27/6
16/7

6/9
20/9

We know the mean we do not know variance

$$y_1, \dots, y_n \mid \sigma^2 \sim \text{IID } N(\underbrace{\theta}_{\text{known}}, \sigma^2)$$

Likelihood

$$p(y \mid \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(y_i - \theta)^2\right\}$$

$$\propto (\sigma^2)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \theta)^2\right\}$$

$$\pi(\sigma^2) \propto (\sigma^2)^{-(\gamma+1)} \exp\left\{-\frac{\delta}{\sigma^2}\right\} \quad z \in \mathbb{R}^+$$

$$\sigma^2 \sim \text{inv Gamma}(\gamma, \delta)$$

$$\underline{f(z) \propto z^{-\gamma} e^{-\delta/z}} \longleftarrow f(y) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} y^{\alpha-1} e^{-\beta y}$$

$$z = \frac{1}{y}$$

$$; f(z) \propto \left(\frac{1}{z}\right)^{\alpha-1} e^{-\beta/z} \left|-\frac{1}{z^2}\right|$$

$$V(z) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)} \quad \alpha > 2 \qquad f(z) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} z^{-(\alpha+1)} e^{-\beta/z}$$

inverse Gamma distr with param. α, β

$$\mathrm{Mode}(z) = \frac{\beta}{\alpha+1} \; ; \; \underline{E(z) = \frac{\beta}{\alpha-1}} \quad \alpha > 1$$

$$\pi(\sigma^2 \mid y) \propto (\sigma^2)^{-\gamma-1-\frac{n}{2}} \exp\left\{-\frac{1}{\sigma^2}\left(\frac{1}{2}\sum(y_i-\theta)^2+\delta\right)\right\}$$

$$\sim \text{inverse Gamma}\left(\gamma+\frac{n}{2},\ \delta+\frac{1}{2}\sum(y_i-\theta)^2\right)$$

$$E(\sigma^2 \mid y) = \frac{2\delta + \sum(y_i-\theta)^2}{2\gamma + n - 2}$$

if δ, γ low $\quad E(\sigma^2 \mid y) \approx \dfrac{\sum(y_i-\theta)^2}{n-2}$

$$\boxed{\sigma^2 =_d \frac{\sigma_0^2 \nu_0}{X} \qquad X \sim \chi^2_{\nu_0}} \equiv \text{Gamma}\left(\frac{\nu_0}{2}, \frac{1}{2}\right)$$

$$\sigma^2 \sim \text{inv-}\chi^2(\nu_0, \sigma_0^2)$$

$$\frac{\sigma^2}{\sigma_0^2 \nu_0} \sim \text{invGamma}\left(\frac{\nu_0}{2}, \frac{1}{2}\right)$$

$$\sigma^2 \sim \text{invGamma}\left(\frac{\nu_0}{2}, \frac{\nu_0 \sigma_0^2}{2}\right)$$

$$\pi(\sigma^2) \propto (\sigma^2)^{-\nu_0/2 - 1} \exp\left\{-\frac{\nu_0 \sigma_0^2}{2\sigma^2}\right\}$$

$$\pi(\sigma^2 | y) \propto (\sigma^2)^{-\left(\frac{\nu_0}{2} + \frac{n}{2}\right) - 1} \exp\left\{-\frac{1}{2\sigma^2}\left(\nu_0 \sigma_0^2 + \sum (y_i - \theta)^2\right)\right\}$$

$$\rightarrow \sigma^2 | y \sim \text{inv-}\chi^2\left(\nu_0 + n, \frac{\nu_0 \sigma_0^2 + \sum (y_i - \theta)^2}{\nu_0 + n}\right)$$

$$\sigma^2 \mid y \sim \text{inv}\,\chi^2\left(\nu_0 + n,\ \frac{\nu_0\sigma_0^2 + \sum (y_i - \theta)^2}{\nu_0 + n}\right)$$

$$E(\sigma^2 \mid y) = \frac{\nu_0\sigma_0^2 + \sum (y_i - \theta)^2}{\nu_0 + n - 2}$$

$$= \frac{\nu_0\sigma_0^2 + n\hat{\sigma}^2}{\nu_0 + n - 2}$$

In General $\sigma^2 \sim \text{inv-}\chi^2(\nu_0, \sigma_0^2)$

$\text{inv. Gamma}\left(\frac{\nu_0}{2}, \frac{\nu_0\sigma_0^2}{2}\right)$

$$\underset{\nu_0 > 2}{E(\sigma^2)} = \frac{\frac{\nu_0\sigma_0^2}{2}}{\frac{\nu_0}{2} - 1} = \frac{\nu_0\sigma_0^2}{\nu_0 - 2}$$