Dynamic behavior of more complicated processes

DINAMICA E CONTROLLO DEI PROCESSI CHIMICI
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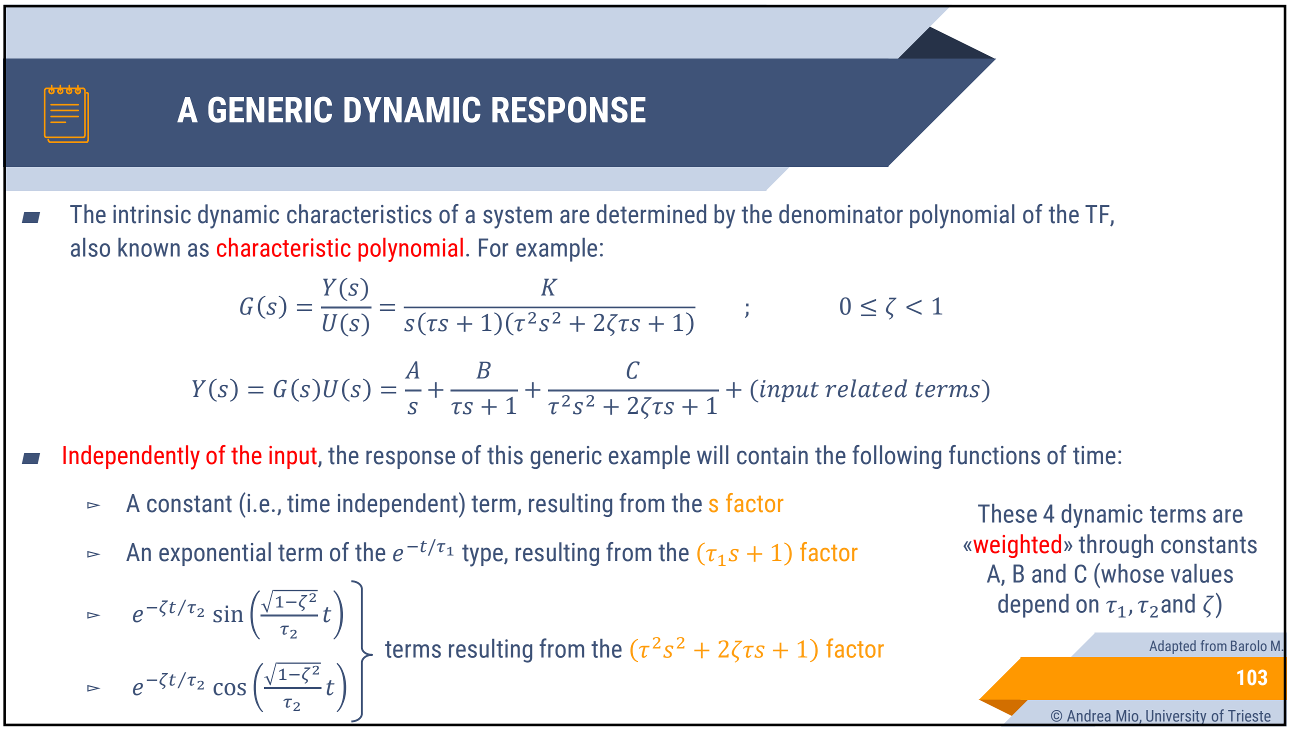


A GENERIC DYNAMIC RESPONSE

- The intrinsic dynamic characteristics of a system are determined by the denominator polynomial of the TF, also known as characteristic polynomial. For example:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{s(\tau s + 1)(\tau^2 s^2 + 2\zeta\tau s + 1)} \quad ; \qquad 0 \le \zeta < 1$$

$$Y(s) = G(s)U(s) = \frac{A}{s} + \frac{B}{\tau s + 1} + \frac{C}{\tau^2 s^2 + 2\zeta\tau s + 1} + (input\ related\ terms)$$

- Independently of the input, the response of this generic example will contain the following functions of time:
  - A constant (i.e., time independent) term, resulting from the s factor
  - An exponential term of the e^{-t/τ_1} type, resulting from the $(\tau_1 s + 1)$ factor
  - $e^{-\zeta t/\tau_2} \sin\left(\frac{\sqrt{1-\zeta^2}}{\tau_2} t\right)$
  - $e^{-\zeta t/\tau_2} \cos\left(\frac{\sqrt{1-\zeta^2}}{\tau_2} t\right)$

$\Big\}$ terms resulting from the $(\tau^2 s^2 + 2\zeta\tau s + 1)$ factor

These 4 dynamic terms are «weighted» through constants A, B and C (whose values depend on τ_1, τ_2 and ζ)

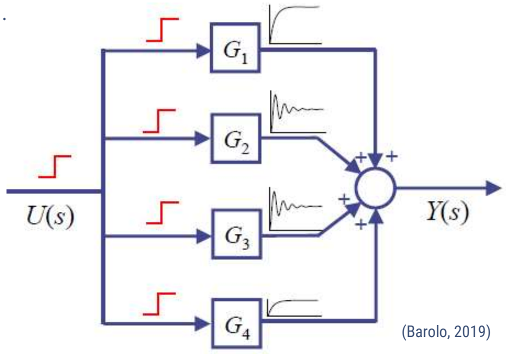
Adapted from Barolo M.

NATURAL MODES AND POLES

$$G(s)=\frac{A}{\tau_1 s+1}+\frac{B}{\tau_2 s+1}+\frac{C}{\tau_3 s+1}+\frac{D}{\tau_4 s+1}+\cdots=G_1(s)+G_2(s)+G_3(s)+G_4(s)+\cdots\rightarrow$$

$$\rightarrow\ Y(s)=G(s)\cdot U(s)=G_1(s)U(s)+G_2(s)U(s)+G_3(s)U(s)+G_4(s)U(s)+\cdot$$

- However complex G(s) might be, it may be thought as the superposition (combination) of simpler dynamic modes
- The τ_i's may be real or complex, with positive or negative real parts
- Each dynamic mode contributes to the overall response through an addendum of G(s)
  - The overall response is the weighted sum of the natural modes, each of which does not depend on the input
- Despite the shape of the overall response will depend on the f.f. U(s), the intrinsic dynamics (which depends on natural modes only) is independent from U(s)
- Notice: If one natural mode (pole) drive one subsystem to instability, the overall response will be unstable!



(Barolo, 2019)

Adapted from Barolo M.

NATURAL MODES AND POLES /cont'd

$p = -\frac{1}{\tau}$

$\frac{1}{\tau} e^{-t/\tau}$

$-p\, e^{+t \cdot p}$

$$G(s)=\frac{Y(s)}{U(s)}=\frac{K}{s(\tau s+1)(\tau^2 s^2+2\zeta\tau s+1)}$$

Additional characteristics of Y(s) response will be determined by the specific forcing function perturbating the system

- Anyway, the intrinsic dynamics of the response is determined by the process, not by the f.f.
  - The intrisic characteristics are called natural modes o «modi naturali» o «risposte naturali»
  - From a practical p.o.v., the natural modes are determined by the roots p_i of the characteristic polynomial
  - In this case: $p_1 = 0$; $p_2 = -\frac{1}{\tau_1}$; $p_3 = -\frac{\zeta}{\tau_2} + j\frac{\sqrt{1-\zeta^2}}{\tau_2}$; $p_4 = -\frac{\zeta}{\tau_2} - j\frac{\sqrt{1-\zeta^2}}{\tau_2}$
  - In the time domain, each mode corresponds to a term of the $[-p_i e^{t\cdot p_i}]$ type

$e^{(a+jb)} = e^{a} \cdot e^{(jb)}$

- The roots of the characteristic polynomial are called poles of the system
  - The natural modes are «weighted» one another depending on the values of parameters $\tau_i, \zeta, \ldots$
- In general, the response is «dominated» by the slowest mode (the one with ↑ τ_i)
  - The relevant $e^{t\cdot p_i}$ decays slower

Adapted from Barolo M.

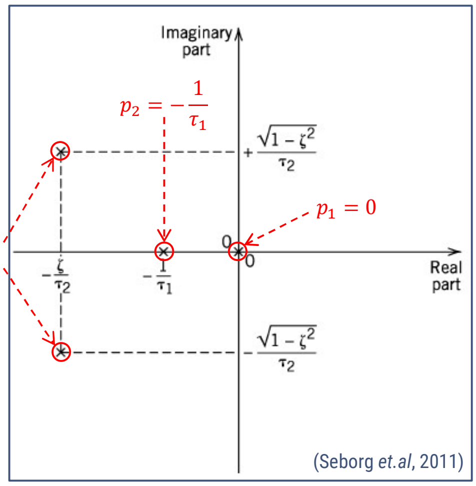
POLES IN THE COMPLEX PLANE

$$G(s) = \frac{K}{s(\tau s + 1)(\tau^2 s^2 + 2\zeta\tau s + 1)} \quad ; 0 \le \zeta < 1$$

- The integrator (p_1) is located on the plane origin
- Any time a complex solution occurs, its conjugate occurs as well ($p_{3,4}$)
- In the time domain, the poles p_i become modes of the $[-p_i e^{t \cdot p_i}]$ type

$$p_{3,4} = -\frac{\zeta}{\tau_2} \pm j\frac{\sqrt{1-\zeta^2}}{\tau_2}$$

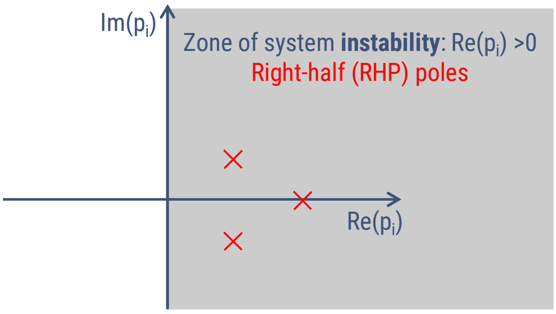
- The speed of response corresponding to a given mode increases as the pole moves further away from the imaginary axis
- Poles with non-null imaginary part ($p_{3,4}$) indicate the existence of oscillations in the response (sine and cosine terms are present)
- Poles with positive real part indicate a response that does not settle to a steady state (unstable systems)



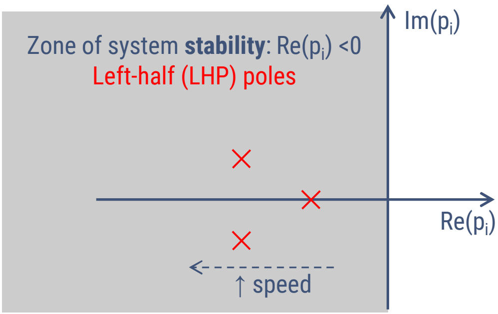
(Seborg *et.al*, 2011)

Adapted from Barolo M.

POLES IN THE COMPLEX PLANE /cont'd



Zone of system **instability**: Re(p_i) >0
Right-half (RHP) poles



Zone of system **stability**: Re(p_i) <0
Left-half (LHP) poles

Oscillating response: Im(p_i) ≠ 0
Complex and conjugated poles
(on right or left half planes)

Adapted from Barolo M.

SYSTEM WITH NUMERATOR DYNAMICS

- FO systems: $\tau \frac{dy(t)}{dt} + y(t) = Ku(t) \quad \rightarrow \quad G(s) = \frac{K}{\tau s + 1} \quad (\tau > 0)$
- Consider the system: $\tau_1 \frac{dy}{dt} + y = K\left(u + \tau_a \frac{du}{dt}\right)$
  - ❖ y,u deviation variables
  - ❖ System initially at s.s.
  - ▻ Lead-lag element o elemento «anticipo-ritardo»
  - ▻ It follows: $\boxed{G(s) = K\frac{\tau_a s + 1}{\tau_1 s + 1}}$ The numerator has its own dynamics! A polynomial in s appears at the numerator
- Consider a different system:

$$\tau_1 \frac{dy}{dt} + y = K\left(u + \frac{1}{\tau_a}\int_0^t u(t^*)dt^*\right)$$

  - ▻ It follows: $\boxed{G(s) = K\frac{\tau_a s + 1}{\tau_a s(\tau_1 s + 1)}}$ The numerator has its own dynamics! There is an integrator term (pole at origin)

The **overall dynamics** of the system is determined not only by the denominator roots (poles), but also by the numerator roots (zeroes)

Adapted from Barolo M.

TRANSFER FUNCTIONS OF DYNAMIC SYSTEMS

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \dots + b_0}{a_n s^n + a_{n-1}s^{n-1} + \dots + a_0}}$$

m≤n

Pole/zero form

$$\boxed{G(s) = \frac{b_m(s - z_1)(s - z_2)\dots(s - z_m)}{a_n(s - p_1)(s - p_2)\dots(s - p_n)}}$$

Gain/time constant form

$$\boxed{G(s) = K\frac{(\tau_a s + 1)(\tau_b s + 1)\dots}{(\tau_1 s + 1)(\tau_2 s + 1)\dots}}$$

Gain K=b_0/a_0=G(0) is highlighted

- It follows that: $z_1 = -\frac{1}{\tau_a};\ z_2 = -\frac{1}{\tau_b};\ \dots \qquad p_1 = -\frac{1}{\tau_1};\ p_2 = -\frac{1}{\tau_2};\ \dots$
- The presence of zeroes does not influence the presence (and location) of the poles
  - ▻ Unless an exact «pole-zero cancellation» occurs
- However, the zeroes have a strong effect on how the natural modes of the system are «weighted» one another
  - ▻ They affect the TF factorization coefficients
- Dominant time constant: the **largest** one (i.e., the slowest natural mode)

Adapted from Barolo M.

STEP RESPONSE OF FO SYSTEMS WITH NUMERATOR DYNAMICS

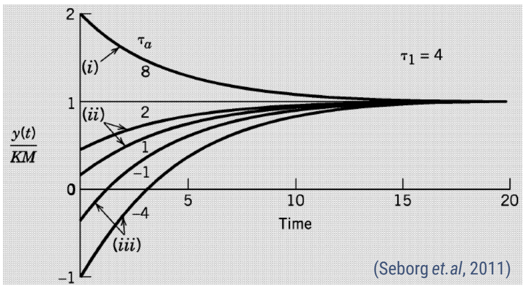
$$u(t) = \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases}$$

$$G(s) = K\frac{\tau_a s + 1}{\tau_1 s + 1}; \quad Y(s) = G(s)U(s); \quad U(s) = \frac{M}{s} \quad \rightarrow \quad \boxed{Y(s) = KM\frac{\tau_a s + 1}{s(\tau_1 s + 1)}}$$

$$y\big|_{t=0^+} = KM(\frac{\tau_a}{\tau_1})$$

$$y\big|_{t=\infty} = KM$$

- Upon inverse transform: $\boxed{y(t) = KM\left[1 - \left(1 - \frac{\tau_a}{\tau_1}\right)e^{-t/\tau_1}\right]}$
- The response depends on the ratio $\frac{\tau_a}{\tau_1}$
  - As $t \rightarrow \infty$ the output settles to KM, regardless of the value of the ratio
  - At $t = 0^+$ the response always shows a «jump» (with respect to the initial s.s.)
    - Whether the jump is «above» or «below» the final value KM depends on the relative values of τ_a and τ_1
  - If $\tau_a = 0$ a typical FO response is obtained
  - If $\tau_a = \tau_1 > 0$ the response is that of a pure gain (pole-zero cancellation)
  - If $\tau_a < 0$ (positive zero), the response starts below zero (i.e., below the initial steady state)



(Seborg *et.al*, 2011)

Step response of a lead/lag element

τ_1: lag time; τ_a: lead time

Adapted from Barolo M.

PHYSICAL REALIZABILITY OF A DYNAMIC SYSTEM

- Generally, a «jump» at t=0 (for a step f.f.) can occur only if the numerator N(s) and the denominator D(s) of G(s) have dynamics of the same order
  - See initial value theorem, for example on: $Y(s) = K\frac{\tau_a s + 1}{(\tau_1 s + 1)} \cdot \frac{M}{s}$
- Real physical systems have D(s) with dynamics of higher order with respect to that of N(s)
  - i.e., they have an intrinsic inertia, which prevents them from instantaneously responding to an input change
  - m < n: physical realizability condition
- A lead/lag response does not exist physically in industrial processes, but it can be obtained as the output of a control system
  - It is possible to build a lead-lag element digitally
  - In order to change the system dynamics, it introduces a «jump» in a manipulated input

Adapted from Barolo M.

STEP RESPONSE OF SO SYSTEMS WITH NUMERATOR DYNAMICS

$$u(t) = \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases}$$

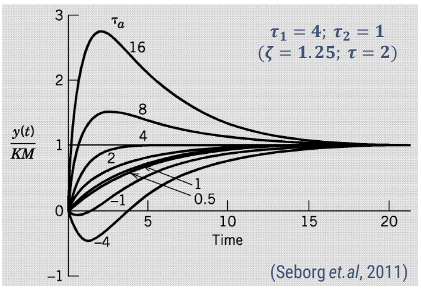
$$G(s) = K\frac{\tau_a s + 1}{(\tau_1 s + 1)(\tau_2 s + 1)}; \quad Y(s) = G(s)U(s); \quad U(s) = \frac{M}{s} \quad \rightarrow \quad \boxed{Y(s) = KM\frac{\tau_a s + 1}{s(\tau_1 s + 1)(\tau_2 s + 1)}}$$

$(\tau_1 > 0;\ \tau_2 > 0)$

- Upon inverse transform: $\boxed{y(t) = KM\left(1 + \frac{\tau_a - \tau_1}{\tau_1 - \tau_2}e^{-t/\tau_1} + \frac{\tau_a - \tau_2}{\tau_2 - \tau_1}e^{-t/\tau_2}\right)}$

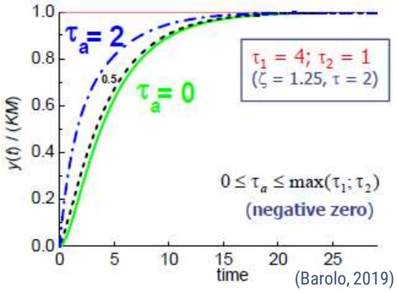
$$y\big|_{t=0^+} = 0$$
$$y\big|_{t=\infty} = KM$$

SO response with numerator dynamics



(Seborg *et.al*, 2011)

SO without/with numerator dynamics



(Barolo, 2019)

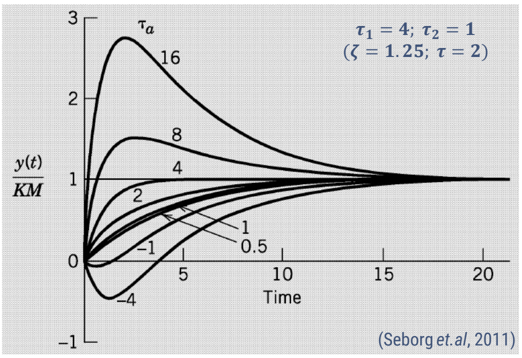
Adapted from Barolo M.

STEP RESPONSE OF SO SYSTEMS WITH NUMERATOR DYNAMICS /cont'd

$$u(t) = \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases}$$

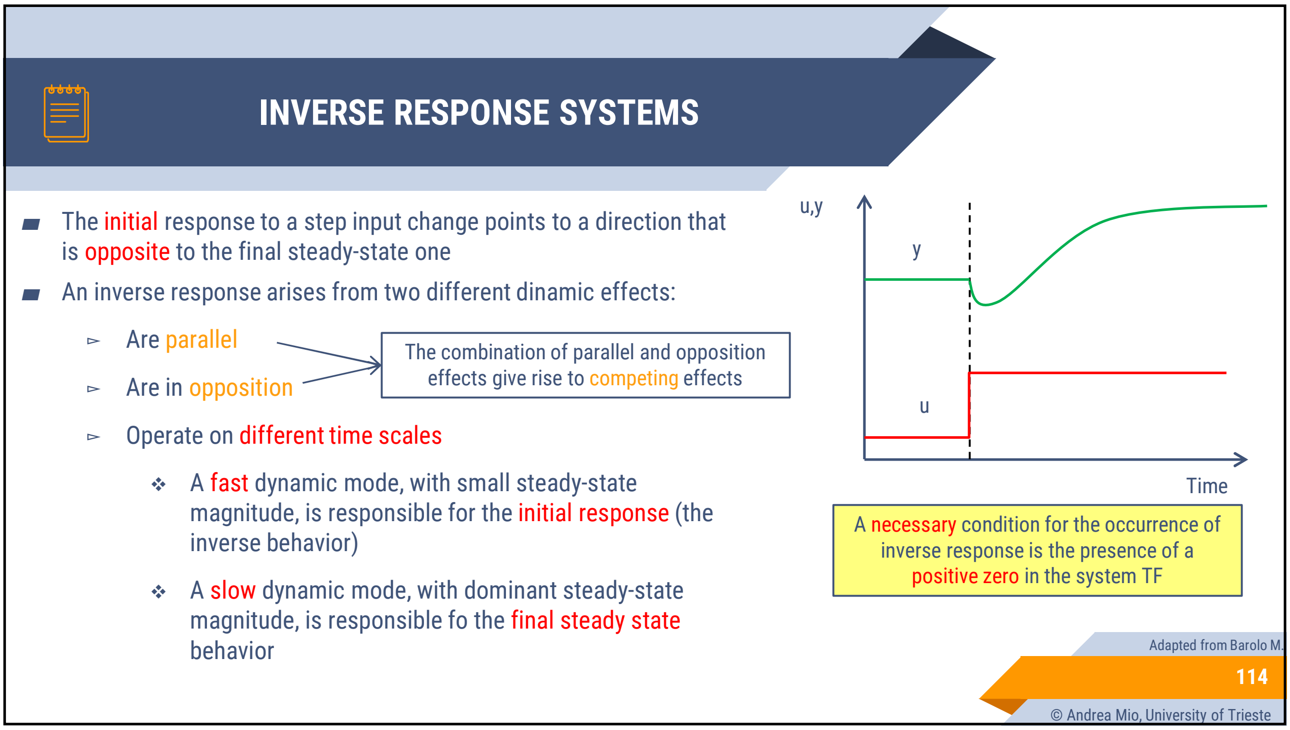
$$\boxed{G(s) = K\frac{\tau_a s + 1}{(\tau_1 s + 1)(\tau_2 s + 1)}}$$

- Compared to a SO system, adding a zero:
  - Does not change either the final value, nor the initial one
  - Does change the way the natural modes are «weighted» to provide the overall response
- For $\tau_a > 0$ (negative zero): the initial response is accelerated with respect to pure SO ($\tau_a = 0$)
  - Generally, this acceleration is independent from the denominator order
- For $\tau_a > \max(\tau_1; \tau_2)$: the initial response is accelerated up to the point that an overshoot occurs
  - The overall response is slower, even though the initial one is faster
- For $\tau_a < 0$ (positive zero): an inverse response occurs (with undershoot)



(Seborg *et.al*, 2011)

Adapted from Barolo M.



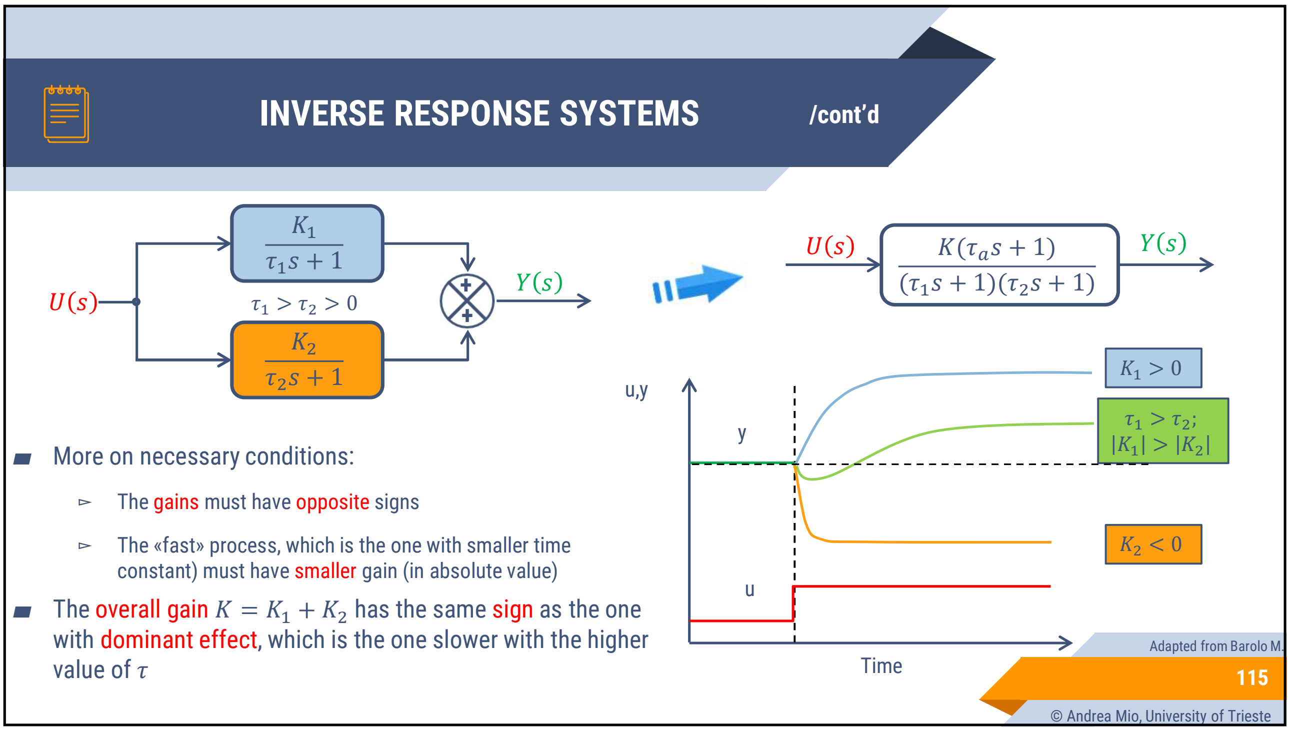
INVERSE RESPONSE SYSTEMS

- The initial response to a step input change points to a direction that is opposite to the final steady-state one
- An inverse response arises from two different dinamic effects:
  - Are parallel
  - Are in opposition
  - Operate on different time scales
    - A fast dynamic mode, with small steady-state magnitude, is responsible for the initial response (the inverse behavior)
    - A slow dynamic mode, with dominant steady-state magnitude, is responsible fo the final steady state behavior

The combination of parallel and opposition effects give rise to competing effects

A necessary condition for the occurrence of inverse response is the presence of a positive zero in the system TF

Adapted from Barolo M.



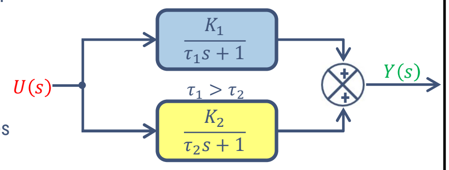
INVERSE RESPONSE SYSTEMS /cont'd

- More on necessary conditions:
  - The gains must have opposite signs
  - The «fast» process, which is the one with smaller time constant) must have smaller gain (in absolute value)
- The overall gain $K = K_1 + K_2$ has the same sign as the one with dominant effect, which is the one slower with the higher value of τ

Adapted from Barolo M.

WHERE CAN WE FIND INVERSE RESPONSE SYSTEMS?

- Open-loop inverse responses of real physical systems
  - Response of the bottom level of a distillation column to a step increase of the reboiler steam flow
    - Long-term effect: more liquid is evaporated from the bottom → the bottom level decreases
    - Short-term effect: more liquid overflows from tray weirs → the bottom level increases
  - Response of the exit temperature in an exothermic catalytic tubular reactor to a step increase of the feed temperature
    - Long-term effect: the catalyst T increases all along the catalyst bed → the T of reactants close to the exit increases → the speed of reaction in this section increases →the product exit T increases
    - Short-term effect: the conversion close to the reactor inlet increase before the catalyst can be heated up → the reactants concentration close to the exit decreases because of fluid transportation →less heat is released close to the exit → exit T decreases



Adapted from Barolo M.