

First-principles («theoretical») dynamic models of chemical processes

DINAMICA E CONTROLLO DEI PROCESSI CHIMICI
A.A. 2019-2020
Corso di Laurea Magistrale in Ingegneria di Processo e dei Materiali



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WHAT'S THE PURPOSE OF DYNAMIC MODEL?

- To improve process **understanding** and for operator **training**
 - ▷ Dynamic simulators can be used as «virtual processes» to train the operators to face both normal and abnormal (emergency) situations
 - ▷ They can be used as Operator Training Systems (OTSs)
 - ❖ The real control system is interfaced to the dynamic simulator
- To **develop** a control strategy for a new process, or to **improve** an existing control strategy
 - ▷ Different control alternatives can be evaluated on the model
 - ❖ E.g., different pairings between controlled variables and manipulated variables
 - ▷ The model can be used for controller tuning (i.e., to assign appropriate values to the control law parameters)
 - ▷ The model itself could be incorporated **within** the control law
- To **optimize** the process operating conditions
 - ▷ Usually, a steady-state model is sufficient

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WHICH KIND OF DYNAMIC MODEL?

■ First-principles (theoretical) models

- ▷ They use the **principles** of chemical engineering, and of chemistry, physics, biology, physiology, pharmacology, ...
 - ❖ They represent the physical knowledge of the system
 - ❖ They can be used in a wide range of operating conditions (model extrapolation)
 - ❖ Their parameters have physical meaning
 - ❖ Often, they are difficult to derive, and can be «slow» from a calculation viewpoint

«Essentially, all models are wrong, but some are useful»

■ Empirical models (data-driven models; black-box models)

- ▷ They are obtained by **fitting** experimental data
 - ❖ They can be used in a limited range of operating conditions (no extrapolation)
 - ❖ Their parameters rarely have physical meaning
 - ❖ They are relatively easy to derive and fast to calculate

Box, G.E.P., and Draper, N.R. (1987). Empirical Model Building and Response Surfaces, John Wiley & Sons, Hoboken, NJ

■ Semi-empirical models (hybrid models; grey-box models)

- ▷ They are a combination of first-principles and empirical models
 - ❖ A compromise of their pros and cons

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FIRST-PRINCIPLES MODELS

■ Basically, they can be derived by using two kinds of equations

1. Conservation laws (mass; energy; momentum)
 - ❖ Ordinary differential equations (ODEs) → concentrated parameter models
 - ❖ Partial differential equations (PDEs) → distributed parameter models
 2. Constitutive equations
 - ❖ Algebraic equations (AEs): thermodynamic equilibrium, mass/energy transport, physical properties, kinetic equations, ...
- ▷ The set of ODEs and AEs results in a **system of differential and algebraic equations (DAEs)**

The model must **not be more complex than required** for the particular application it is developed for.

Fit-all-purpose models do not exist!

■ First-principles models represent an abstraction of reality

- ▷ They can incorporate macroscopic and/or microscopic characteristics of the real system
- ▷ The level of details to be included in the model depends on the required use of the model
- ▷ A compromise between accuracy and complexity is needed
 - ❖ The more the model is accurate, the more is complex
 - ❖ The more the model is complex, the more it costs to develop, the more difficult it is to use
- ▷ Some simplifications usually need to be considered

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DEGREES OF FREEDOM OF A DYNAMIC MODEL

- Number N_F of degrees of freedom (d.o.f.) of a set of equations

$$N_F = N_V - N_E$$

N_V = number of model variables

N_E = number of **independent** equations

- $N_F = 0$: the model is **exactly specified**
 - The set of equations has a solution
- $N_F > 0$: the model is **underspecified**
 - There are more variables than equations
 - The N_E equations have an infinite number of solutions, because N_F variables can be assigned arbitrarily
- $N_F < 0$: the model is **overspecified**
 - There are more equations than variables
 - The set of equations has no solution

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HOW TO FIND THE NUMBER OF DEGREES OF FREEDOM

1. List all quantities in the model that are **known** constants (or parameters whose values can be determined)
 - E.g., from equipment size, physical properties
2. Determine the number N_E of **independent equations** and the number N_V of **process variables**
 - **Time t is not a process variable**: it is neither an input nor an output
3. Calculate the number of **d.o.f.**: $N_F = N_V - N_E$
4. Identify the N_E **output variables** that will be calculated using the process model
5. Identify the N_F input variables that must be assigned in order to **saturate** the N_F d.o.f.
 - They may be either manipulated variables or disturbances

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EXAMPLES: writing the dynamic models of some systems

- Stirred and heated tank, constant volume (SEMD, p.21)
- Stirred and heated tank, variable volume (SEMD, p. 22)
- Stirred tank, constant volume, with electrical resistance having non-negligible thermal capacitance (SEMD, p.23)
- Suggestion for independent study
 - ▷ Models for other systems (SEMD, p. 24-30)
 - ▷ Numerical integration of sets of differential equations (Matlab; SEMD Appendix C, p.480-482)

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Stirred and heated tank, constant volume

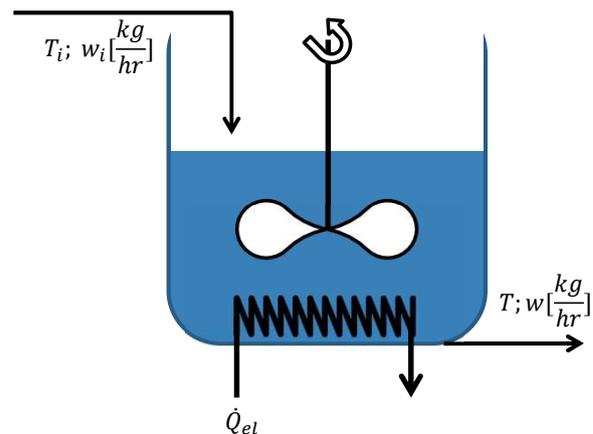
Assumptions:

1. Perfect mixing $\rightarrow T_{\text{out}}(t) = T = T_{\text{internal}}(t)$
2. $w_i = w \rightarrow$ no accumulation
3. Constant $c_p \rightarrow$ independent on T
4. Constant $\rho \rightarrow$ independent on T
5. No heat losses with surroundings
6. $\dot{Q}_{el}(t) = \dot{Q}_{liq}(t) \rightarrow$ Electrical power dissipated is transferred continuously to the system

■ Dynamic model? $\rightarrow T(t)? V(t)?$

$$\frac{dT}{dt} = \frac{\dot{Q}_{liq}}{\rho V c_p} + \frac{w_i(T_i - T)}{\rho V}$$

- Parameters: $N_P = \rho, c_p, V = 3$
- Variables: $N_V = w_i, T_i, \dot{Q}_{el}, T = 4$
- Equations: $N_E = 1$
- D.o.f.: $N_F = N_V - N_E = 3$



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Stirred and heated tank, variable volume

Assumptions:

1. Perfect mixing → $T_{out}(t) = T = T_{internal}(t)$
2. $w_i \neq w \rightarrow$ accumulation!!!!
3. Constant $C_p \rightarrow$ independent on T
4. Constant $\rho \rightarrow$ independent on T
5. No heat losses with surroundings
6. $\dot{Q}_{el}(t) = \dot{Q}_{liq}(t) \rightarrow$ Electrical power dissipated is transferred continuously to the system

Dynamic model? → $T(t)? V(t)?$

$$\frac{dT}{dt} = \frac{\dot{Q}_{liq}}{\rho V c_p} + \frac{w_i(T_i - T)}{\rho V}$$

$$\frac{dV}{dt} = \frac{w_i - w}{\rho}$$

- Parameters: $N_p = \rho, c_p = 2$
- Variables: $N_V = w_i, w, T_i, Q_{el}, T, V = 6$
- Equations: $N_E = 2$
- D.o.f.: $N_F = N_V - N_E = 4$

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Stirred and heated tank, constant volume, resistance thermal capacitance

Assumptions:

1. Perfect mixing → $T_{out}(t) = T = T_{internal}(t)$
2. $w_i = w \rightarrow$ no accumulation
3. Constant $C_p \rightarrow$ independent on T
4. Constant $\rho \rightarrow$ independent on T
5. No heat losses with surroundings
6. $\dot{Q}_{el}(t) \neq \dot{Q}_{liq}(t) \rightarrow$ Electrical power dissipated is **NOT** transferred continuously to the system

Dynamic model? → $T(t)? V(t)?$

$$\frac{dT}{dt} = \frac{h_e S_e (T_e - T)}{\rho V c_p} + \frac{w (T_i - T)}{\rho V}$$

$$\frac{dT_e}{dt} = \frac{\dot{Q}_{el}}{c_{p,e} M_e} - \frac{h_e S_e (T_e - T)}{c_{p,e} M_e}$$

- Parameters: $N_p = \rho, c_p, V, c_{p,e}, M_e, S_e, h_e = 7$
- Variables: $N_V = w_i, T_i, Q_{el}, T, T_e = 5$
- Equations: $N_E = 2$
- D.o.f.: $N_F = N_V - N_E = 3$

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