Dynamic behavior of first-order and second-order processes

DINAMICA E CONTROLLO DEI PROCESSI CHIMICI
A.A. 2019-2020
Corso di Laurea Magistrale in Ingegneria di Processo e dei Materiali

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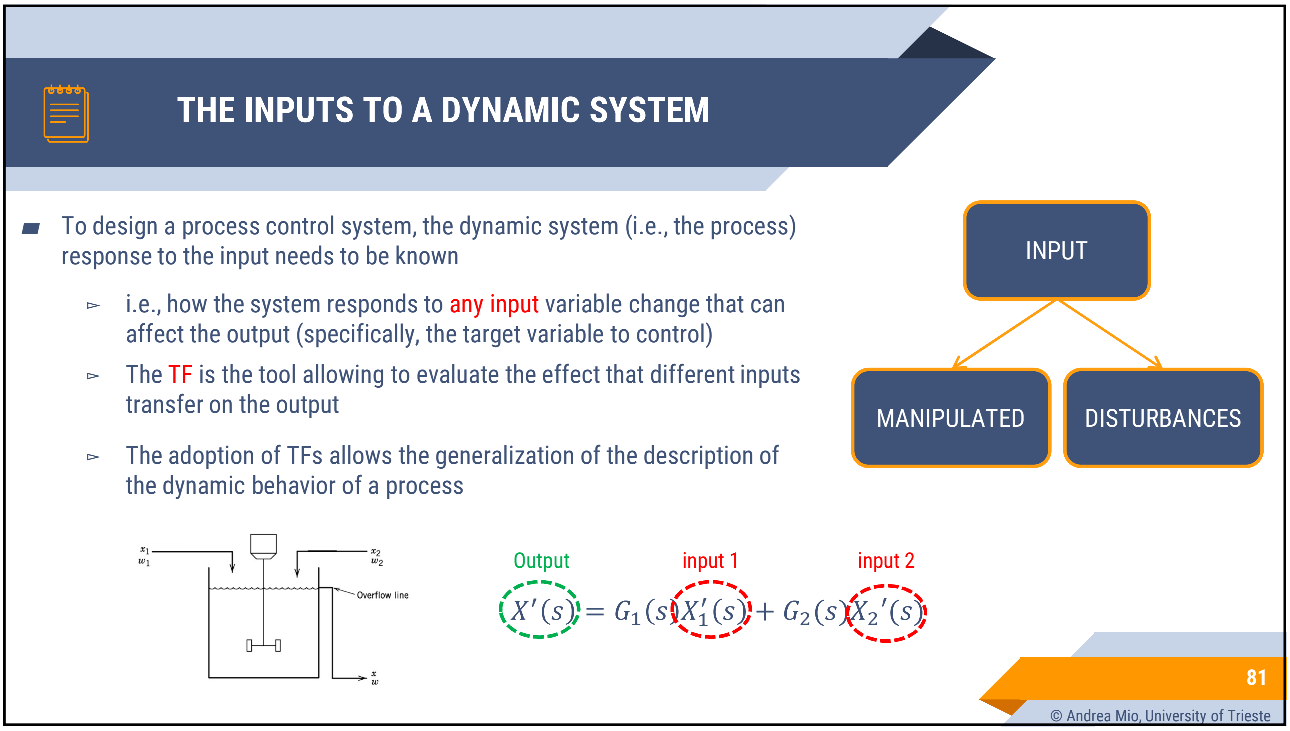


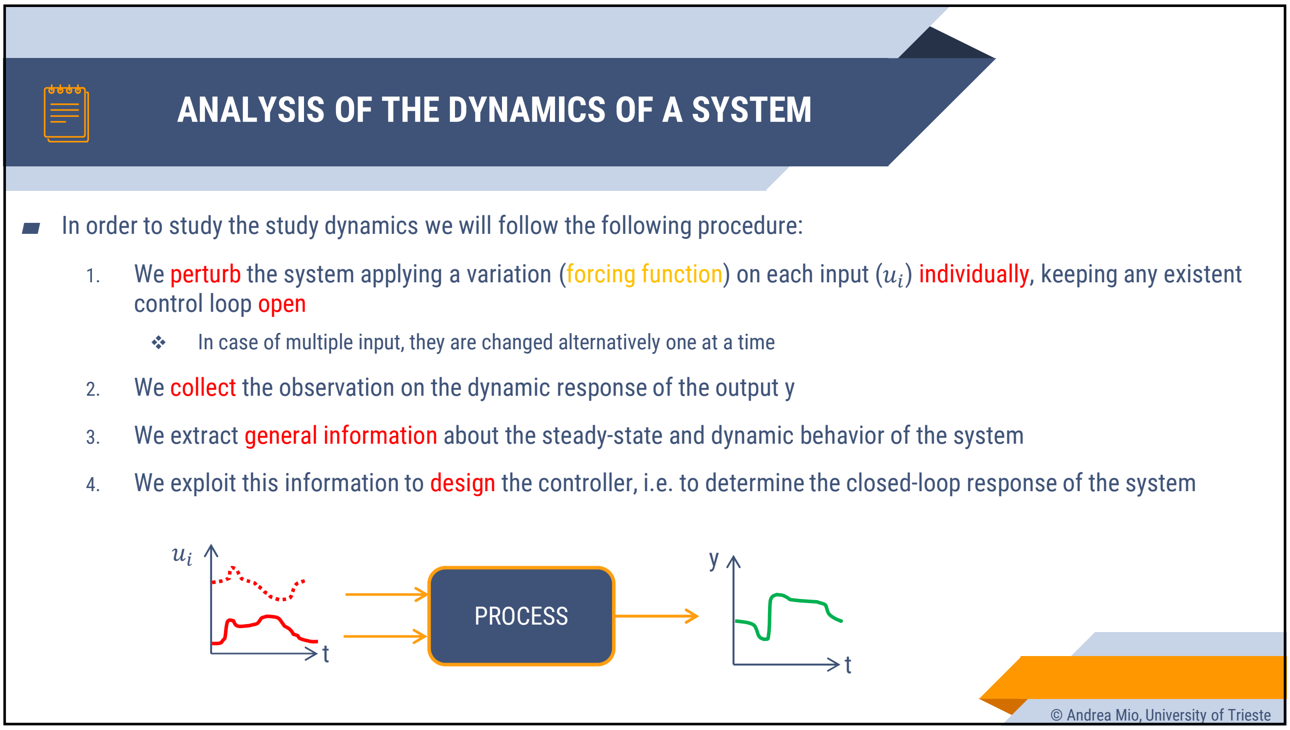
THE INPUTS TO A DYNAMIC SYSTEM

- To design a process control system, the dynamic system (i.e., the process) response to the input needs to be known
  - i.e., how the system responds to any input variable change that can affect the output (specifically, the target variable to control)
  - The TF is the tool allowing to evaluate the effect that different inputs transfer on the output
  - The adoption of TFs allows the generalization of the description of the dynamic behavior of a process

INPUT
MANIPULATED
DISTURBANCES

Output   input 1   input 2

$$\boxed{X'(s)} = G_1(s)\boxed{X_1'(s)} + G_2(s)\boxed{X_2'(s)}$$



ANALYSIS OF THE DYNAMICS OF A SYSTEM

- In order to study the study dynamics we will follow the following procedure:
  1. We perturb the system applying a variation (forcing function) on each input (u_i) individually, keeping any existent control loop open
     - In case of multiple input, they are changed alternatively one at a time
  2. We collect the observation on the dynamic response of the output y
  3. We extract general information about the steady-state and dynamic behavior of the system
  4. We exploit this information to design the controller, i.e. to determine the closed-loop response of the system

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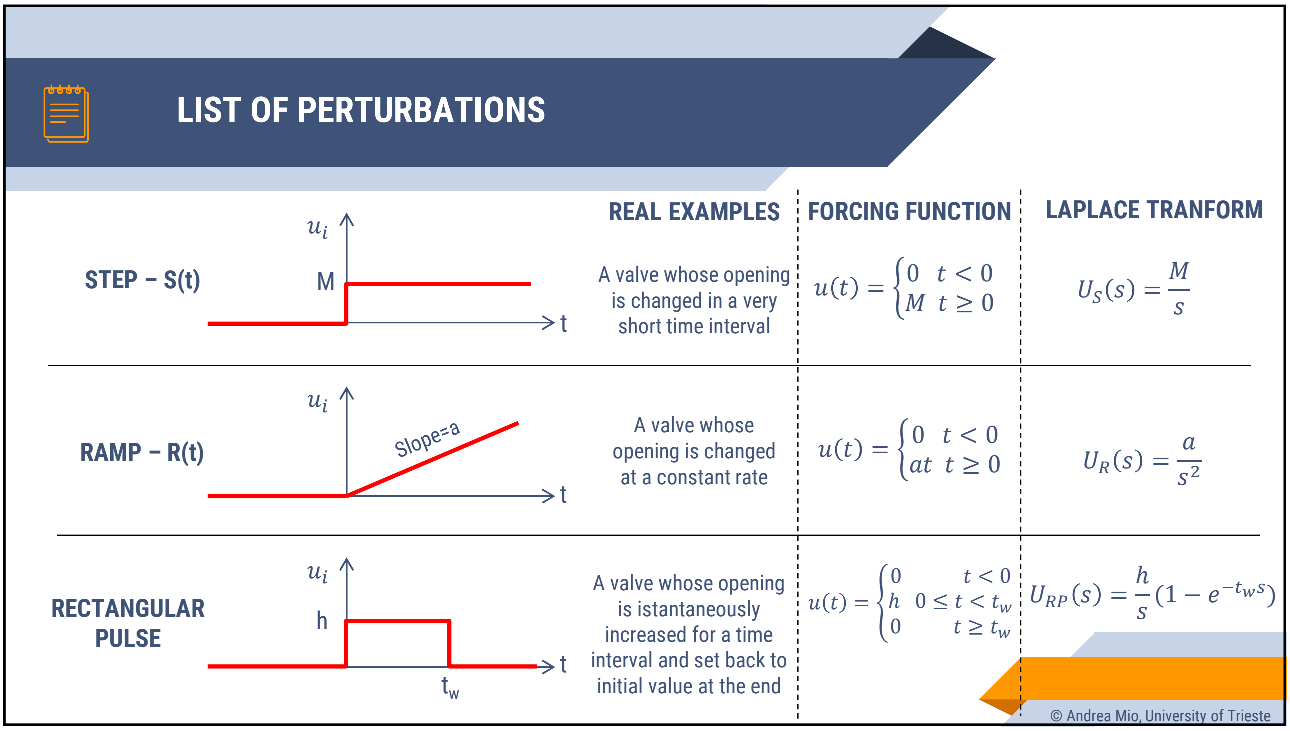

LIST OF PERTURBATIONS

		REAL EXAMPLES	FORCING FUNCTION	LAPLACE TRANFORM
STEP – S(t)		A valve whose opening is changed in a very short time interval	$u(t) = \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases}$	$U_S(s) = \frac{M}{s}$
RAMP – R(t)		A valve whose opening is changed at a constant rate	$u(t) = \begin{cases} 0 & t < 0 \\ at & t \geq 0 \end{cases}$	$U_R(s) = \frac{a}{s^2}$
RECTANGULAR PULSE		A valve whose opening is istantaneously increased for a time interval and set back to initial value at the end	$u(t) = \begin{cases} 0 & t < 0 \\ h & 0 \leq t < t_w \\ 0 & t \geq t_w \end{cases}$	$U_{RP}(s) = \frac{h}{s}(1 - e^{-t_w s})$

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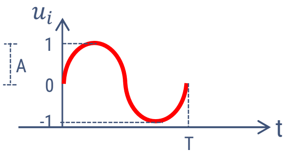
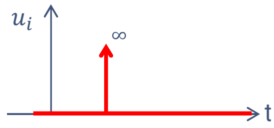
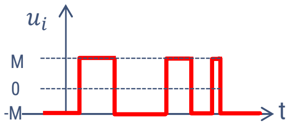
LIST OF PERTURBATIONS /cont'd

		REAL EXAMPLES	FORCING FUNCTION	LAPLACE TRANFORM
SINUSOIDAL – sin(t)		Daily variation of cooling water temperaure	$u(t)=\begin{cases}0 & t<0\\ A\sin\omega t & t\geq 0\end{cases}$	$U_{sin}(s)=\frac{A\omega}{s^2+\omega^2}$
UNIT PULSE – UP(t)		Difficult to obtain; it can be approximated by a tracer injection for RTD studies	$u(t)=\begin{cases}0 & t<0\\ \infty & 0\leq t<t_w\\ 0 & t\geq t_w\end{cases}$ With $t_w \to 0$	$U_{UP}(s)=1$
RANDOM		Usually characterized with statistical term (mean, st.dev.)		

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FIRST-ORDER SYSTEMS

- They are constituted by a 1°-order differential equation with $a_0 \neq 0$:

$$\frac{a_1}{a_0}\frac{dy(t)}{dt}+y(t)=\frac{b_0}{a_0}u(t) \quad \rightarrow \quad \boxed{\tau\frac{dy(t)}{dt}+y(t)=Ku(t)}$$

Called: first-order lag

Notice: the speed of variation of the output depends on the current output value itself!
(in deviation variable terms)

$$\boxed{G(s)=\frac{Y(s)}{U(s)}=\frac{K}{\tau s+1}}$$

- K: steady-stete gain of the system [output u.m./input u.m.]
- $\tau > 0$: time constant of the system [time]

- Assume that u(t) is at steady-state and at time t=0 a perturbation occurs
- We need to take into account two different types of perturbations
  - Step input change: $U_S(s)=\frac{M}{s}$
  - Sinusoidal input change: $U_{sin}(s)=\frac{A\omega}{s^2+\omega^2}$

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STEP RESPONSE OF FO SYSTEMS

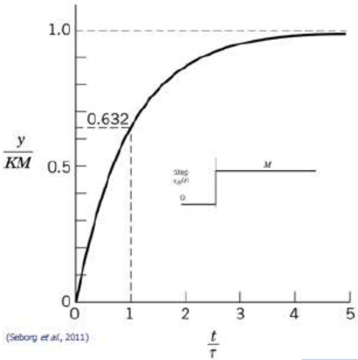
$$u(t) = \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases}$$

$$G(s) = \frac{K}{\tau s + 1}; \qquad Y(s) = G(s)U(s); \qquad U(s) = \frac{M}{s} \qquad \rightarrow \qquad \boxed{Y(s) = \frac{KM}{s(\tau s + 1)}}$$

$$y\Big|_{t=0^+} = 0$$
$$y\Big|_{t=\infty} = KM$$

Upon inverse transform: $\boxed{y(t) = KM\left(1 - e^{\frac{-t}{\tau}}\right)}$

- The process starts responding to the perturbation instantaneously
- The speed of change y is maximum at t=0, then it progressively decreases to 0
- A new steady-state is attained only as $t \rightarrow \infty$
- It only takes a time equal to 4-5 time constants for the transient to die out
- τ provides an indication on the speed of response: $\uparrow \tau \rightarrow$ slow response
- K indicates how far the new steady state value of y is from the original one: $\uparrow$ K $\rightarrow \uparrow$ amplification of input variation onto the output



(Seborg *et al.*, 2011)

t	*y*/ (*KM*)
0	0
τ	0.6321
2τ	0.8647
3τ	0.9502
4τ	0.9817
5τ	0.9933

RESPONSE OF FO SYSTEMS TO A SINUSOIDAL

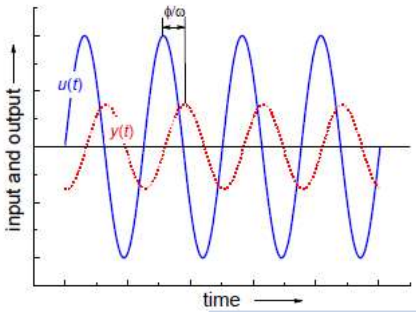
$$u(t) = \begin{cases} 0 & t < 0 \\ A \sin \omega t & t \geq 0 \end{cases}$$

$$G(s) = \frac{K}{\tau s + 1}; \qquad Y(s) = G(s)U(s); \qquad U(s) = \frac{A\omega}{s^2 + \omega^2} \qquad \rightarrow \qquad \boxed{Y(s) = \frac{KA\omega}{(\tau s + 1)(s^2 + \omega^2)}}$$

Upon inverse transform: $\boxed{y(t) = \frac{KA\omega\tau}{\omega^2\tau^2 + 1} e^{\frac{-t}{\tau}} + \frac{KA}{\sqrt{\omega^2\tau^2 + 1}} \sin(\omega t + \phi)}$

$\phi = \tan^{-1}(-\omega t)$ =phase angle [rad]

- An initial transient exists, which dies out after a few τ
- Then the response is purely sinusoidal (periodic), as the input perturbation
- The amplitude of the response sinusoid is different from that (A) of the forcing function; It depends on:
  - The process: K, τ
  - The forcing function, namely its angular frequency ω (hence on frequency f or period $T = 2\pi/\omega = 1/f$)
- The system response lags behind the input
  - The phase shift ϕ depends on the dynamic characteristics of the system (τ) and of those of the forcing function (ω)



EFFECT OF FREQUENCY

$$u(t) = A\sin(\omega t)$$

$$\boxed{y(t) = \frac{KA\omega\tau}{\omega^2\tau^2+1}e^{\frac{-t}{\tau}} + \frac{KA}{\sqrt{\omega^2\tau^2+1}}\sin(\omega t+\phi)} \quad \phi = \tan^{-1}(-\omega t) = \text{phase angle [rad]}$$

- What's the influence of the input frequency f (hence its angular frequency ω) on the response?
- The ratio between the response amplitude and the input amplitude is given by:

Amplitude ratio $AR = \dfrac{amplitude\ response}{amplitude\ input} = \dfrac{K}{\sqrt{\omega^2\tau^2+1}}$

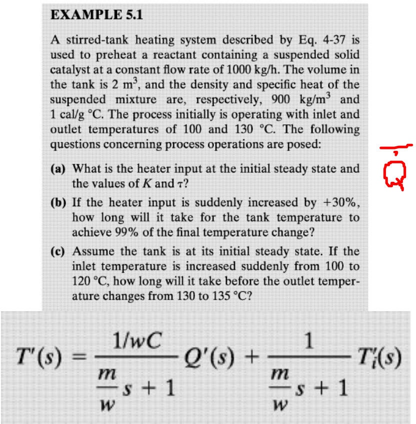
- ❖ $\uparrow in.\omega \rightarrow\downarrow out.ampl$
- ❖ $\uparrow\uparrow\uparrow in.\omega \rightarrow response$ flattens, i.e. the input is filtered by the process
- ❖ The same holds true if the process has a ↑↑↑ τ

- The phase shift (lag) ϕ between the sine waves is maximum $(-\pi/2rad)$ when the input frequency is very large
- The input frequency can be denoted as «large» of «smalle» depending on the dynamic characteristic of the process (τ)

What matters is the product $\omega\tau$

EXAMPLES

- 4° ed., Example 5.1, p. 71; 3° ed, 5.1 p.78



EXAMPLE 5.1

A stirred-tank heating system described by Eq. 4-37 is used to preheat a reactant containing a suspended solid catalyst at a constant flow rate of 1000 kg/h. The volume in the tank is 2 m^3, and the density and specific heat of the suspended mixture are, respectively, 900 kg/m^3 and 1 cal/g °C. The process initially is operating with inlet and outlet temperatures of 100 and 130 °C. The following questions concerning process operations are posed:

(a) What is the heater input at the initial steady state and the values of K and τ?

(b) If the heater input is suddenly increased by +30%, how long will it take for the tank temperature to achieve 99% of the final temperature change?

(c) Assume the tank is at its initial steady state. If the inlet temperature is increased suddenly from 100 to 120 °C, how long will it take before the outlet temperature changes from 130 to 135 °C?

$$T'(s) = \frac{1/wC}{\frac{m}{w}s+1}Q'(s) + \frac{1}{\frac{m}{w}s+1}T_i'(s)$$

$\bar{Q}$?