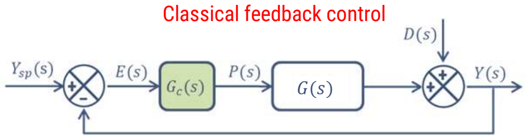
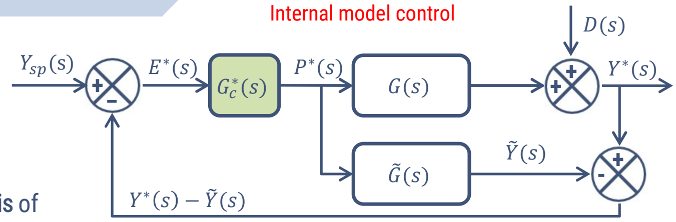
SYNTHESIS BY INTERNAL MODEL CONTROL (IMC)

Classical feedback control



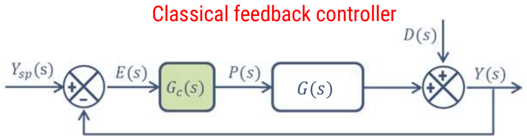
Internal model control



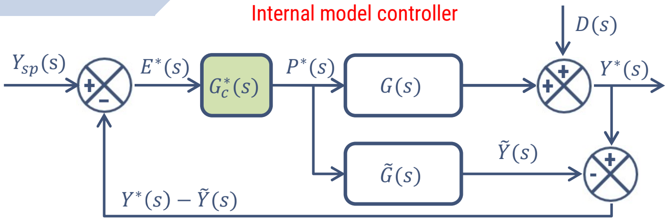
- Also IMC has been originally developed for the **synthesis** of a model-based feedback controller
  - as for DS, it can lead to **analytical tuning rules** for PID controllers (used very frequently in industrial applications)
  - DS and IMC can be made identical if the controller design parameters are selected consistently
- With respect to DS, IMC allows one to consider in a more systematic fashion model uncertainty and the presence of disturbances (which is neglected in DS)
- The internal model is used to estimate the controlled variable from the manipulated variable at each time instant. Their value may be different because of:
  - Modeling errors (i.e., process/model mismatch: $\tilde{G}(s) \neq G(s)$)
  - Unknown disturbances ($D(s) \neq 0$), that are not accounted by the model

IMC – COMPARISON WITH A STANDARD FB CONTROLLER

Classical feedback controller



Internal model controller



- When are the two resulting architectures equivalent?
  - If they are equivalent, all feedback control theory is valid for the IMC controller, and IMC can be implemented using a standard feedback architecture
- IMC feedback controller ($G_c^*(s)$) $\quad Y^* = D + GP^* = D + G(G_c^* E^*) = D + GG_c^* E^*$

$$E^* = Y_{sp} - (Y^* - \tilde{Y}) = Y_{sp} - (Y^* - P^*\tilde{G}) = (Y_{sp} - Y^*) + G_c^*\tilde{G}E^* \rightarrow \quad E^* = \frac{Y_{sp} - Y^*}{1 - G_c^*\tilde{G}} \quad \Rightarrow \quad Y^* = D + GG_c^* \frac{Y_{sp} - Y^*}{1 - G_c^*\tilde{G}} \qquad (1)$$

- Standard FB controller ($G_c(s)$) $\quad Y = D + GP = D + G(G_c E) = D + GG_c E \quad ; \quad E = Y_{sp} - Y$

$$\Rightarrow \quad Y = D + GG_c(Y_{sp} - Y) \qquad (2)$$

IMC – COMPARISON WITH A STANDARD FB CONTROLLER /cont'd

- The two controllers are equivalent if: $Y^* = Y$

$$D + GG_c^* \frac{Y_{sp} - Y^*}{1 - G_c^* \tilde{G}} = D + GG_c(Y_{sp} - Y) \Rightarrow G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} \quad (3)$$

- Even if the architecture of an IMC controller seems different from that of a conventional FB controller, any IMC controller G_c^* can be made equivalent to a standard feedback controller G_c
  - But the standard feedback controller includes a model and is not necessarily a PID controller

- What is the response $Y^*(s)$ of a system controlled via IMC?
  - By solving (1) with respect to $Y^* = Y$, it follows: $\Rightarrow Y = \frac{GG_c^*}{1 + G_c^*(G - \tilde{G})} Y_{sp} + \frac{1 - \tilde{G}G_c^*}{1 + G_c^*(G - \tilde{G})} D$
  - If the model is perfect ($G = \tilde{G}$): $\Rightarrow Y = GG_c^* Y_{sp} + (1 - GG_c^*) D$

How do we design G_c^*, i.e. the IMC controller gives this response?

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DESIGN OF THE IMC CONTROLLER G_c^*

- Following the experience with DS, we design the control law so as to include the **process model inverse** and a **filter**

1. **The process model is factored: $\tilde{G} = \tilde{G}_+ \tilde{G}_-$**

$\tilde{G}_+$
- Includes the non-invertible part
  - Dead times -> unreazability
  - Positive zeroes -> instability
- The gain is set equal to 1

$\tilde{G}_-$
- Includes the invertible part
- The gain is set equal to that of the «overall» model

2. **The IMC controller is designed as:** $G_c^* = \frac{1}{\tilde{G}_-} f$

  - Hence, the controller is the inverse (of the «good», i.e. invertible part) of the model, which guarantees that the controller is a **stable dynamic system**
  - And it is combined with a **filter** f (which provides robustness to the controller): $f = \frac{1}{(\tau_c s + 1)^r}$
    - The low-pass filter order is chosen so as to obtain a physically realizable controller. Often, choosing r=1 is enough; otherwise one should increase r
    - τ_c is the **filter time constant**; the filter has a unit steady-state gain

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IMC CONTROLLER RESPONSE FOR A PERFECT MODEL

$\tilde{G} = G \quad ; \quad \begin{cases} Y = G_c^* G Y_{sp} + (1 - G_c^* G) D \\ G_c^* = \dfrac{1}{\tilde{G}_-} f \end{cases}$

Closed-loop response for an IMC-controlled system

TF of the IMC controller

$$Y = \left(\frac{1}{\tilde{G}_-} f\right)(G_+ G_-) Y_{sp} + \left[1 - \left(\frac{1}{\tilde{G}_-} f\right)(G_+ G_-)\right] D \quad \rightarrow \quad Y = f\tilde{G}_+ Y_{sp} + (1 - f\tilde{G}_+) D;$$

For D=0

CLTF of an IMC-controlled system to a set-point change

$$\frac{Y(s)}{Y_{sp}(s)} = f\tilde{G}_+ \qquad (4)$$

Hence, the closed-loop response is equal to the **filter dynamics**, combined with the dynamics of the non-invertible part of the process

- The controllers from **IMC and DS can be designed so as to provide identical responses**
  - The DS response $[Y/Y_{sp}]_d$ is set equal to the IMC response (4), and then controller G_c obtained from DS is «converted» into an IMC controller using (3) $\quad G_c = \dfrac{G_c^*}{1 - G_c^* \tilde{G}}$
  - What differs between the DS and IMC approaches is the rationale:
    - In DS, $[Y/Y_{sp}]$ is assigned → G_c
    - In IMC, G_c^* is assigned → $[Y/Y_{sp}]$

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IMC – THE PROCESS MODEL

- FOPDT system $\quad \tilde{G}(s) = \dfrac{K_p e^{-\theta_p s}}{\tau_p s + 1}$
  - A FO Padè approximation for the dead time may be used:

$$e^{-\theta_p s} \cong \frac{1 - \frac{\theta_p}{2}s}{1 + \frac{\theta_p}{2}s} \rightarrow \tilde{G} = \frac{K_p\left(1 - \frac{\theta_p}{2}s\right)}{(\tau_p s + 1)\left(1 + \frac{\theta_p}{2}s\right)}$$

$$\tilde{G}_+ = \left(1 - \frac{\theta_p}{2}s\right); \qquad \tilde{G}_- = \frac{K_p}{(\tau_p s + 1)\left(1 + \frac{\theta_p}{2}s\right)}$$

  - IMC controller synthesis: $G_c^* = \dfrac{1}{\tilde{G}_-} f \quad ; \quad f = \dfrac{1}{(\tau_c s + 1)^r}$ For r=1

$$G_c^* = \frac{(\tau_p s + 1)\left(1 + \frac{\theta_p}{2}s\right)}{K_p} \times \frac{1}{(\tau_c s + 1)}$$

  - Transformation into a standard feedback controller: $\quad G_c = \dfrac{G_c^*}{1 - G_c^* \tilde{G}}$

$$G_c = \frac{\dfrac{(\tau_p s + 1)\left(1 + \frac{\theta_p}{2}s\right)}{(\tau_c s + 1)K_p}}{1 - \dfrac{(\tau_p s + 1)\left(1 + \frac{\theta_p}{2}s\right)}{(\tau_c s + 1)K_p} \times \dfrac{K_p\left(1 - \frac{\theta_p}{2}s\right)}{(\tau_p s + 1)\left(1 + \frac{\theta_p}{2}s\right)}} = \frac{\dfrac{(\tau_p s + 1)\left(1 + \frac{\theta_p}{2}s\right)}{(\tau_c s + 1)K_p}}{\dfrac{(\tau_c s + 1) - \left(1 - \frac{\theta_p}{2}s\right)}{(\tau_c s + 1)}} = \frac{(\tau_p s + 1)\left(1 + \frac{\theta_p}{2}s\right)}{K_p\left(\tau_c + \frac{\theta_p}{2}\right)s}$$

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IMC – THE PROCESS MODEL /cont'd

$$\tilde{G}(s) = \frac{K_p e^{-\theta_p s}}{\tau_p s + 1}$$

- Resulting standard feedback controller: $G_c = \dfrac{(\tau_p s + 1)\left(1 + \frac{\theta_p}{2}s\right)}{K_p\left(\tau_c + \frac{\theta_p}{2}\right)s}$

$$G_c = \frac{1}{K_p\left(\tau_c + \frac{\theta_p}{2}\right)} \times \frac{1 + \left(\tau_p + \frac{\theta_p}{2}\right)s + \tau_p\frac{\theta_p}{2}s^2}{s} = \frac{1}{K_p\left(\tau_c + \frac{\theta_p}{2}\right)} \times \frac{\left(\tau_p + \frac{\theta_p}{2}\right)}{\left(\tau_p + \frac{\theta_p}{2}\right)} \times \frac{1 + \left(\tau_p + \frac{\theta_p}{2}\right)s + \tau_p\frac{\theta_p}{2}s^2}{s}$$

$$G_c = \frac{\left(\tau_p + \frac{\theta_p}{2}\right)}{K_p\left(\tau_c + \frac{\theta_p}{2}\right)} \times \frac{1 + \left(\tau_p + \frac{\theta_p}{2}\right)s + \tau_p\frac{\theta_p}{2}s^2}{\left(\tau_p + \frac{\theta_p}{2}\right)s} = \boxed{\frac{1}{K_p} \times \frac{\frac{\tau_p}{\theta_p/2} + 1}{\frac{\tau_c}{\theta_p/2} + 1} \times \frac{1 + \left(\tau_p + \frac{\theta_p}{2}\right)s + \tau_p\frac{\theta_p}{2}s^2}{\left(\tau_p + \frac{\theta_p}{2}\right)s}}$$

- Equivalence with a PID structure: $G_c = K_c\left(1 + \dfrac{1}{\tau_I s} + \tau_D s\right) = K_c\left(\dfrac{1 + \tau_I s + \tau_I \tau_D s^2}{\tau_I s}\right)$

$$K_c = \frac{1}{K_p} \times \frac{\frac{\tau_p}{\theta_p/2} + 1}{\frac{\tau_c}{\theta_p/2} + 1} \quad ; \quad \tau_I = \tau_p + \frac{\theta_p}{2} \quad ; \quad \tau_D = \frac{\tau_p}{1 + 2\frac{\tau_p}{\theta_p}}$$

Tuning relations

If the system is FOPTD and the filter is FO, then the IMC controller is equivalent to a PID controller with assigned tuning

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IMC – CHOOSING τ_c

- If the dead time is approximated by a Taylor series expansion, a different expression for the controller settings is obtained
  - In particular, a PI controller is obtained instead of a PID (see SEMD, Example 12.2, p.206)
- The filter time constant τ_c is the design parameter of the IMC controller
  - $\uparrow \tau_c \rightarrow \downarrow K_c$ the controller is more conservative
  - τ_c has a meaning that resembles a closed-loop time constant (see Eq.(4))
  - For a system with dominant time constant τ_{dom}, typically a reasonable choice for τ_c is $\theta_p < \tau_c < \tau_{dom}$
- The IMC **analytical tuning relations** are usually very effective
  - Very used **industrially**, particularly for FOPDT systems
  - For systems with more complex dynamics:
    - See SEMD, Table 12.1 p.207
    - For some models, two different tunings are reported: this is related to how the dead time is approximated
    - For systems with complex dynamics, a different filter f is used to obtain a PID structure

Required Performance	Tuning recommendation for FOPDT systems
Default	$\tau_c = \theta_p$
Aggressive control	$\tau_c = 0.5\theta_p$
Smooth response	$\tau_c = 1.5\theta_p$
Robust control	$\tau_c = 3.0\theta_p$

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ANALYTICAL RELATIONS FROM IMC

Table 12.1 IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

Case	Model	K_cK	τ_I	τ_D
A	$\frac{K}{\tau s + 1}$	$\frac{\tau}{\tau_c}$	τ	—
B	$\frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\tau_1 + \tau_2}{\tau_c}$	$\tau_1 + \tau_2$	$\frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$
C	$\frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{2\zeta\tau}{\tau_c}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
D	$\frac{K(-\beta s + 1)}{\tau^2 s^2 + 2\zeta\tau s + 1}, \beta > 0$	$\frac{2\zeta\tau}{\tau_c + \beta}$	$2\zeta\tau$	$\frac{\tau}{2\zeta}$
E	$\frac{K}{s}$	$\frac{2}{\tau_c}$	$2\tau_c$	—
F	$\frac{K}{s(\tau s + 1)}$	$\frac{2\tau_c + \tau}{\tau_c^2}$	$2\tau_c + \tau$	$\frac{2\tau_c \tau}{2\tau_c + \tau}$
G	$\frac{Ke^{-\theta s}}{\tau s + 1}$	$\frac{\tau}{\tau_c + \theta}$	τ	—
H	$\frac{Ke^{-\theta s}}{\tau s + 1}$	$\frac{\tau + \frac{\theta}{2}}{\tau_c + \frac{\theta}{2}}$	$\tau + \frac{\theta}{2}$	$\frac{\tau\theta}{2\tau + \theta}$

(Seborg *et al.*, 2011)

- Notice:
  - For all models, K corresponds to the product $K_p K_m K_V$
  - For FOPDT or SOPDT systems with $\theta_p/\tau_p \ll 1$ (lag-dominant system), to avoid a very sluggish response the IMC tuning for regulatory control is changed by letting: $\tau_I = \min[\tau_p; 4(\tau_c + \theta_p)]$

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ANALYTICAL RELATIONS FROM IMC /cont'd

Table 11.1 IMC-Based PID Controller Settings for $G_c(s)$ (Chien and Fruehauf, 1990)

Case	Model	K_cK	τ_I	τ_D
I	$\frac{K(\tau_3 s + 1)e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\tau_1 + \tau_2 - \tau_3}{\tau_c + \theta}$	$\tau_1 + \tau_2 - \tau_3$	$\frac{\tau_1\tau_2 - (\tau_1 + \tau_2 - \tau_3)\tau_3}{\tau_1 + \tau_2 - \tau_3}$
J	$\frac{K(\tau_3 s + 1)e^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{2\zeta\tau - \tau_3}{\tau_c + \theta}$	$2\zeta\tau - \tau_3$	$\frac{\tau^2 - (2\zeta\tau - \tau_3)\tau_3}{2\zeta\tau - \tau_3}$
K	$\frac{K(-\tau_3 s + 1)e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{\tau_1 + \tau_2 + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}{\tau_c + \tau_3 + \theta}$	$\tau_1 + \tau_2 + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}$	$\frac{\tau_3\theta}{\tau_c + \tau_3 + \theta} + \frac{\tau_1\tau_2}{\tau_1 + \tau_2 + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}$
L	$\frac{K(-\tau_3 s + 1)e^{-\theta s}}{\tau^2 s^2 + 2\zeta\tau s + 1}$	$\frac{2\zeta\tau + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}{\tau_c + \tau_3 + \theta}$	$2\zeta\tau + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}$	$\frac{\tau_3\theta}{\tau_c + \tau_3 + \theta} + \frac{\tau^2}{2\zeta\tau + \frac{\tau_3\theta}{\tau_c + \tau_3 + \theta}}$
M	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c + \theta}{(\tau_c + \theta)^2}$	$2\tau_c + \theta$	—
N	$\frac{Ke^{-\theta s}}{s}$	$\frac{2\tau_c + \theta}{\left(\tau_c + \frac{\theta}{2}\right)^2}$	$2\tau_c + \theta$	$\frac{\tau_c\theta + \frac{\theta^2}{4}}{2\tau_c + \theta}$
O	$\frac{Ke^{-\theta s}}{s(\tau s + 1)}$	$\frac{2\tau_c + \tau + \theta}{(\tau_c + \theta)^2}$	$2\tau_c + \tau + \theta$	$\frac{(2\tau_c + \theta)\tau}{2\tau_c + \tau + \theta}$

(Seborg *et al.*, 2011)

Adapted from Barolo M

INTERNAL MODEL CONTROL – CONCLUDING REMARKS

- The IMC method is used **to design** (not tu tune!) a controller $G_c^*(s)$, which is based on a model of the process to be controlled
  - This controller is equivalent to a conventional feedback controller: $G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}}$
- The rationale used to develop the controller is:
  1. A model of the process is first identified (at open-loop)
  2. The $G_c^*(s)$ controller is forced to be the inverse of the invertible part of the model, with the additional superposition of an appropriate filter
  3. The system response trajectory is eventually determined
- In general, the structure of the resulting $G_c(s)$ controller is not a PID
- However, if…
  - The filter is chosen appropriately
  - And the process dynamics is relatively simple (e.g., FOPDT, SOPDT, integrator)

Then the resulting controller is a PID with assigned tuning

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INTERNAL MODEL CONTROL – CONCLUDING REMARKS /cont'd

- Therefore, the IMC method can be also interpreted as a methodology to obtain analytical tuning relations for a PID controller
- Characteristics of the PID_{IMC} tuning:
  - The tuning achieved is usually much more effective than that obtained by DS
  - It depends on the open-loop process parameters $(K_p; \tau_p; \theta_p)$
  - It has a single adjustable parameter (τ_c) instead of three $(K_c; \tau_I; \tau_D)$
  - τ_c has a **physical meaning**
  - By appropriately choosing τ_c, a tradeoff between controller performance and robustness is achieved (aggressiveness vs. conservativeness)
- IMC is among the most employed methods to obtain **analytical tuning relations** for PID controllers
  - In this perspective, we speak about **model-based** PID tuning
  - By appropriately defining the filter, analytical relations can be obtained also for systems with very complex dynamics

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