INTEGRAL CONTROL (I)

- The controller action does **not depend** on the **instantaneous** value of the error, but on the **time profile** of the error (a sort of «memory» of the controller action)

$$p(t) = \bar{p} + \frac{1}{\tau_I}\int_0^t \varepsilon(t^*)dt^*$$

τ_I = integral time or reset time >0 [time units]
= «tempo integrale»

$$\frac{dp'}{dt} = \frac{1}{\tau_I}\varepsilon(t)$$

- The integral action can remove the offset
  - The control action p(t) **changes** as long as the **integral term changes**, i.e. as long as an error $\varepsilon(t) \neq 0$ exists
    - Only if the error is zero and steady the controller output stops changing
  - At steady state and with zero error, the **integral term is not zero**, but it is constant instead
    - It contains the entire «history» of past errors
    - It can be interpreted as the indirect way through which bias is adjusted to compensate for a sustained disturbance (or for a set-point variation)

$$G_c(s) = \frac{P'(s)}{E(s)} = \frac{1}{\tau_I s}$$

$$E(s) \rightarrow \boxed{\frac{1}{\tau_I s}} \rightarrow P'(s)$$

The controller is a dynamic system
Namely, it is **a pure integrator**

Adapted from Barolo M.

PROPORTIONAL-INTEGRAL CONTROL (PI)

- The integral action, used alone, does not provide a **prompt** control action
  - The integral contribution «builds up» little by little, which means that the **error must remain non-zero** for a long time in order for p to change sufficiently
- Virtually, the I action is always coupled to the P action

$$p(t) = \bar{p} + K_c\left(\varepsilon(t) + \frac{1}{\tau_I}\int_0^t \varepsilon(t^*)dt^*\right)$$

$$G_c(s) = \frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right) = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)$$

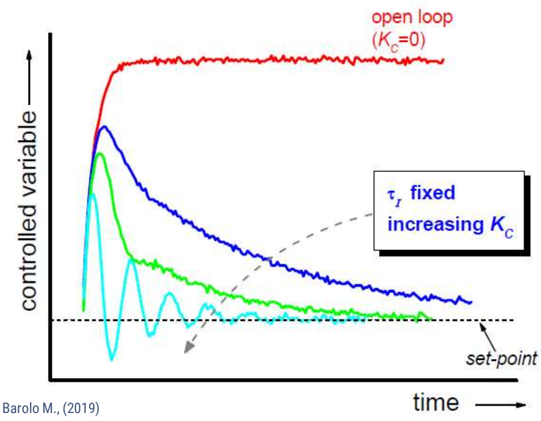
$$E(s) \rightarrow \boxed{K_c\frac{\tau_I s + 1}{\tau_I s}} \rightarrow P'(s)$$

Adapted from Barolo M.

RESPONSE OF A PI CONTROLLER

Typical response to a step disturbance – effect of Kc

$$p(t) = \bar{p} + K_c\left(\varepsilon(t) + \frac{1}{\tau_I}\int_0^t \varepsilon(t^*)dt^*\right)$$



Barolo M., (2019)

- The offset is always removed
  - Increasing Kc fastens the response, but the system may oscillate

Warning
For large values of the gain, the closed-loop system may become unstable!!!

Adapted from Barolo M.

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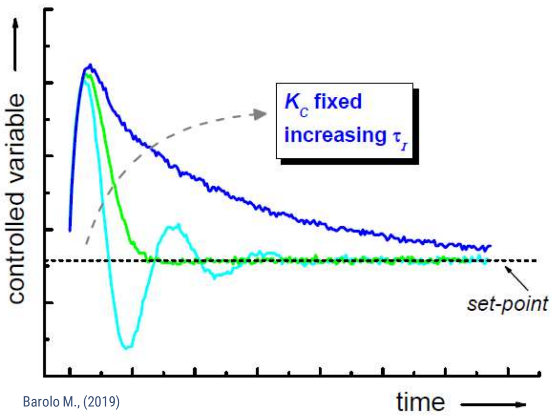
RESPONSE OF A PI CONTROLLER /cont'd

Typical response to a step disturbance – effect of τ_I

$$p(t) = \bar{p} + K_c\left(\varepsilon(t) + \frac{1}{\tau_I}\int_0^t \varepsilon(t^*)dt^*\right)$$



Barolo M., (2019)

- The offset is always removed
  - Increasing τ_I dampens the oscillations, but the response is slower

Warning
For small values of the integral time, the closed-loop system may become unstable!!!

Adapted from Barolo M.

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PROS & CONS of PI CONTROL

- It can **remove the offset**
  - Adopted whenever the offset is not acceptable
- **Fasten** the response
  - It can significantly speed up the system response with respect to the open-loop one

- **Tuning is more complex** than in P-only control
  - The values of two adjustable parameters (Kc and τ_I) have to be identified
- The system response may become **oscillatory**
  - Bad tuning may even bring the system to **instability**
- The integral action may **saturate** (reset windup)
  - The technology to avoid windup exists

About 90% of the conventional industrial controller are based on the PI logic

Adapted from Barolo M.

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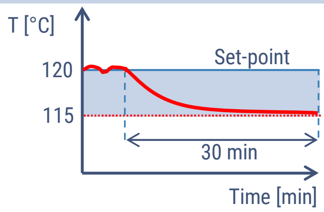
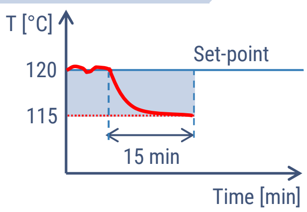
DERIVATIVE CONTROL (D)





Ideal derivative action
D control law

$$p(t) = \bar{p} + \tau_D \frac{d\varepsilon(t)}{dt}$$

τ_D = derivative time >0

- The control action **depends of the time derivative** of the error
  - The controller acts by **anticipating** the future behavior of the process
  - **it cannot remove the offset**
    - If the error is **constant** (not necessarily zero), then also the control action is **constant**
    - This is why the D action is usually **coupled to the PI action** (sometimes to P only)
  - It is recommended for **slow loops** (large θ_p/τ_p ratio) with **noise-free measurements**
    - Typical slow loops: temperature (which is also almost noise-free); composition
    - Typical fast loops: flow (very noisy); liquid pressure

Adapted from Barolo M.

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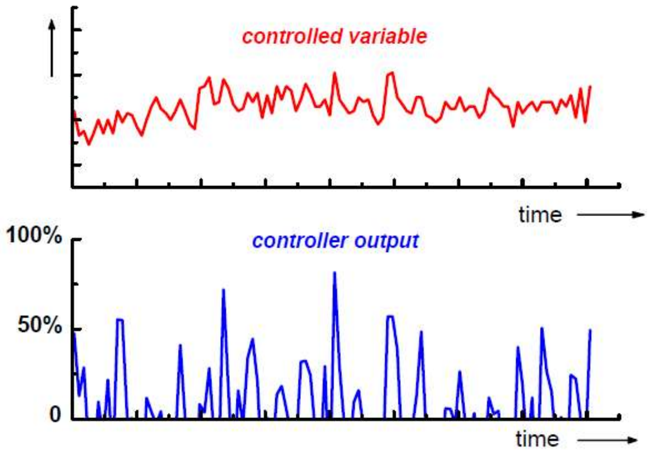
WHY NOISE-FREE MEASUREMENTS?

- The derivative action requires to derive a measurement with respect to time



$$p(t) = \bar{p} + \tau_D \frac{d\varepsilon(t)}{dt}$$

$$\frac{d\varepsilon(t)}{dt} \cong \left.\frac{d\varepsilon}{dt}\right|_{t=t_k} \cong \frac{\varepsilon_k - \varepsilon_{k-1}}{t_k - t_{k-1}} = \frac{\varepsilon_k - \varepsilon_{k-1}}{\Delta t}$$

- If the noise level is high, the valve opens and closes very frequently. Possible solutions are:
  - filter the measurement
  - attenuate the derivative action
  - remove the derivative action

Adapted from Barolo M.

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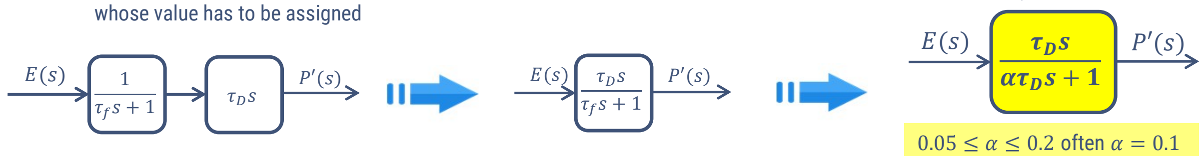
TRANSFER FUNCTION OF D ACTION

$$p(t) = \bar{p} + \tau_D \frac{d\varepsilon(t)}{dt}$$

- The derivative action tends to «stabilize» the response
  - The anticipatory control action **counteracts the oscillations** as soon as they generate, thus attenuating them
  - The settling time usually **decreses**
- The TF of an «ideal» D controller is $\quad G_c(s) = \frac{P'(s)}{E(s)} = \tau_D s$
  - It is physically unrealizable
    - When the error is subject to a step change, it would give rise to $p'(t) \to \infty$
  - In commercial implementations, it is usually coupled to a derivative filter, i.e. a FO system with time constant τ_f, whose value has to be assigned



$0.05 \le \alpha \le 0.2$ often $\alpha = 0.1$

  - The filter attenuates the sensitivity of the control action to high-frequency measurement noise

Adapted from Barolo M.

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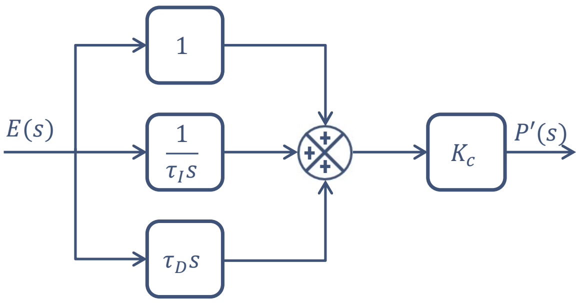
PROPORTIONAL-INTEGRAL-DERIVATIVE CONTROL (PID)

$$p(t) = \bar{p} + K_c\left(\varepsilon(t) + \frac{1}{\tau_I}\int_0^t \varepsilon(t^*)dt^* + \tau_D\frac{d\varepsilon(t)}{dt}\right)$$

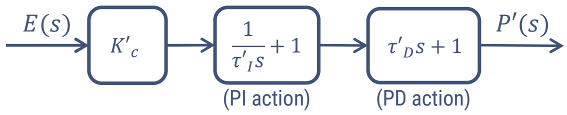
Control law for a PID controller

$$G_c(s) = \frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Parallel form
(ideal; non interacting)



Serial form
(real; interacting)



Expanded form

$$p(t) = \bar{p} + K_c\varepsilon(t) + K_I\int_0^t \varepsilon(t^*)dt^* + K_D\frac{d\varepsilon(t)}{dt}$$

It can be demonstrated that the three forms are equivalent, provided that the tuning parameters are properly «converted» into one another

Adapted from Barolo M.

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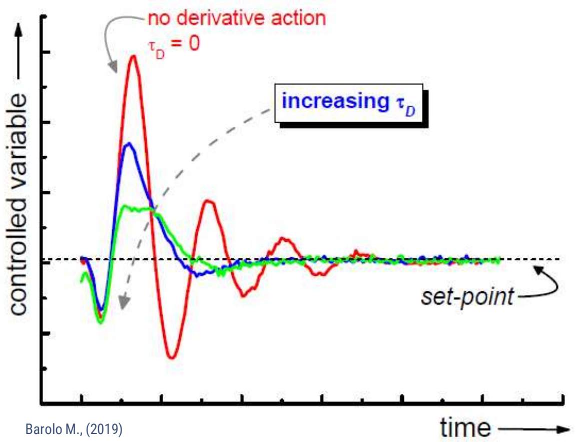
RESPONSE OF A PID CONTROLLER

Typical response to a step disturbance



Barolo M., (2019)

$$p(t) = \bar{p} + K_c\left(\varepsilon(t) + \frac{1}{\tau_I}\int_0^t \varepsilon(t^*)dt^* + \tau_D\frac{d\varepsilon(t)}{dt}\right)$$

- Increasing $\boldsymbol{\tau_D}$ ($\uparrow \boldsymbol{\tau_D}$)
  - The oscillations determined by the integral action are attenuated (↓**oscillations**)
  - The process response is faster (↓**settling time**)
  - If τ_D is too large, the response worsens ($\uparrow\uparrow\uparrow \boldsymbol{\tau_D} \rightarrow$ **worst response**)

Warning
Noisy measurements can degrade the controller performance

Adapted from Barolo M.

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PROS & CONS of PID CONTROL

- The derivative action can increase the speed of response of the closed-loop system
- Oscillations can be attenuated with respect to PI control

- Tuning is even more complex than in PI controllers
  - The values of three adjustable parameters (Kc, τ_I and τ_D) have to be assigned
- The derivative action can amplify measurement noise
  - «Wear-and tear» of the control valve

Avoid using D action when the controlled variable measurement is noisy or when the loop is not slow

Adapted from Barolo M.

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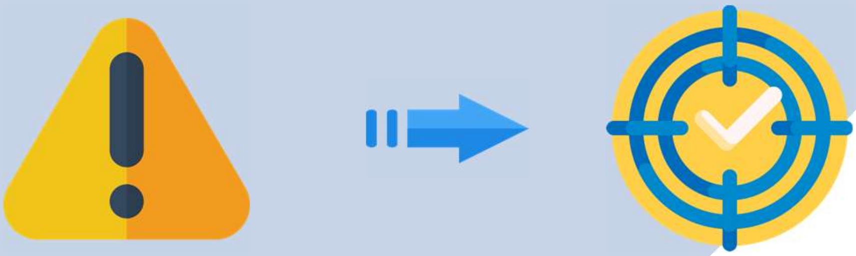

ISSUES RELATED TO PRACTICAL IMPLEMENTATION OF PID CONTROLLERS

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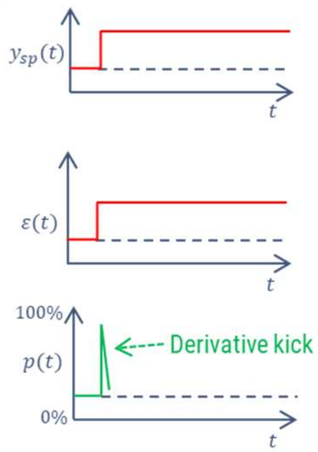
THE DERIVATIVE KICK

- If the set-point is changed in a PID controller, a step error change is generated
  - A drastic change (spike) in the controller output p(t) is obtained because of derivative action
- Instead of deriving the error, it is convenient to derive the measurement

$$\frac{d\varepsilon(t)}{dt} = \frac{dy_{sp}(t)}{dt} - \frac{dy_m(t)}{dt}$$

  - In the time intervals where the set-point is contant: $\frac{d\varepsilon(t)}{dt} = -\frac{dy_m(t)}{dt}$
  - Giving: $$p(t) - \bar{p} = K_c\left(\varepsilon(t) + \frac{1}{\tau_I}\int_0^t \varepsilon(t^*)dt^* - \tau_D\frac{dy_m(t)}{dt}\right)$$



- A smaller peak may still be present due to the contribution of the P action (proportional kick)
  - This can be avoided **filtering** the set-point profile using a FO system

Adapted from Barolo M.

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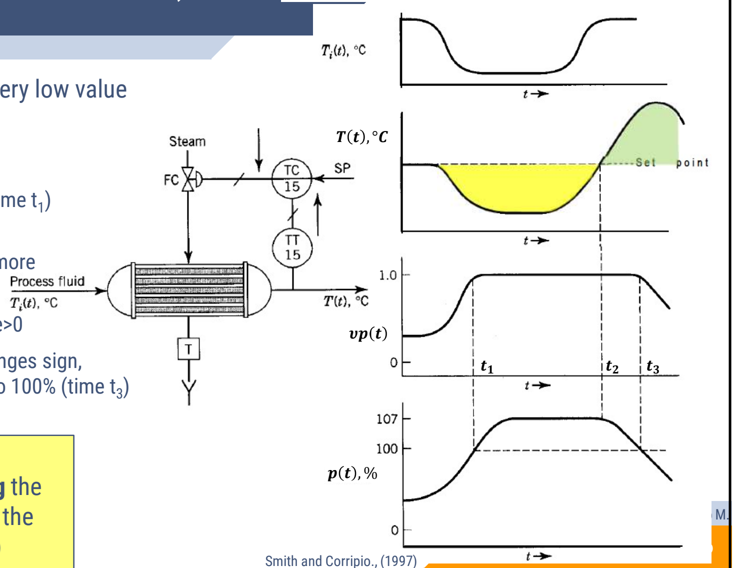
RESET WINDUP (INTEGRAL WINDUP)

$$p(t) - \bar{p} = K_c\left(\varepsilon(t) + \frac{1}{\tau_I}\int_0^t \varepsilon(t^*)dt^*\right)$$

- Assume that the inlet T_i drops to an unusually very low value
  - The outlet temperature T(t) decreases
  - The controller opens the steam valve (p(t) increases)
  - The valve keeps opening until it becomes wide open (time t_1)
  - Since e>0, the controller keeps integrating the error; p(t) increases above 100%, but the valve cannot open more
  - If Ti starts to increase, the outlet temperature starts to increase as well, but for some time the error is still e>0
  - At t=t_2, the outlet T crosses the set-point, the error changes sign, but the valve remains wide open untile p(t) gets back to 100% (time t_3)



Smith and Corripio., (1997)

Notice
What matters is **stopping** the **integration**, not limiting the controller output p(t)

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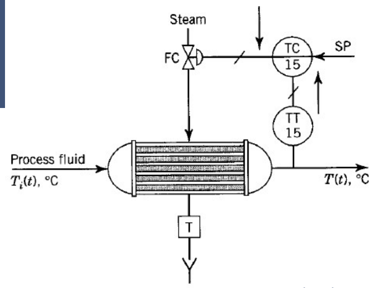
DIRECT-ACTING AND REVERSE-ACTING CONTROLLERS

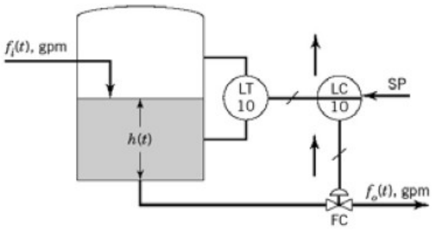
$$p(t) = \bar{p} + K_c[y_{sp}(t) - y_m(t)]$$

The controller gain Kc may be positive or negative



Smith and Corripio., (1997)

- The definition of **direct-acting** or **reverse-acting** controller refers to the **relation between the measured variable (y_m) and the controller output** ($p(t)$)
- Example: assume that $K_c > 0$
  - If $y_m \downarrow$ → $\varepsilon(t) > 0$ → $p(t) \uparrow$ → reverse acting control
  - The opposite behavior arises when $K_c < 0$
- The controller action is selected depending on the control valve action and on the process characteristics
  - Air-to-open (fail-closed)
  - Air-to-close (fail-open)

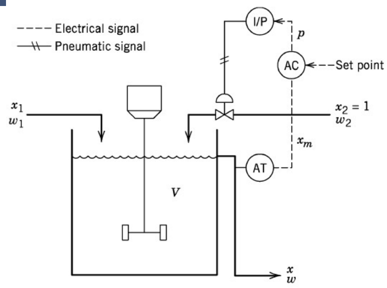


Adapted from Barolo M.

DIRECT-ACTING AND REVERSE-ACTING CONTROLLERS /cont'd

- It is essential to assign the correct controller action

General Guideline
The overall product of the gains for all the components in the feedback loop must be positive



Smith and Corripio., (2006)

- However, be aware of non self-regulating systems!

	Air-to open valve (direct-acting; $K_V > 0$)	Air-to close valve (reverse-acting; $K_V < 0$)
$(K_P \cdot K_m) > 0$	Reverse-acting PID (Kc>0)	Direct-acting PID (Kc<0)
$(K_P \cdot K_m) < 0$	Direct-acting PID (Kc<0)	Reverse-acting PID (Kc>0)

Km is the measurement sensor gain

Adapted from Barolo M.