Solutions to the Test of Mathematics

February 11th, 2020

Name:.....................................**Surname:**................................

Matriculation number:........................

1. How many ways are there of colouring 5 boxes with 3 colours, black, white and green, in such a way that at least two colors are used?

   Solution: $3^5 - 3 = 243 - 3 = 240$.

2. Determine the domain of the following real-valued function:

$$y = f(x) = log(arcsin(x^2 + x)).$$

   Solution: $0 < x^2 + x \leq 1 \Leftrightarrow \left(\frac{-1-\sqrt{5}}{2} \leq x < -1\right)\ or\ \left(0 < x \leq \frac{-1+\sqrt{5}}{2}\right)$

3. Consider the real-valued function defined as follows:

$$y = f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 0 \\ -x^3 - 2 & \text{if } x > 0 \end{cases}.$$

   Determine the inverse function $x = f^{-1}(y)$.

   Solution:

$$x = f^{-1}(y) = \begin{cases} -\sqrt{y+1} & \text{if } y \geq -1 \\ \sqrt[3]{-2-y} & \text{if } y < -2 \end{cases}.$$

4. Determine the following limit:

$$\lim_{x \to +\infty} x^{\frac{1}{x}}.$$

   Solution:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\frac{log\, x}{x}} = e^0 = 1.$$

5. Study the following function and draw its graph (just consider the first derivative):

$$f(x) = arctan\left(\frac{x}{1+x^2}\right).$$

   Solution; Odd function. Domain $A = \mathbb{R}$. $\lim_{x \to \infty} f(x) = 0$.

$$f'(x) = \frac{1}{1 + \left(\frac{x}{1+x^2}\right)^2} \frac{1-x^2}{1+x^2}.$$

   $f'(x) = 0 \Leftrightarrow x = \pm 1$. $f'(x) > 0 \Leftrightarrow -1 < x < 1$. -1 point of absolute minimum, 1 point of absolute maximum.

6. Determine the following indefinite integral:

$$\int \frac{x}{\sqrt{2x^2+3}}\, dx.$$

   Solution:

$$\int \frac{x}{\sqrt{2x^2+3}}\, dx = \frac{1}{2}\sqrt{2x^2+3}.$$

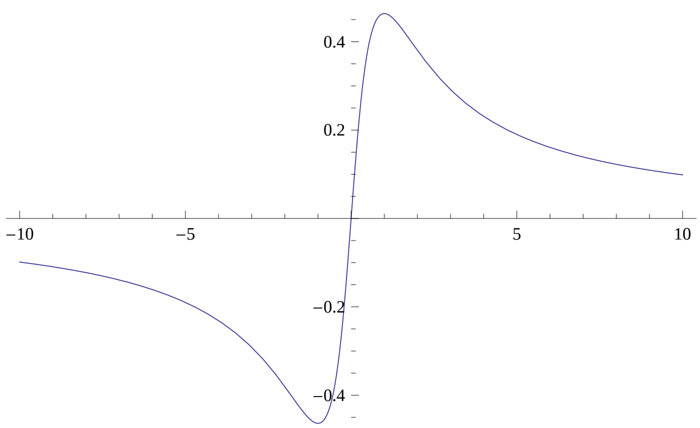

Figure 1: Graph of $f(x) = f(x) = arctan\left(\frac{x}{1+x^2}\right)$.