Esercizi sull'integrale e sulla somma di convoluzione

1. Si determini la risposta del sistema di figura 1 quando $x(t)$ e $h(t)$ sono:

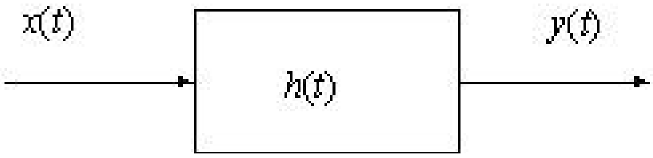

Fig. 1

a) $x(t)=e^{-at}u(t)$, $h(t)=e^{-bt}u(t)$ (considerare sia $a \neq b$ sia $a = b$)

b) $x(t)=u(t)-2u(t-2)+u(t-5)$, $h(t)=e^{2t}u(1-t)$

c) $x(t)=e^{-3t}u(t)$, $h(t)=u(t-1)$

d) $x(t)=e^{-2t}u(t+2)+e^{3t}u(-t+2)$, $h(t)=e^{t}u(t-1)$

e) $x(t)=\begin{cases} e^{t} & t<0 \\ e^{5t}-2e^{-t} & t>0 \end{cases}$, $h(t)$ come in fig. 2

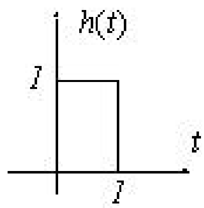

Fig. 2

f) $x(t)$ e $h(t)$ come in figura 3

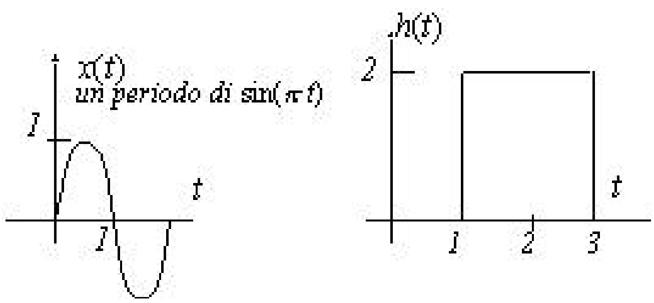

Fig. 3

g) $x(t)$ come in figura 4, $h(t)=u(-2-t)$

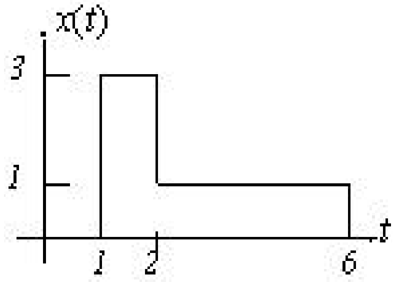

Fig.4

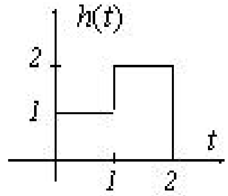

Fig. 5

h) $x(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$
$h(t)$ come in figura 5

i) $x(t)$ e $h(t)$ come in figura 6
j) $x(t)$ e $h(t)$ come in figura 7
k) $x(t)$ e $h(t)$ come in figura 8
l) $x(t)$ e $h(t)$ come in figura 9

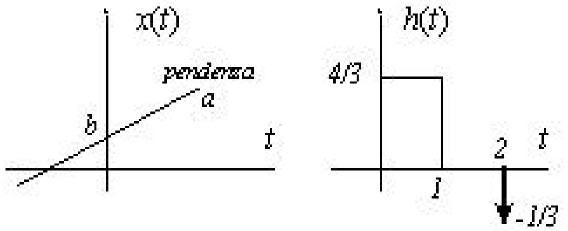

Fig. 6

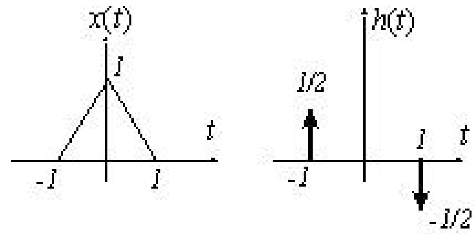

Fig. 7

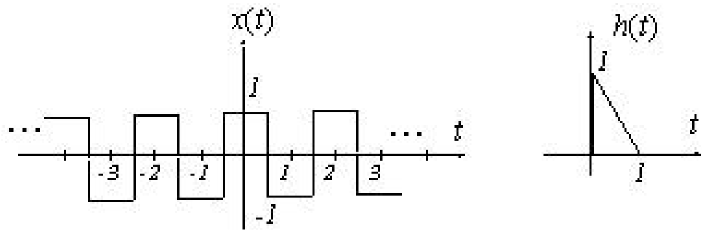

Fig. 8

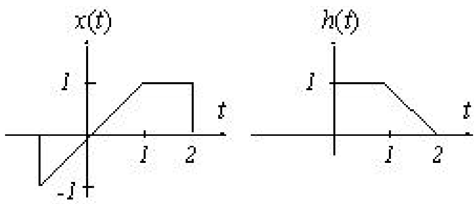

Fig. 9

2. Si determini la risposta del sistema di figura 10 quando $x[n]$ e $h[n]$ sono:

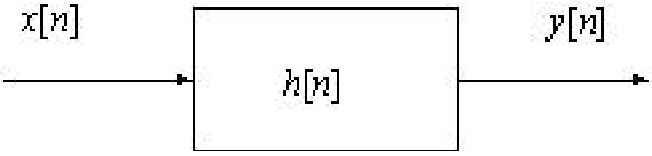

Fig. 10

a) $x[n] = a^n u[n]$ $\quad (a \neq b)$
$h[n] = b^n u[n]$

b) $x[n] = h[n] = a^n u[n]$

c) $x[n] = 2^n u[-n]$
$h[n] = u[n]$

d) $x[n] = (-1)^n \{u[-n] - u[-n-8]\}$
$h[n] = u[n] - u[n-8]$

e) $x[n]$ e $h[n]$ come in figura 11.

f) $x[n]$ e $h[n]$ come in figura 12.

g) $x[n]$ e $h[n]$ come in figura 13

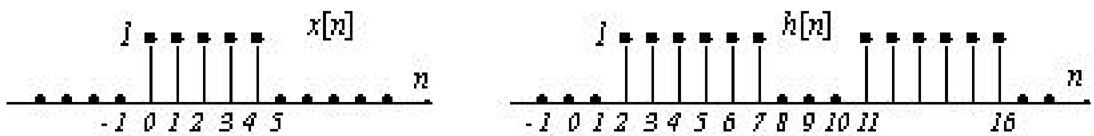

Fig. 11

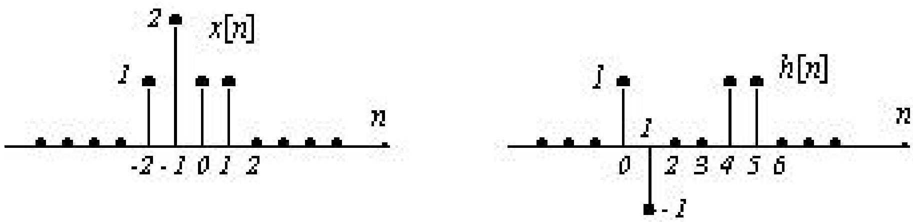

Fig. 12

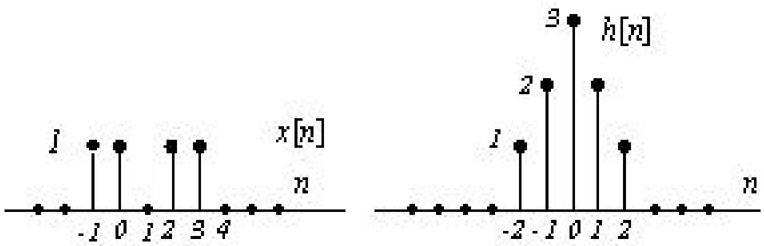

Fig. 13

h) $x[n] = 1$ per tutti i valori di n

$h[n] = \left(\frac{1}{2}\right)^n u[n] + 4^n u[-n-1]$

i) $x[n] = u[n] - u[-n]$

$h[n] = \left(\frac{1}{2}\right)^n u[n] + 4^n u[-n-1]$

j) $x[n] = \left(-\frac{1}{2}\right)^n u[n-4]$

$h[n] = 4^n u[2-n]$

3. In un sistema LTI tempo continuo il segnale di ingresso $x(t)$ e quello di uscita $y(t)$ sono legati dalla relazione:

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2) d\tau$$

Determinare la risposta impulsiva del sistema e la risposta al segnale di ingresso di figura 14

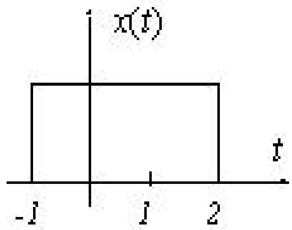

Fig. 14

4. Un sistema lineare, ma non tempo invariante, è caratterizzato dalla risposta $h[k,n]$ all'impulso unitario centrato in n=k , vale a dire a $\delta[n-k]$. Per ognuna delle seguenti espressioni di $h[k,n]$ individuare una relazione esplicita tra l'uscita $y[n]$ e l'ingresso $x[n]$ del relativo sistema.

a) $h[k,n] = \begin{cases} \delta[n-k] & \text{per } k \text{ pari} \\ 0 & \text{per } k \text{ dispari} \end{cases}$

b) $h[k,n] = \delta[2n-k]$

c) $h[k,n] = ku[n-k]$

d) $h[k,n] = k\delta[n-2k] + 3k\delta[n-k]$

e) $h[k,n] = \begin{cases} \delta[n-k+1] & \text{per } k \text{ pari} \\ 5u[n-k] & \text{per } k \text{ dispari} \end{cases}$