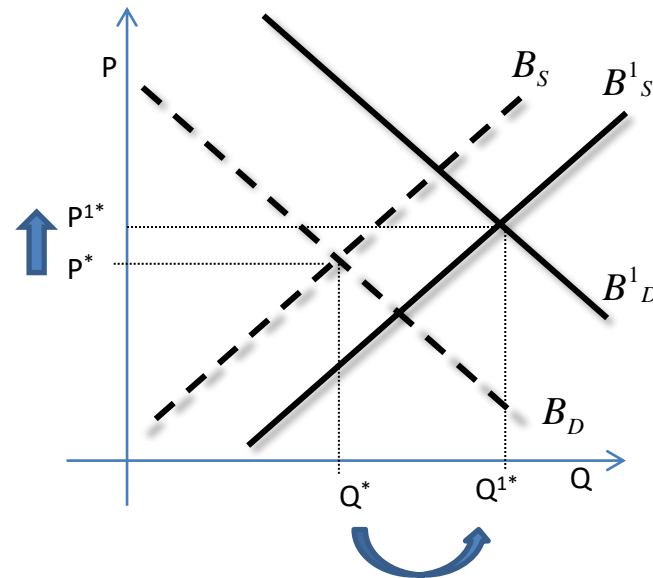
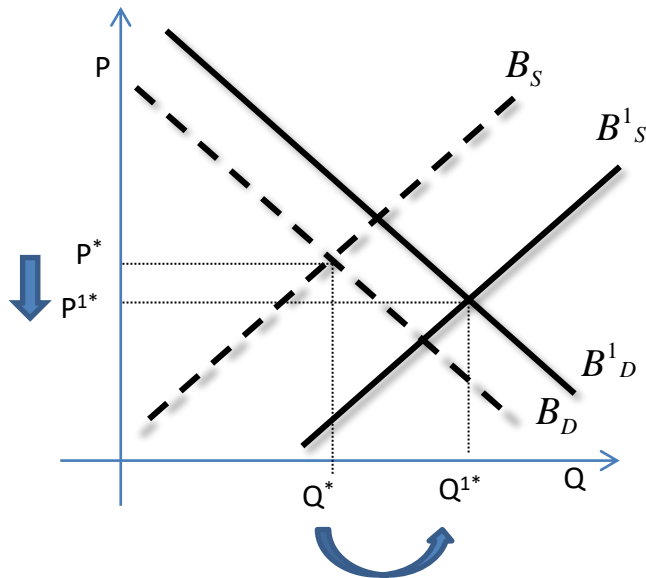
- An increase in expected inflation affects simultaneously demand (decrease of expected return) and supply (cheaper borrowing)
- IR will increase (prices fall)
- Effect on quantity is not readily predictable



DEMAND AND SUPPLY FRAMEWORK

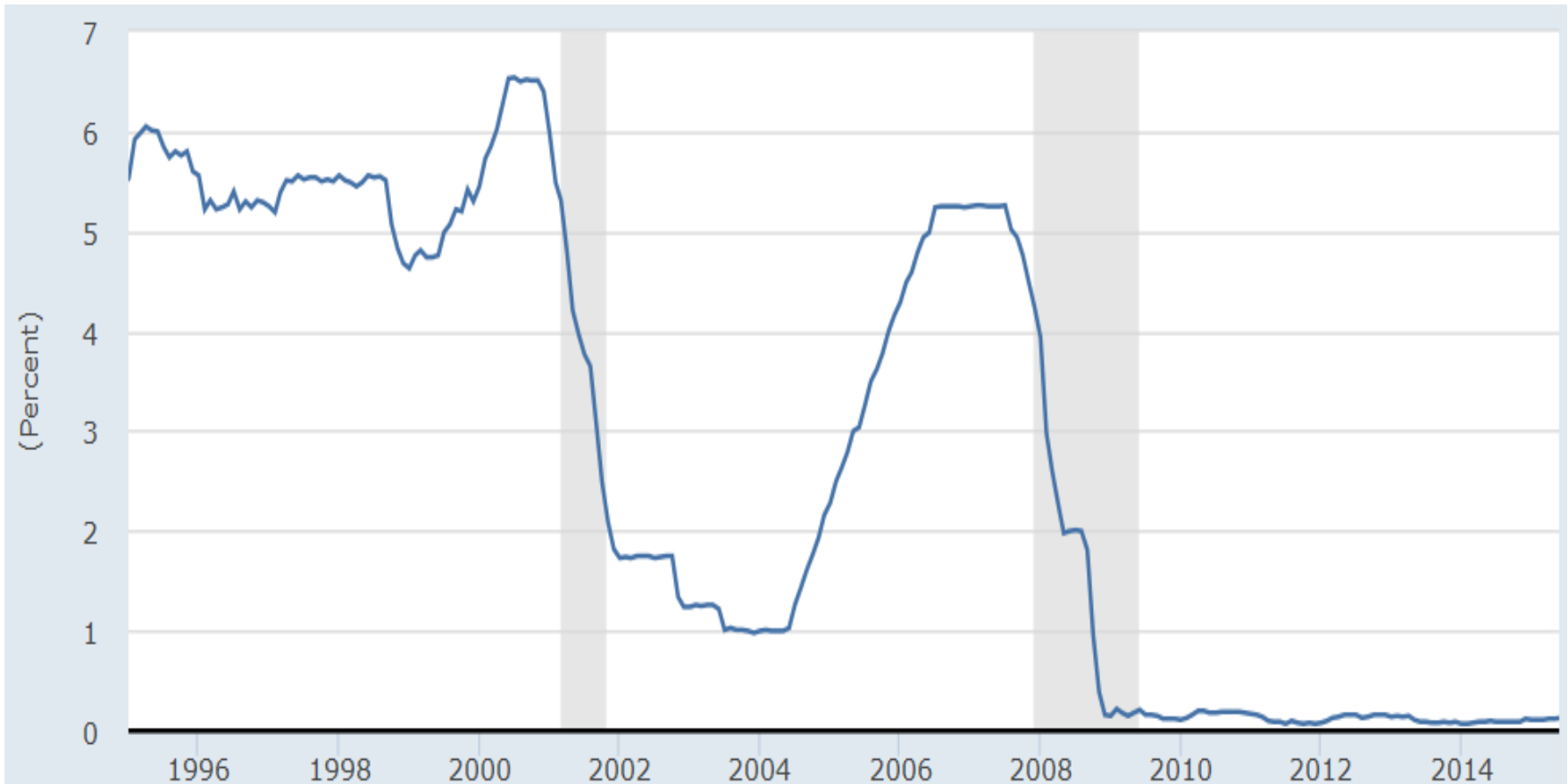
Changes in IR due to business cycles:

- An economic expansion affects simultaneously demand (increase of wealth) and supply (greater expected returns on investments)
- Quantity will increase
- IR can increase or decrease (usually, increase – and decrease during recessions)



DEMAND AND SUPPLY FRAMEWORK

US interbank rates and economic cycles, FRED



LIQUIDITY PREFERENCE FRAMEWORK

When CBs increase the money supply, IR should decline, but:

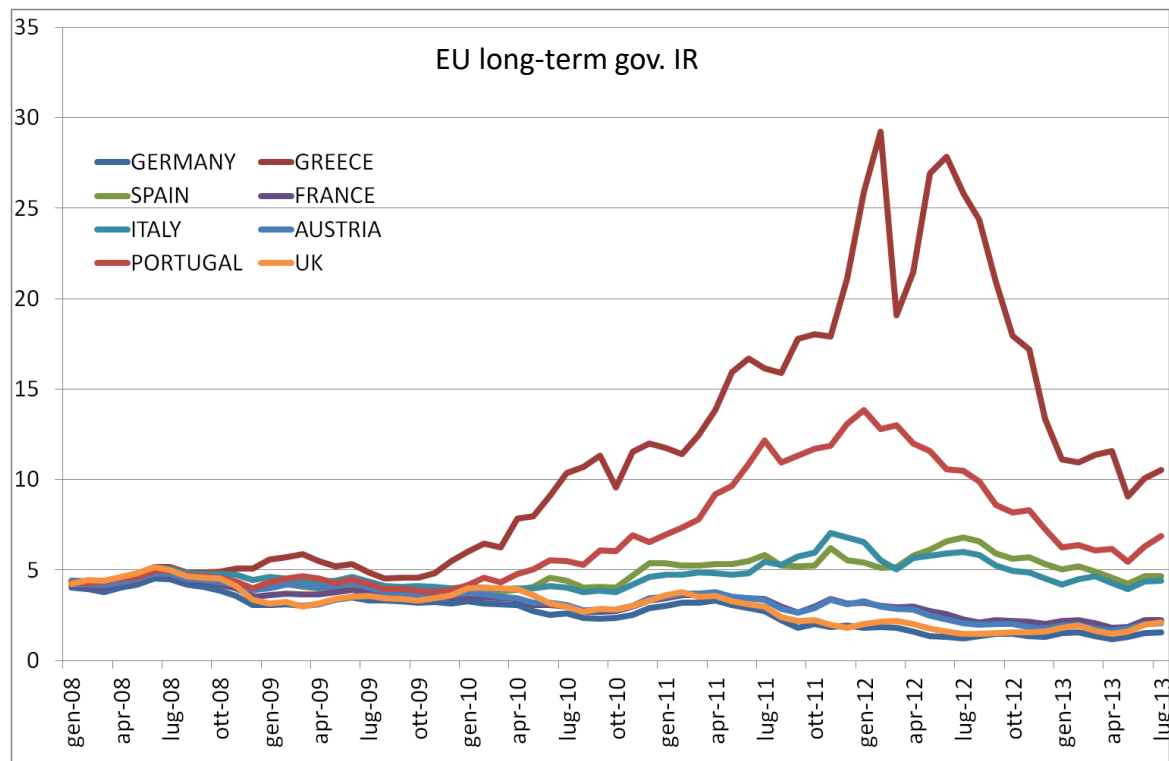
- Immediate **liquidity effect** reducing IR
- Economic stimulus: more income (**income effect**) and IR, but it takes time to have effects (wages, investments, ...)
- More inflation (**price-level effect**) and IR, but it takes time to adjust prices of goods and services
- More expected inflation (**expected-inflation effect**) and IR, with speed of effects depending on people's speed of adjusting expectations
- Result:
 - If the liquidity effect is dominant, sharp reduction in IR, then recovery up to a smaller final value
 - If the liquidity effect is insufficient, sharp reduction in IR, then recovery up to a higher final value
 - If the liquidity effect is marginal, people adapt their expectations on inflation and the reduction in IR does not take place, and final IR are higher immediately



RISK AND IR

IR differ also for bonds with equal duration because of default risk:

- government bonds were considered risk-free, yet only few of them now are
- the higher the risk the bigger the risk premium (spread)
- specialised firms (rating agencies) provide judgment over borrowers' default-risk (investment grade VS junk/high yield bonds)
- IR differ also for liquidity risk (adding to the risk premium)
- Finally, some bonds have tax incentives (municipal bonds, Italy's gov., ...)



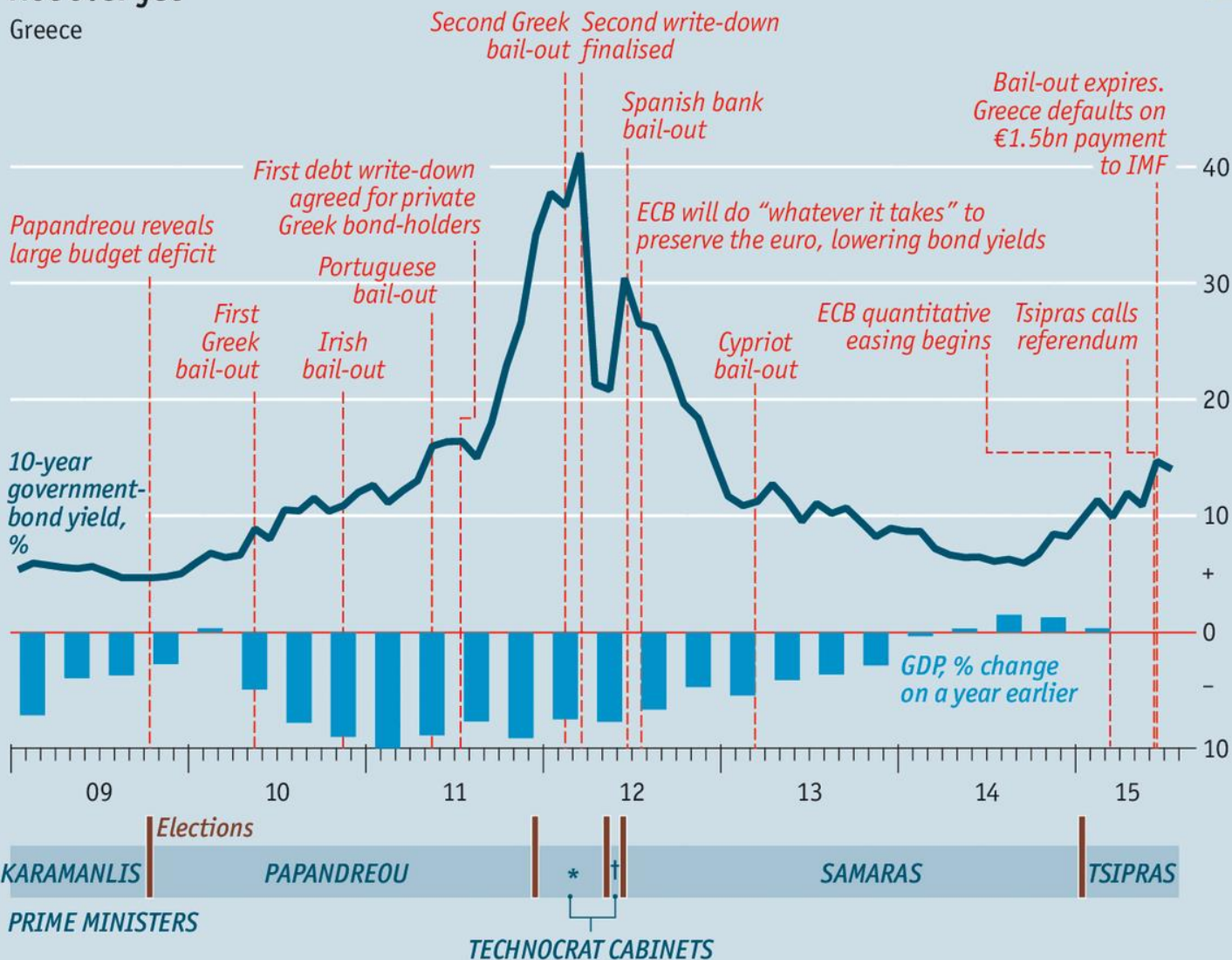
Source: ECB

RISK AND IR

Not over yet

Greece

1



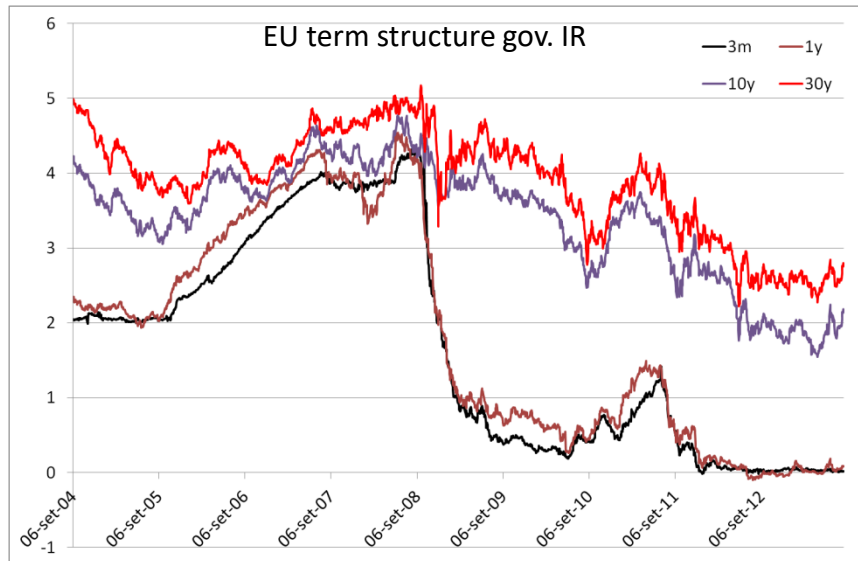
Sources: Thomson Reuters; Eurostat

led by *Lucas Papademos and †Panagiotis Pikrammenos

TERM STRUCTURE OF IR

IR differ also based on bonds' maturity:

- Differences in IR can be plotted at different maturities to derive the term structure of IR (yield curve)
- Usually yield curves are upward-sloping, meaning that longer maturities are charged with higher IR
- Flat or even downward-sloping or inverted yield curves are rare



Source: ECB

- Different maturities move similarly
- When short-term IR are high, inversion is more likely
- Inverted yield curves seem to anticipate recessions ('81, '91, 2000, '07), steep upward curves are associated with economic booms

TERM STRUCTURE OF IR

Three theories for explaining the term structure of IR:

Expectations theory

- If bonds at different maturities are perfect substitutes, their expected return must be equal
- $$(1 + i_{n,0})^n = (1 + i_{1,0})(1 + i_{1,1}^e) \cdot \dots \cdot (1 + i_{1,n-1}^e) \rightarrow i_{n,0} \approx \frac{i_{1,0} + i_{1,1}^e + \dots + i_{1,n-1}^e}{n}$$
- Predicts flat curves, whereas instead are usually upward-sloping (worked... until 1915)

Market segmentation theory

- Bonds at different maturities are not substitutes and each has a specific market, as well as each investor has a preferred maturity
- Together with interest-rate risk aversion, explains why longer investments require a risk premium
- Does not explain why IR move together along time
- Does not explain why with high short-term IR inversion is more likely

TERM STRUCTURE OF IR

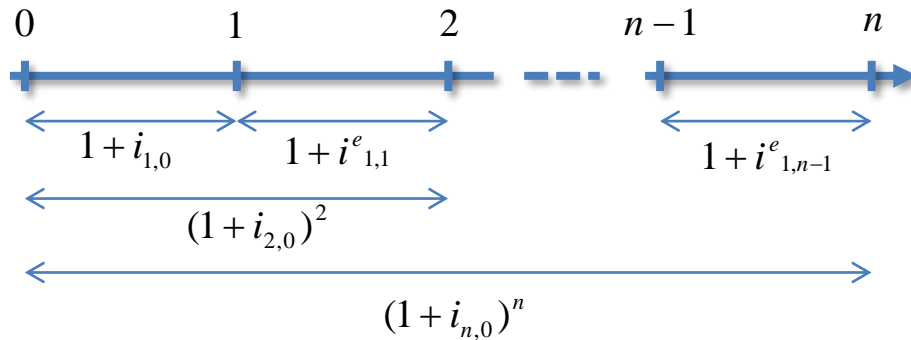
Liquidity premium theory

- Combines the other two in a comprehensive way
- Adds to expectations theory a liquidity premium for longer term bonds that is subject to market (demand, supply) conditions for that segment
- Bonds are substitutes as long as investors' preferences are compensated with a term (liquidity) premium that is always positive and grows as maturity gets longer
- $$i_{n,0} \approx \frac{i_{1,0} + i^e_{1,1} + \dots + i^e_{1,n-1}}{n} + l_{n,0}$$
- Explains inverted term structures: when future expectations on short-term IR are of a wide fall, so that their average is not balanced even by a positive liquidity premium (more likely when short-term rates are high)
- Support empirical evidence that:
 - Term structure is a predictor of business cycles and inflation
 - Term structure is less reliable for intermediate movements

TERM STRUCTURE OF IR

Forward and spot rates:

- Term structures allow to measure expected IR



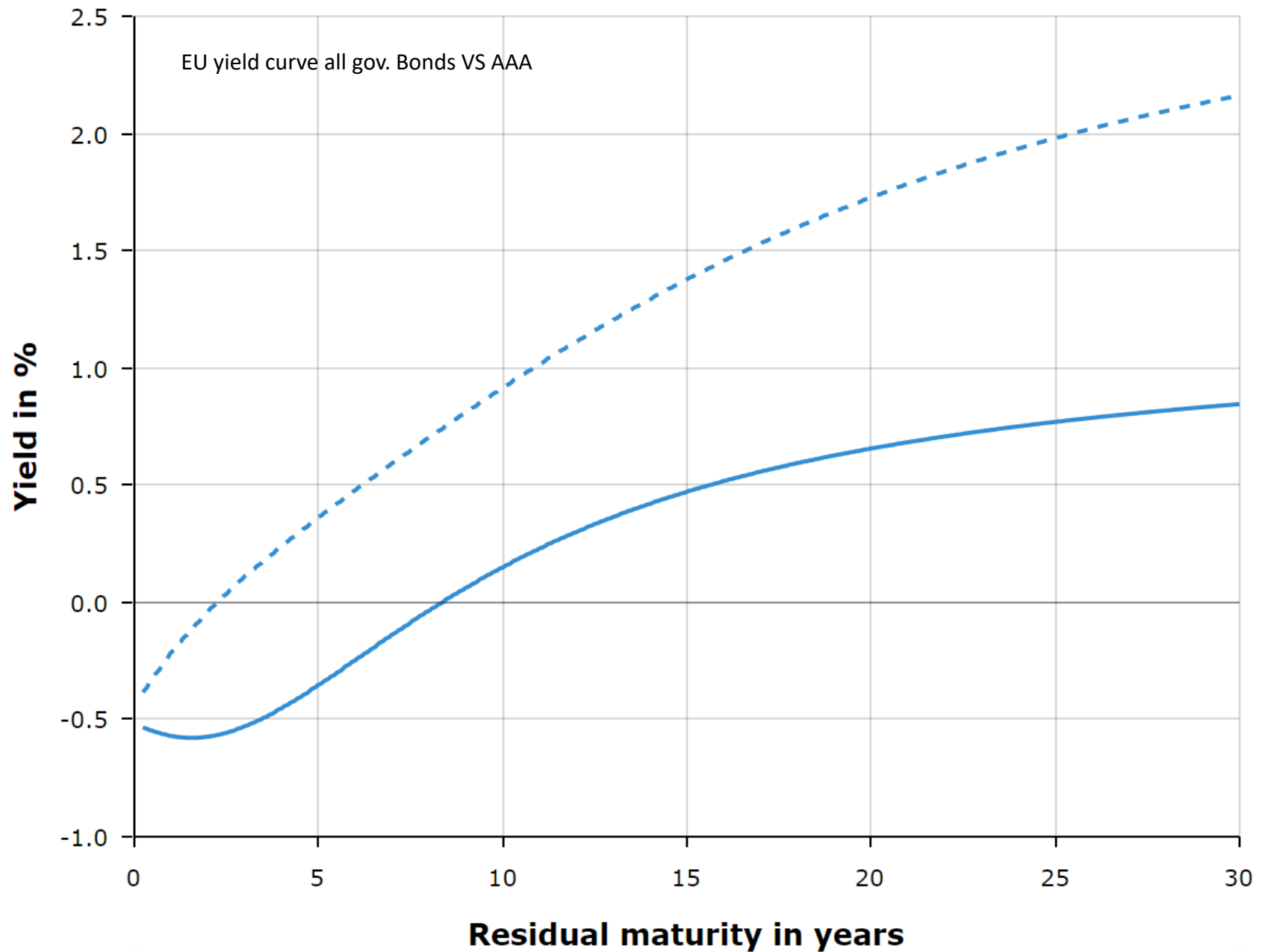
- Expected future IR are forward rates, in contrast to spot rates
- Knowing spot IR we can derive market expectations

F.i.: $i_{1,1}^e = \frac{(1 + i_{2,0})^2}{1 + i_{1,0}} - 1$ or, generalising: $i_{1,k}^e = \frac{(1 + i_{k+1,0})^{k+1}}{(1 + i_{k,0})^k} - 1$

- Including liquidity premiums:

$$i_{1,k}^e = \frac{(1 + i_{k+1,0} - l_{k+1,0})^{k+1}}{(1 + i_{k,0} - l_{k,0})^k} - 1$$

TERM STRUCTURE OF IR



EXAMPLES

1. A selling agent needs a car for his/her job. It is worth 20,000 € today and will allow to earn 15,000 € every year for three years. A three-year loan to buy the car is available at 50% annual interest rate, paid in fixed installments:

-Is it worth it?

-Is the charged IR 'ethical'?

a) Loan's installment: $R = 20,000 \cdot \alpha_{3-50\%} = 14,210.53$

b) Financial and economic plan:

	0	1	2	3
Inflow		15,000.00	15,000.00	15,000.00
Outflow		-14,210.53	-14,210.53	-14,210.53
Net flow		789.47	789.47	789.47
Loan	20,000.00	15,789.47	9,473.68	0.00
Earnings		15,000.00	15,000.00	15,000.00
Interests		-6,000.00	-4,736.84	-2,842.11
Profit/loss		9,000.00	10,263.16	12,157.89

EXAMPLES

2. What is the price effect on the following bonds of market IR increasing from 4% to 4.25%?

a) zero-coupon bond due in 3y for 2,000 with a YTM of 5%

b) bond due in 5y for 3,000 with an annual coupon of 3% and a YTM of 6%

c) a portfolio made of 40% of the bond sub-a) and 60% of the bond sub-b)

d) what if IR drop from 4% to 3% on all three alternatives?

$$\text{a) } DUR = 3 \quad \% \Delta P \approx -3 \cdot \frac{0.25\%}{1 + 4\%} = -0.72\%$$

$$\text{b) } DUR = \left(\sum_{t=1}^5 t \cdot \frac{90}{1.04^t} + 5 \cdot \frac{3,000}{1.04^5} \right) / \left(\sum_{t=1}^5 \frac{90}{1.04^t} + \frac{3,000}{1.04^5} \right) = 4.71 \quad \% \Delta P \approx -4.71 \cdot \frac{0.25\%}{1 + 4\%} = -1.13\%$$

$$\text{c) } DUR = 3 \cdot 40\% + 4.71 \cdot 60\% = 4.03 \quad \% \Delta P \approx -4.03 \cdot \frac{0.25\%}{1 + 4\%} = -0.97\%$$

$$\text{d) } \% \Delta P_1 \approx -3 \cdot \frac{-1\%}{1 + 4\%} = 2.88\% \quad \% \Delta P_2 \approx -4.71 \cdot \frac{-1\%}{1 + 4\%} = 4.53\% \quad \% \Delta P_3 \approx -4.03 \cdot \frac{-1\%}{1 + 4\%} = 3.87\%$$

EXAMPLES

3. The following graph compares T-BILLS, expected inflation and CPI (net of food and energy).
Comments?

