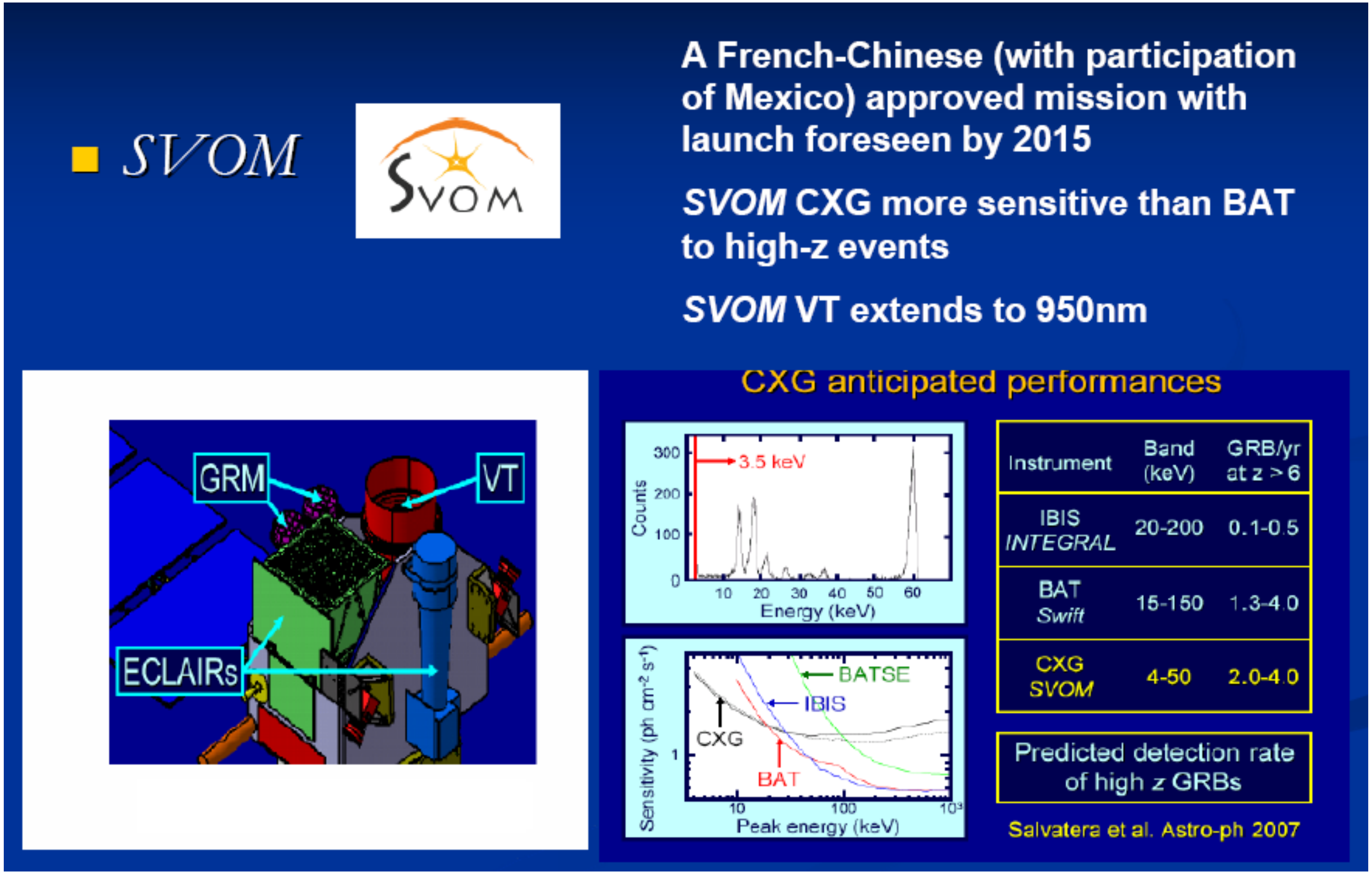
Astrofisica Nucleare e Subnucleare
High Redshift GRBs and GW?

SVOM



- *SVOM*

A French-Chinese (with participation of Mexico) approved mission with launch foreseen by 2015

SVOM CXG more sensitive than BAT to high-z events

SVOM VT extends to 950nm

CXG anticipated performances

Instrument	Band (keV)	GRB/yr at z > 6
IBIS *INTEGRAL*	20-200	0.1-0.5
BAT *Swift*	15-150	1.3-4.0
CXG *SVOM*	4-50	2.0-4.0

Predicted detection rate of high *z* GRBs

Salvatera et al. Astro-ph 2007

Scientific rationale of the SVOM mission

GRB phenomenon
- Diversity and unity of GRBs

GRB physics
- Acceleration and nature of the relativistic jet
- Radiation processes
- The early afterglow and the reverse shock

GRB progenitors
- The GRB-supernova connection
- Short GRB progenitors

- *Cosmology*
  - Cosmological lighthouses (absorption systems)
  - Host galaxies
  - Tracing star formation
  - Re-ionization of the universe
  - Cosmological parameters

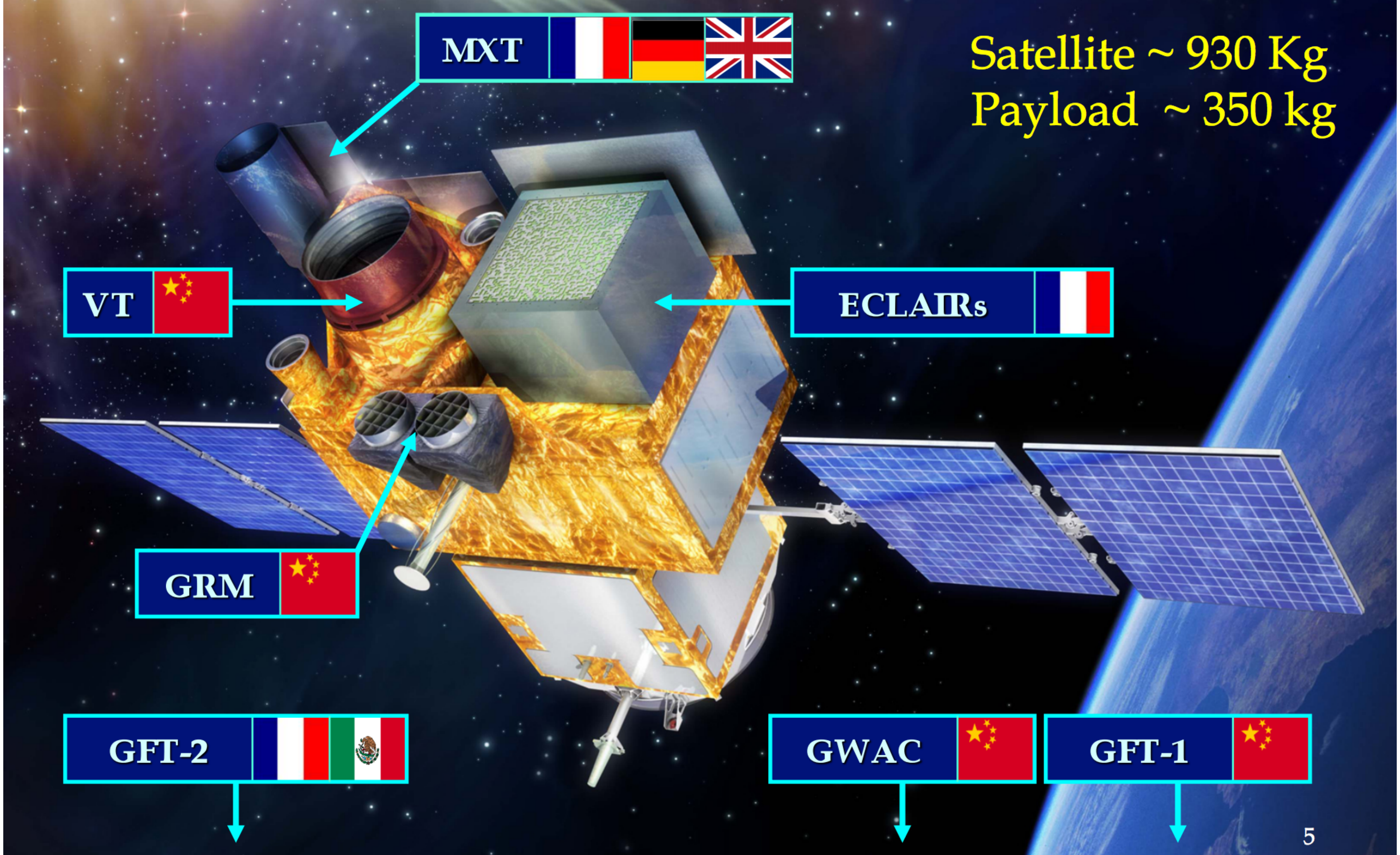
- *Fundamental Physics*
  - Origin of High-Energy Cosmic Rays
  - Probing Lorentz invariance
  - Short GRBs and gravitational waves

SVOM in context at the beginning of the next decade

- SVOM is mini-satellite class mission (< 1000kg)
- SVOM will provide ~80 GRB/yr. It will explore the area of soft GRBs and X-ray Flashes (above 4 keV), and the prompt optical emission with a good sensitivity.
- We aim at measuring the redshift of >50% of SVOM GRBs
- SVOM will operate in the era of advanced GW detectors, providing the opportunity to search for correlations between GW and GRBs
- SVOM GRBs will benefit from follow-up with a new generation of astronomical instruments: JWST, SKA, CTA, LSST, etc.

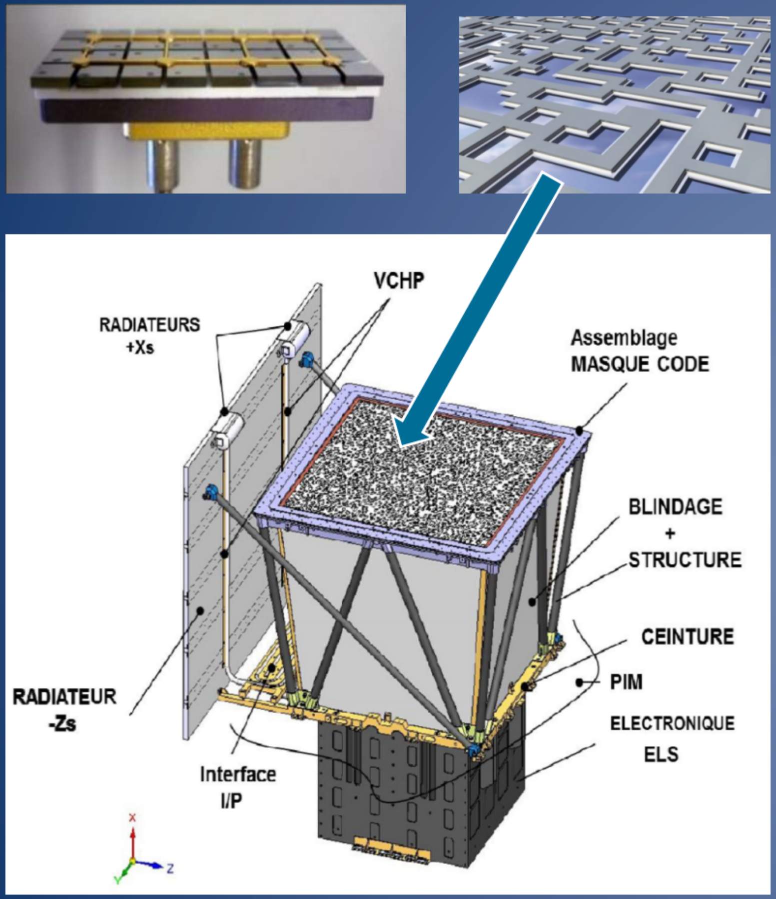
SVOM scientific instrument arrangement

Satellite ~ 930 Kg
Payload ~ 350 kg



Rome - December 2, 2014

In space : ECLAIRs – The trigger camera



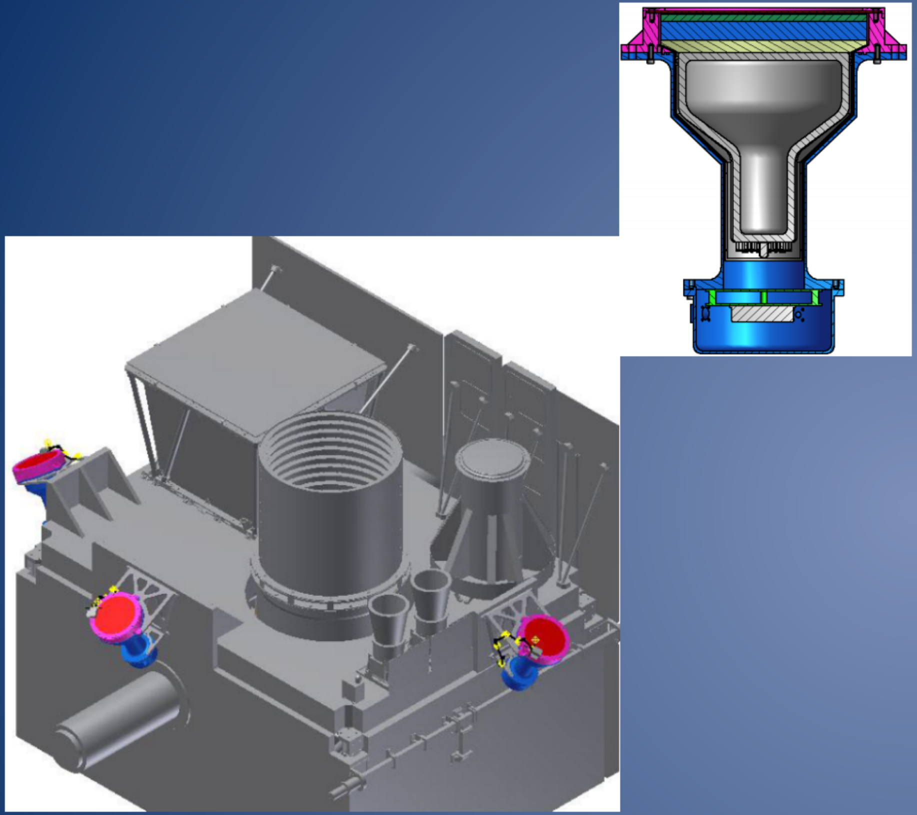
Main characteristics

Coded mask telescope
Wide FOV : 2 Sr
6400 CdTe - 1024 cm^2
4 keV – 150 keV

Anticipated performances

Loc. accuracy < 16 arcmin
4 arcmin for bright bursts
80 GRBs / year

In space : GRM – The Gamma Ray Monitor



Main characteristics

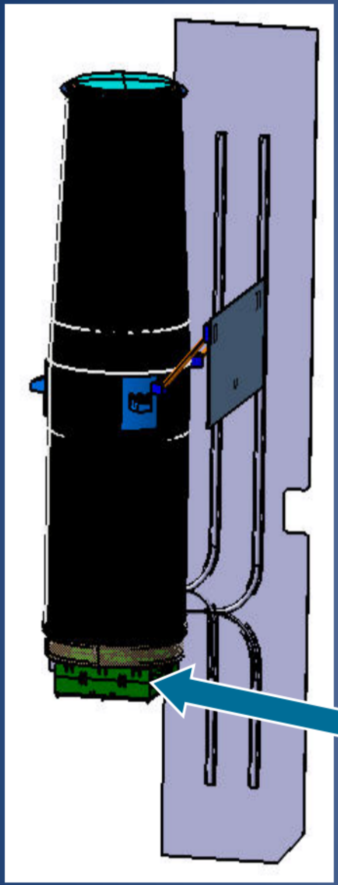
3 NaI detectors, 200 cm^2 each
Thickness 1,5 cm
FOV : 3x2 sr
15 keV – 5 MeV

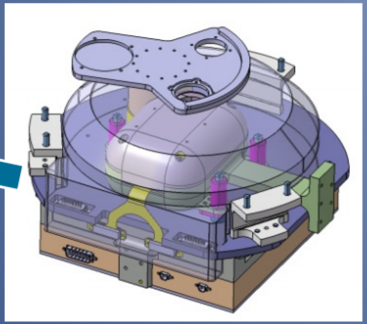
Anticipated performances

Loc. accuracy ~ 15 x15°(in 2.6 sr)
110 GRBs/yr

Enlargement of the FoV to enhance the detection rate of short bursts (which are expected to be candidate sources of GW bursts)

In space : MXT – The Multi-channel X-ray Telescope





Main characteristics

MCP "Lobster eyes" X-ray optic
FOV ~ 1 deg^2
256 x256 PN CCD
0.2 keV – 10 keV

Anticipated performances

~50 cm^2 at 1 keV
Loc. accuracy < 1 arcmin
20 arcsec for bright GRB
~ 70 GRBs/yr

SVOM unique capabilities for GRB studies

- Low energy threshold at 4 keV to detect soft GRBs
- Measure of GRB prompt emission over 6 decades in energy, from 1 to ~10^6 eV.
- Good sensitivity to short GRBs with GRM and ECLAIRs (soft bump)
- Many consecutive orbits with the same pointing, allowing the detection of hour long transients, like the 15000 sec long GRB 111209A at z=0.677
- Good sensitivity of VT, providing accurate GRB positions for >70% of the bursts. Dedicated NIR & vis. ground follow-up telescopes increase this fraction to >80%
- Large fraction of the afterglows seen by both MXT and VT.
- GRBs well located for ground-based follow-up

SVOM and highly redshifted GRBs

at the beginning of the next decade

- We expect to detect ~5 GRBs/yr at redshift $z>5$ with ECLAIRs.

- We aim to quickly identify high-z GRBs, thanks to the pointing strategy of SVOM, the sensitivity of VT, and fast NIR follow-up on the ground.

- This strategy will permit to set up an efficient Follow-up Program performing the optical spectroscopy of most of highly redshifted afterglows, allowing crucial scientific studies.

- Highly redshifted GRBs allow studying the young universe:
  - Gas and dust in young galaxies
  - Reionization of the IGM → GRB II session, Wednesday
  - Star formation rate
  - Search for GRBs from Population III stars (challenging)
    (rare, energetic, possibly very long like GRB111209A, with no detectable host)

SVOM and Gravity Waves

at the beginning of the next decade

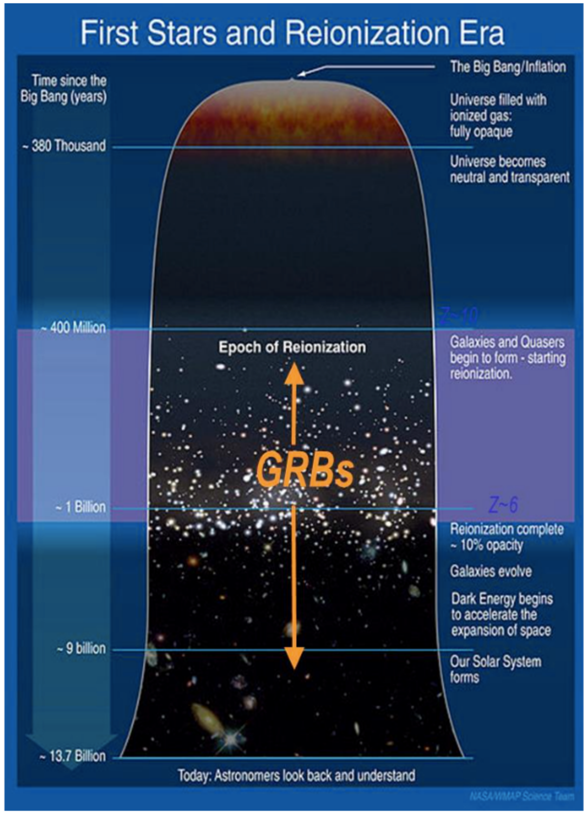
- Coordinated searches of GWs and short GRBs may confirm or dismiss the favorite scenario for short GRBs: the coalescence of two compact objects
- From 2023 the Size of the GW error boxes will be several degrees^2
- **Coincident events:** within the horizon of GW detectors (~400 Mpc), with assumption of 50 BNS/yr, we expect in 5 years of operation.
  - ~3 events in ECLAIRs FOV
  - ~9 events in GRM FOV
- **Follow-up:** within the same assumptions, we expect ~15 events in 5 years of operation that can be followed quickly with SVOM instruments, and particularly with the MXT (<6hours) and with the GWACs

SVOM

https://svom.cnes.fr/en/SVOM/index.htm
http://irfu.cea.fr/Projets/SVOM/svom.html

THESEUS

Gamma-Ray Bursts are the most luminous and remote phenomena in the Universe, with isotropic-equivalent radiated energies in X-gamma rays up to more than ***10^{54} erg released in a few tens of seconds*** *and a redshift distribution extending to at least* ***z = 9-10****. Thus, they are in principle very powerful tools for cosmology*



Two flavours of GRB cosmology:

- ❑ using GRBs to investigate the expansion rate and geometry of the Universe, thus getting clues to "dark energy" properties and evolution
- ❑ GRBs as tools for exploring the early Universe at the end of the "dark ages" (re-ionization, first stars, star formation rate and metallicity evolution in the first billion of years) –> The THESEUS mission concept

THESEUS

THESEUS Requirements

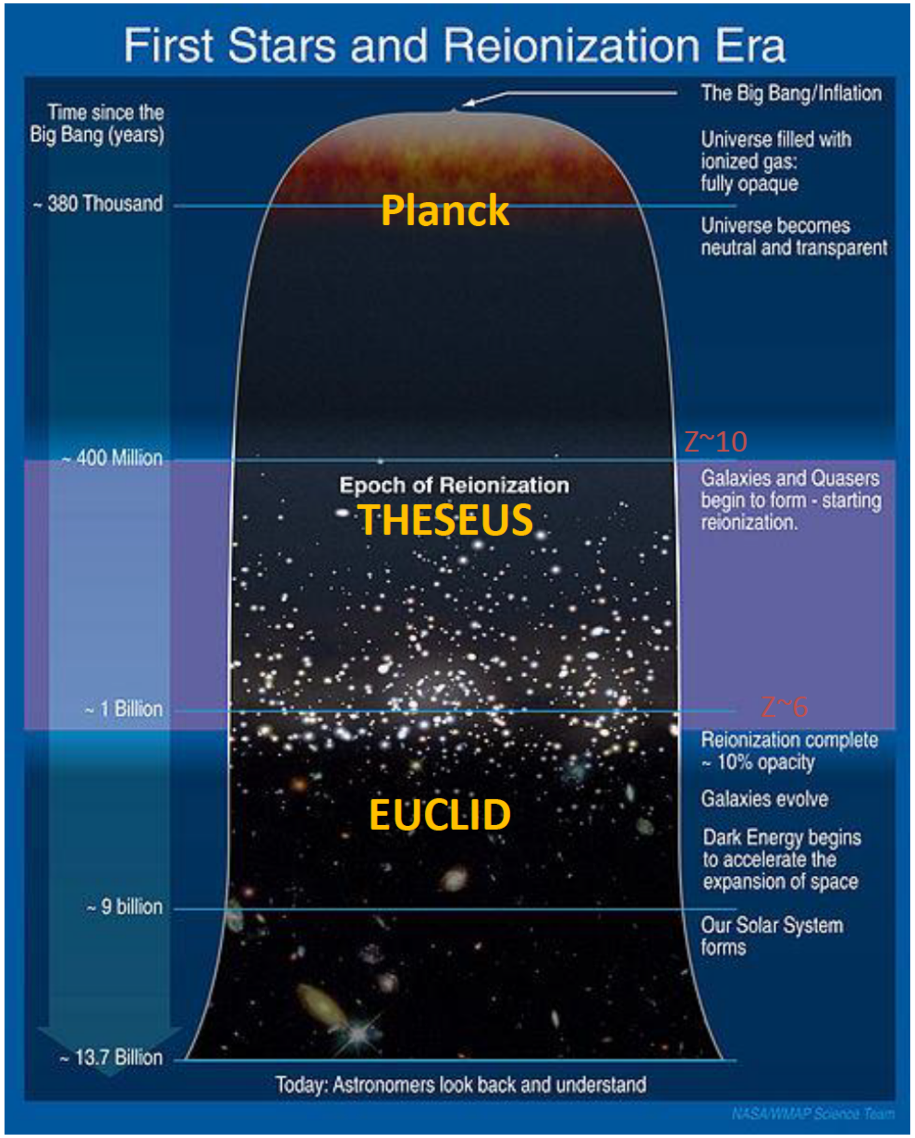
- A full exploration of the early Universe requires the detection of a factor 10 more GRBs (about 80-100) than currently available at z>6
- As supported by intensive simulation efforts (e.g. Ghirlanda+15 MNRAS) a high detection rate of high redshift GRBs requires a *soft* **and** *sensitive* (down to 10^{-9} erg/cm^2/s) *wide field* high-energy trigger, with precise and reliable localization techniques (< 2 arc min)
- In order to efficiently classify and filter the trigger a broad band spectral coverage is needed at high energies
- In order to *identify, classify and study* the high-z GRB counterparts, an near-infrared (due to cosmological Ly-alpha suppression) telescope is needed on board. It will provide accurate positions, GRB redshifts, and GRB afterglows *spectra* (R~1000).
- An agile and autonomous platform (Swift-like) is required in order to point at the GRB position *quickly;* In addition the GRB information has to be down-linked to Earth for rapid follow-up by large ground based facilities.

THESEUS: Main scientific goals

A) Exploring the Early Universe (cosmic dawn and reionization era) by unveiling the Gamma-Ray Burst (GRBs) population in the first billion years

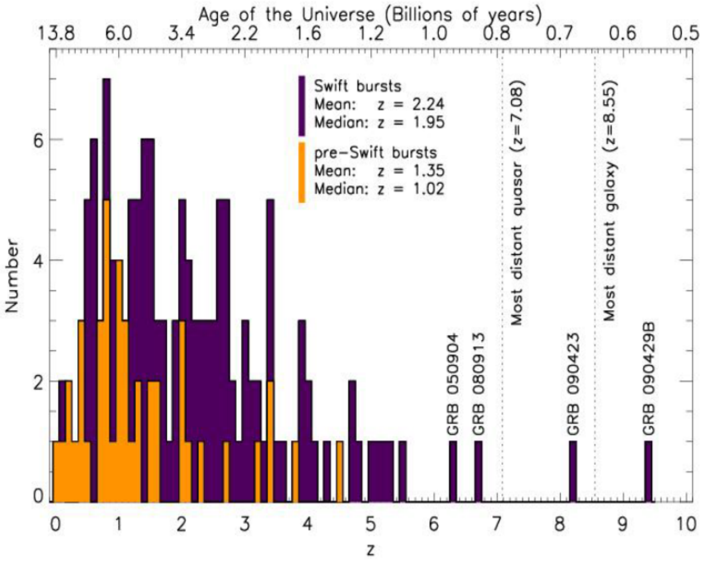
The study of the Universe before and during the epoch of reionization represents one of the major themes for the next generation of space and ground–based observational facilities. Many questions about the first phases of structure formation in the early Universe will still be open in the late 2020s:

- ***When and how did first stars/galaxies form?***
- ***What are their properties? When and how fast was the Universe enriched with metals?***
- ***How did reionization proceed?***

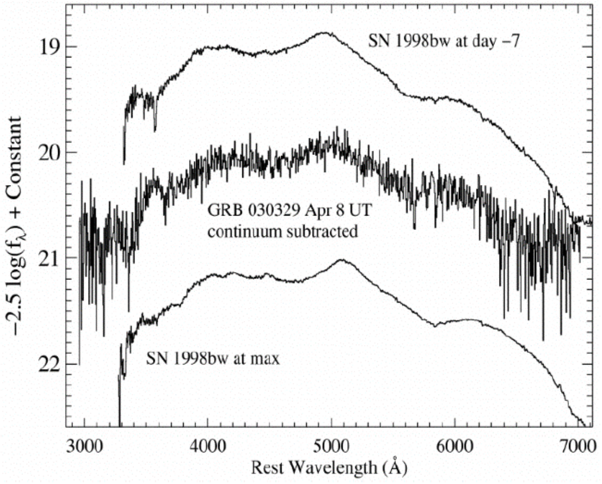


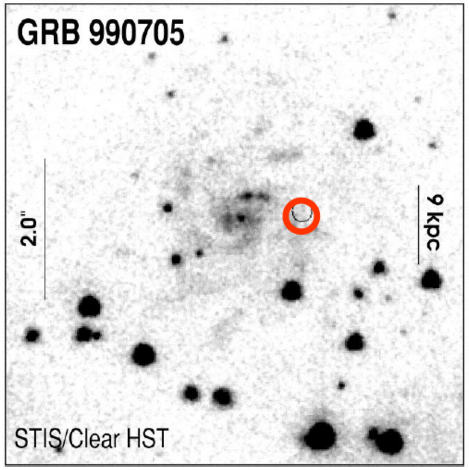
Shedding light on the early Universe with GRBs

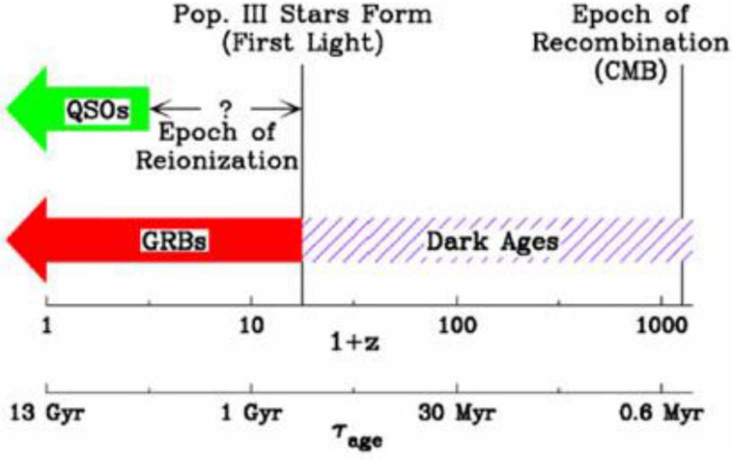
Because of their huge luminosities, mostly emitted in the X and gamma-rays, their redshift distribution extending at least to z ~9 and their association with explosive death of massive stars and star forming regions, GRBs are unique and powerful tools for investigating the early Universe: **SFR evolution, physics of re-ionization, galaxies metallicity evolution and luminosity function, first generation (pop III) stars**



GRBs in Cosmological Context



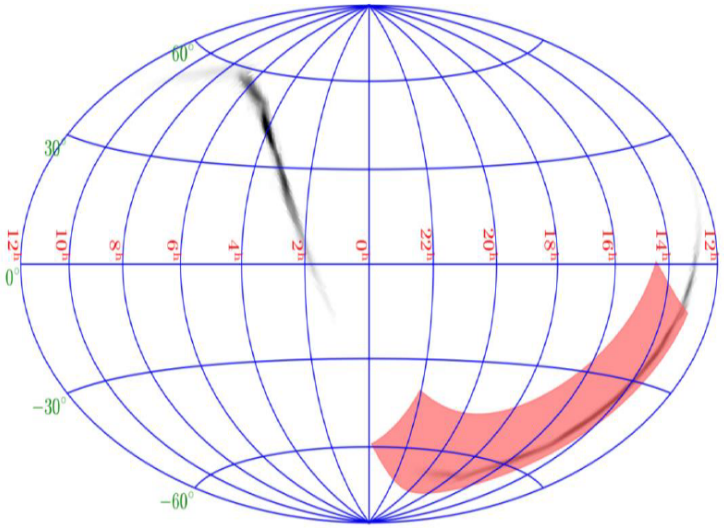




Lamb and Reichart (2000)

B) Perform an unprecedented deep monitoring of the soft X-ray transient Universe in order to:

- **Locate and identify the electromagnetic counterparts to sources of gravitational radiation and neutrinos, which may be routinely detected in the late '20s / early '30s by next generation facilities like aLIGO/aVirgo, eLISA, ET, or Km3NET;**



- Provide real-time triggers and accurate (~1 arcmin within a few seconds; ~1'' within a few minutes) **high-energy transients for follow-up with next-generation optical-NIR (E-ELT, JWST if still operating), radio (SKA), X-rays (ATHENA), TeV (CTA) telescopes; synergy with LSST**

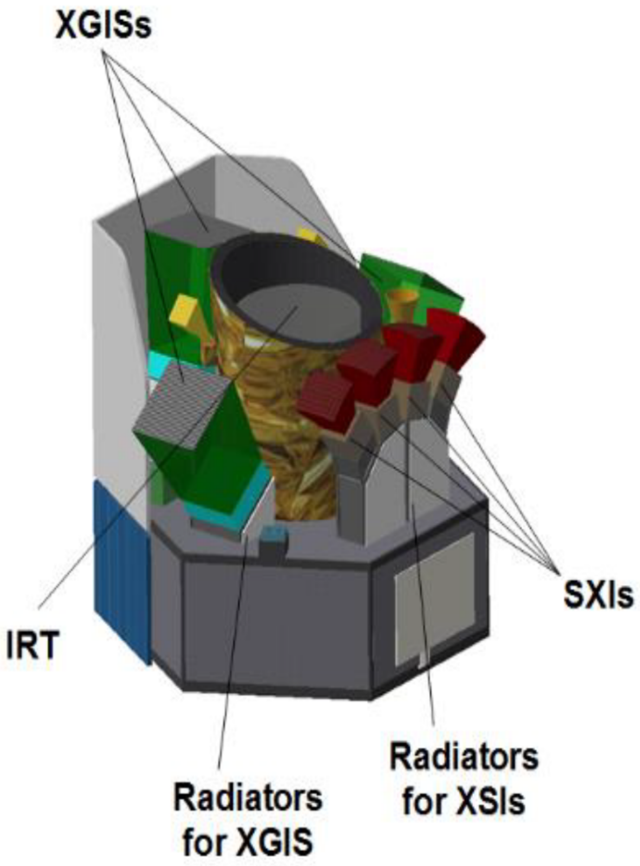
- Provide a fundamental step forward in the comprehension of the physics of various classes of transients and **fill the present gap in the discovery space of new classes of transients events**

Transient type	**SXI Rate**
GW sources	0.03-33 yr^1
SN shock breakout	4 yr^{-1}
Tidal Disruptions Events	50 yr^{-1}
Thermonuclear bursts	35 day^{-1}
Novae	250 yr^{-1}
Dwarf novae	30 day^{-1}
Stellar flares	400 yr^{-1}
Stellar super flares	200 yr^{-1}

probe GRB physics

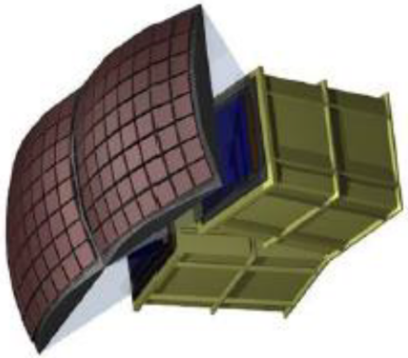
THESEUS payload

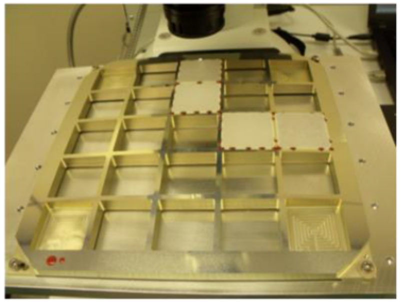
- **Soft X-ray Imager (SXI):** a set of four sensitive lobster-eye telescopes observing in 0.3 - 5 keV band, total FOV of ~1sr with source location accuracy < 1-2’;
- **X-Gamma rays Imaging Spectrometer (XGIS,)**: 3 coded-mask X-gamma ray cameras using bars of Silicon diodes coupled with CsI crystal scintillators observing in 2 keV – 10 MeV band, a FOV of ~1sr, overlapping the SXI, with ~5’ source location accuracy;
- **InfraRed Telescope (IRT)**: a 0.7m class IR telescope observing in the 0.7 – 1.8 μm band, providing a 10’x10’ FOV, with both imaging and moderate resolution spectroscopy capabilities

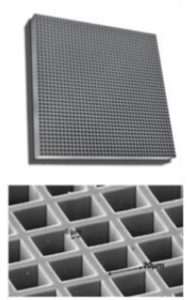


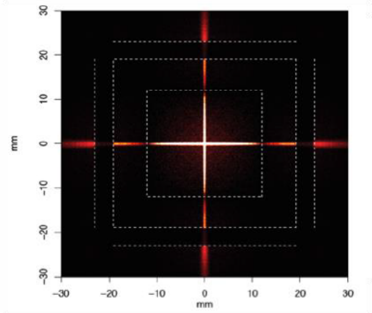
LEO (< 5°, ~600 km)
Rapid slewing bus
Prompt downlink

The Soft X-ray Imager (SXI)









4 DUs, each has a 31 x 26 degree FoV

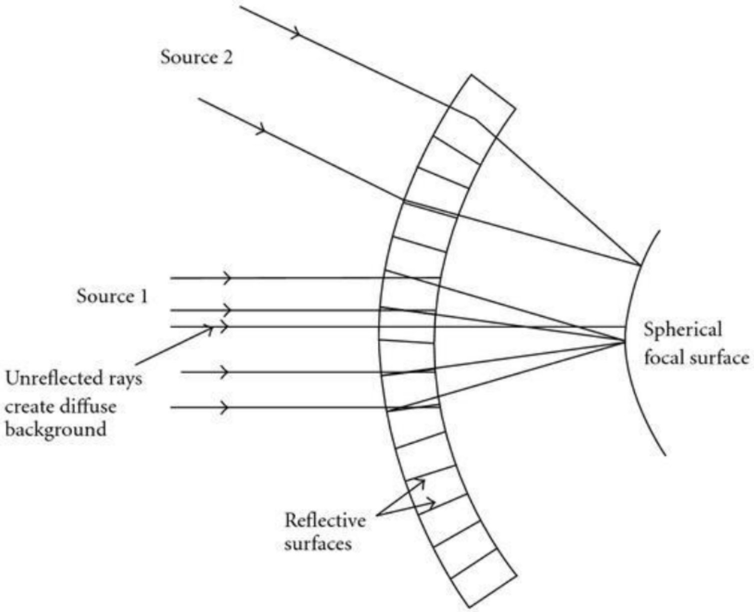
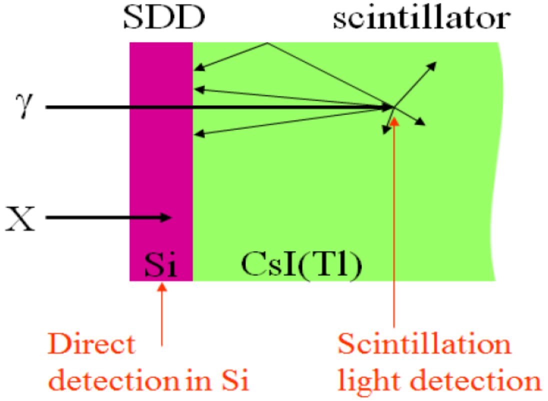
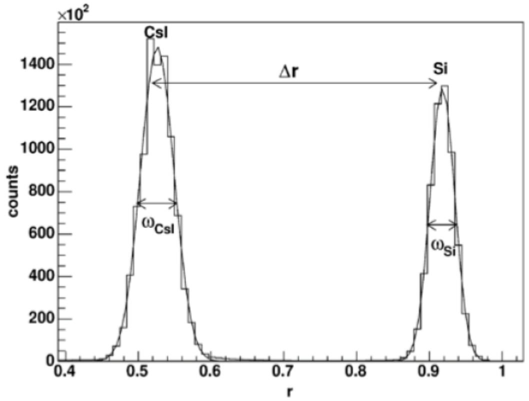


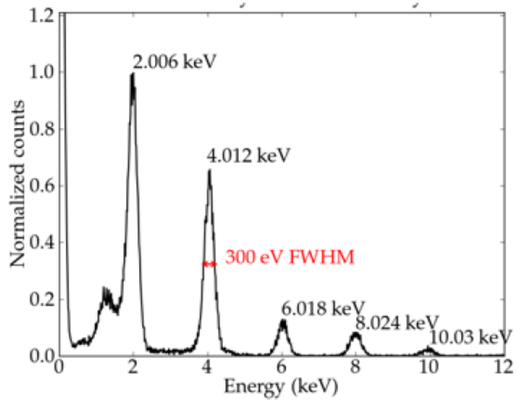
Table 4 : : SXI detector unit main physical characteristics

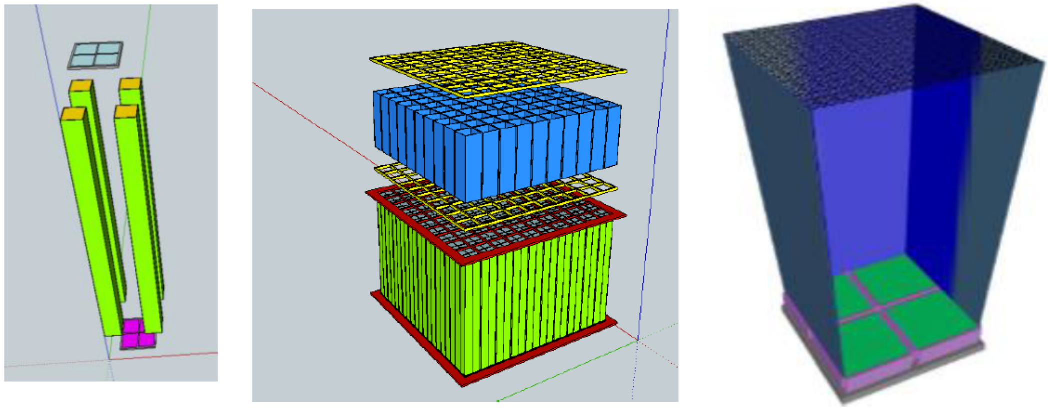
Energy band (keV)	0.3-5
Telescope type:	Lobster eye
Optics aperture (mm2)	320x320
Optics configuration	8x8 square pore MCPs
MCP size (mm2)	40x40
Focal length (mm)	300
Focal plane shape	spherical
Focal plane detectors	CCD array
Size of each CCD (mm2)	81.2x67.7
Pixel size (µm)	18
Pixel Number	4510 x 3758 per CCD
Number of CCDs	4
Field of View (square deg)	~1sr
Angular accuracy (best, worst) (arcsec)	(<10, 105)
Power [W]	27,8
Mass [kg]	40

The X-Gamma-rays spectrometer (XGS)





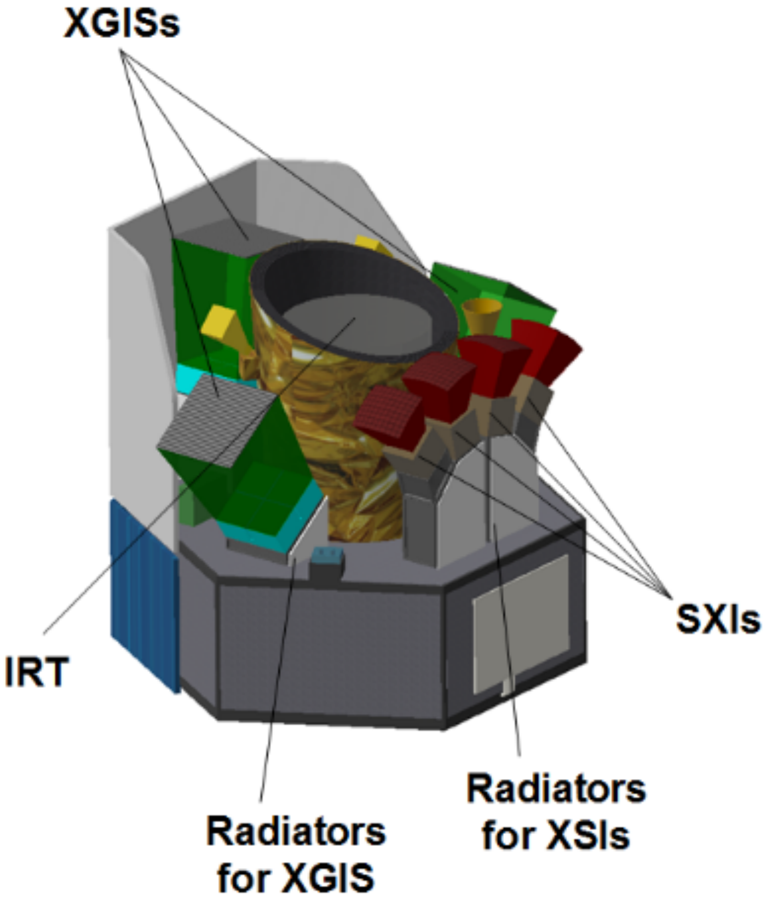




Energy band	*2 keV – 20 MeV*
# detection plane modules	*4*
# of detector pixel / module	*32x32*
pixel size (= mask element size)	*5x5 mm*
Low-energy detector (2-30 keV)	*Silicon Drift Detector 450 μm thick*
High energy detector (> 30 keV)	*CsI(Tl) (3 cm thick)*
Discrimination Si/CsI(Tl) detection	*Pulse shape analysis*
Dimension [cm]	*50x50x85*
Power [W]	*30,0*
Mass [kg]	*37.3*

	2-30 keV	30-150 keV	>150 keV
Fully coded FOV	9 x 9 deg²		
Half sens. FOV	50 x 50 deg²	50 x 50 deg² (FWHM)	
Total FOV	64 x 64 deg²	85 x 85 deg² (FWZR)	2π sr
Ang. res	25 arcmin		
Source location accuracy	~5 arcmin (for >6 σ source)		
Energy res	200 eV FWHM @ 6 keV	18 % FWHM @ 60 keV	6 % FWHM @ 500 keV
Timing res.	1 μsec	1 μsec	1 μsec

THESEUS



https://www.isdc.unige.ch/theseus/

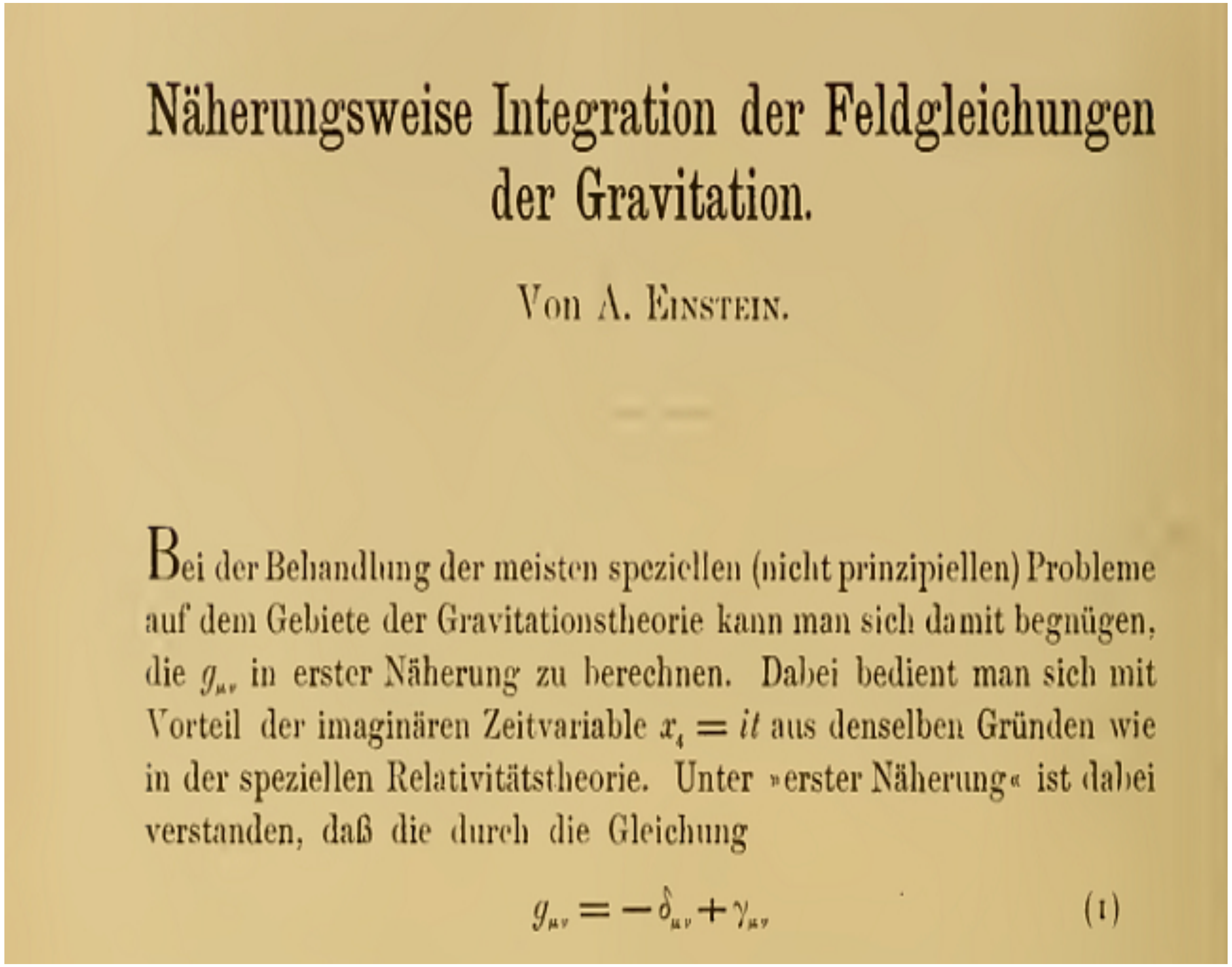
Astrofisica Nucleare e Subnucleare

Gravitational Waves

Exercise #3

- Find recent information on the status of LIGO and Virgo
- Find the status of eLISA GW observatory

Le onde gravitazionali?



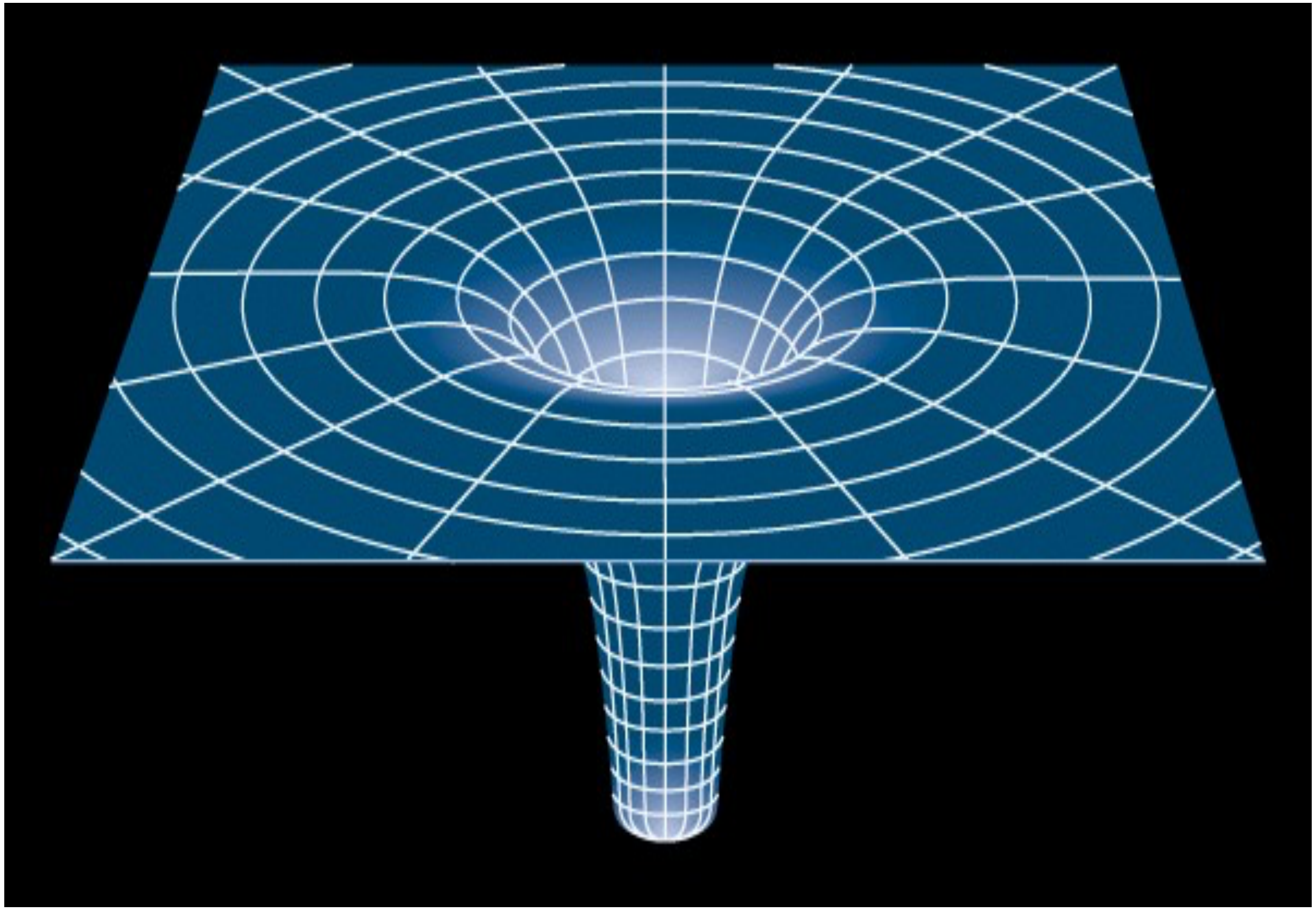
Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

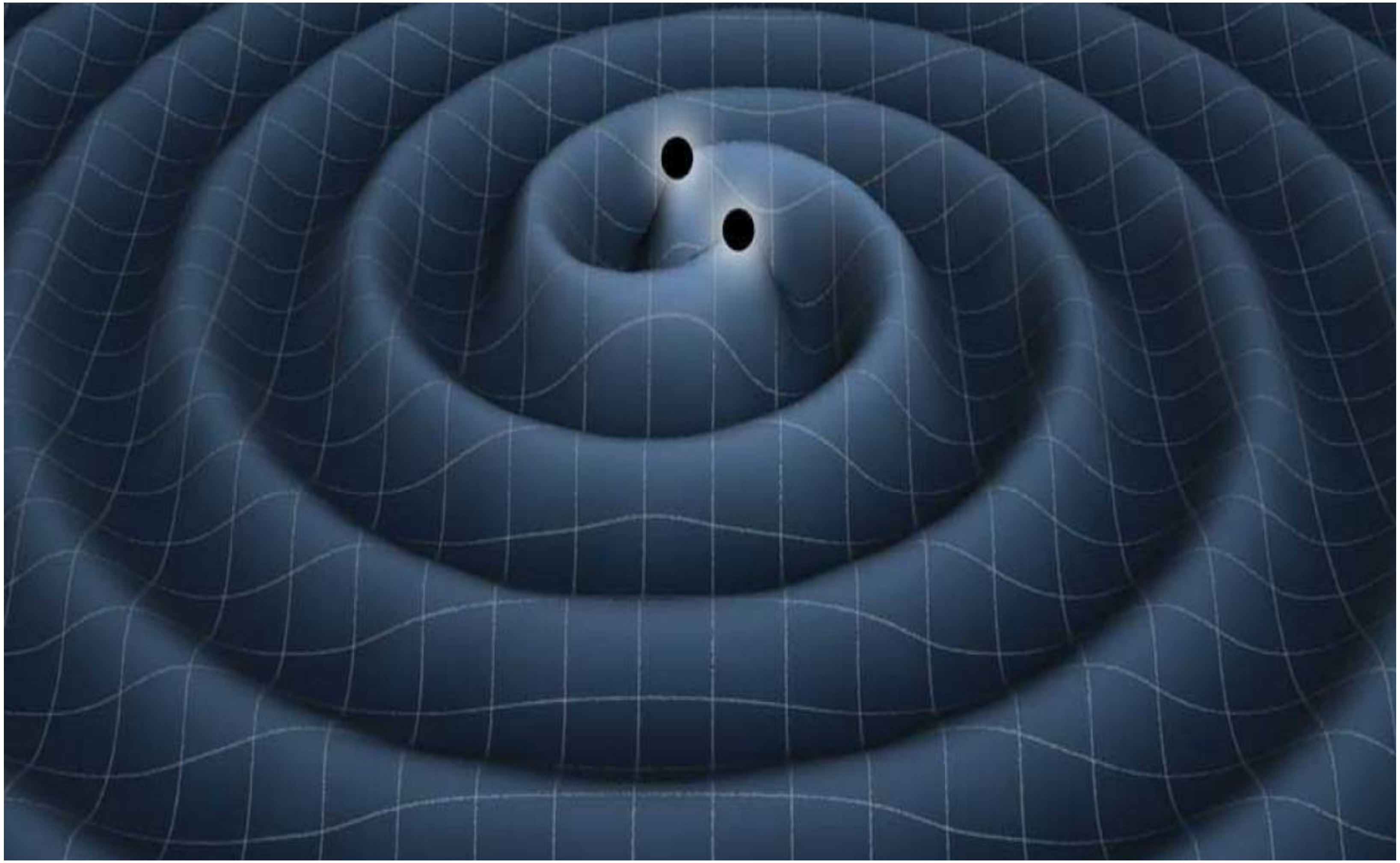
Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

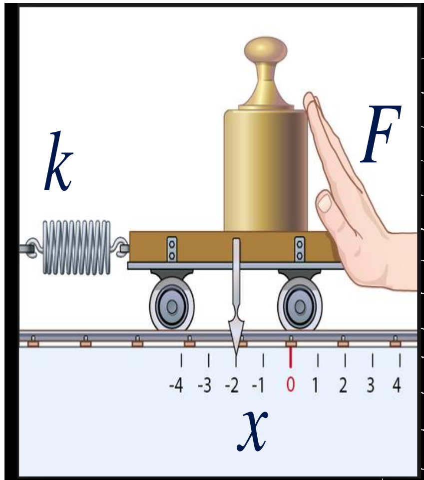
$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

Lo spazio-tempo

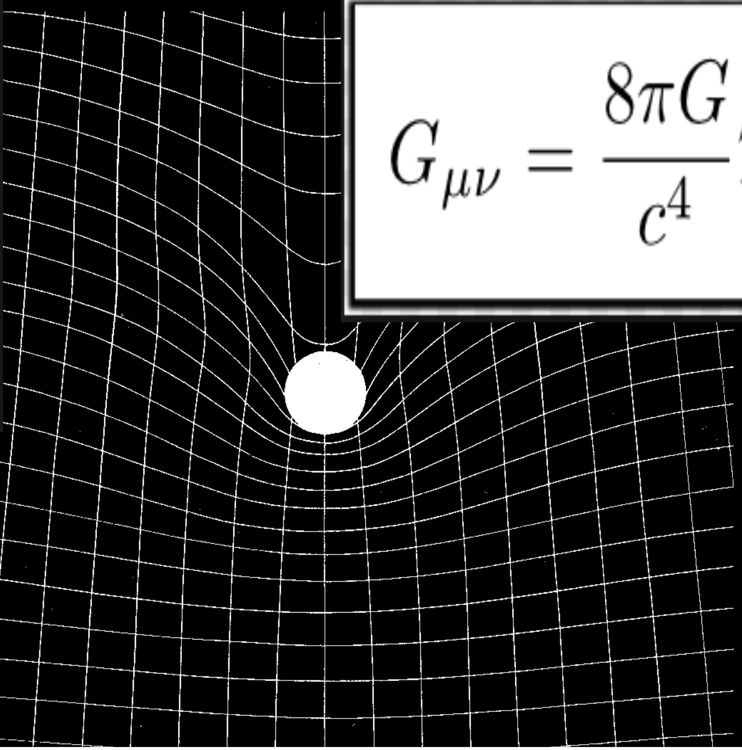


Le onde gravitazionali?





$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



$$F = -kx$$

$$F \Leftrightarrow T_{\mu\nu}$$

$$x \Leftrightarrow G_{\mu\nu}$$

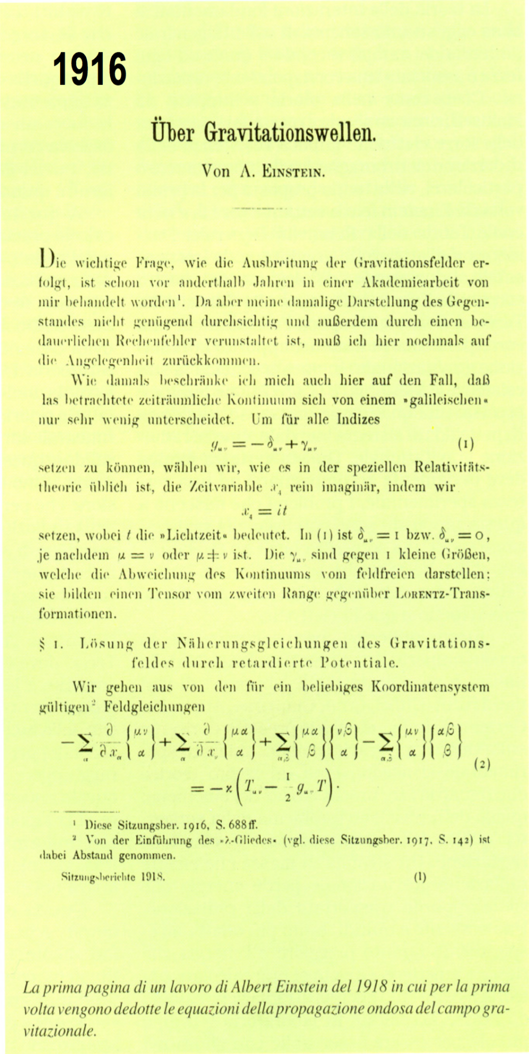
$$k \Leftrightarrow \frac{c^4}{8\pi G}$$

$$c = 299\ 792\ 458\ m/s = 3 \times 10^8\ m/s$$

$$G = 0{,}000\ 000\ 000\ 066\ 7\ \frac{m^3}{kg\ s^2} = 6{,}67 \times 10^{-11}\ \frac{m^3}{kg\ s^2}$$

$k \approx 10^{43}\ kg\ s^{-2}$ STIFF!

Onde gravitazionali



1916

Über Gravitationswellen.

Von A. Einstein.

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem »galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

setzen zu können, wählen wir, wie es in der speziellen Relativitätstheorie üblich ist, die Zeitvariable x_4 rein imaginär, indem wir

$$x_4 = it$$

setzen, wobei t die »Lichtzeit« bedeutet. In (1) ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$ ist. Die $\gamma_{\mu\nu}$ sind gegen 1 kleine Größen, welche die Abweichung des Kontinuums vom feldfreien darstellen; sie bilden einen Tensor vom zweiten Range gegenüber Lorentz-Transformationen.

§ 1. Lösung der Näherungsgleichungen des Gravitationsfeldes durch retardierte Potentiale.

Wir gehen aus von den für ein beliebiges Koordinatensystem gültigen[2] Feldgleichungen

$$-\sum_{\alpha} \frac{\partial}{\partial x_\alpha} \begin{Bmatrix} \mu\nu \\ \alpha \end{Bmatrix} + \sum_{\alpha} \frac{\partial}{\partial x_\nu} \begin{Bmatrix} \mu\alpha \\ \alpha \end{Bmatrix} + \sum_{\alpha\beta} \begin{Bmatrix} \mu\alpha \\ \beta \end{Bmatrix} \begin{Bmatrix} \nu\beta \\ \alpha \end{Bmatrix} - \sum_{\alpha\beta} \begin{Bmatrix} \mu\nu \\ \alpha \end{Bmatrix} \begin{Bmatrix} \alpha\beta \\ \beta \end{Bmatrix}$$
$$= -\varkappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \qquad (2)$$

---

[1] Diese Sitzungsber. 1916, S. 688ff.

[2] Von der Einführung des »λ-Gliedes« (vgl. diese Sitzungsber. 1917, S. 142) ist dabei Abstand genommen.

La prima pagina di un lavoro di Albert Einstein del 1918 in cui per la prima volta vengono dedotte le equazioni della propagazione ondosa del campo gravitazionale.

Weak field approximation

$$g_{\mu\nu} = g^{o}_{\mu\nu} + h_{\mu\nu}$$

$$|h_{\mu\nu}| << 1$$

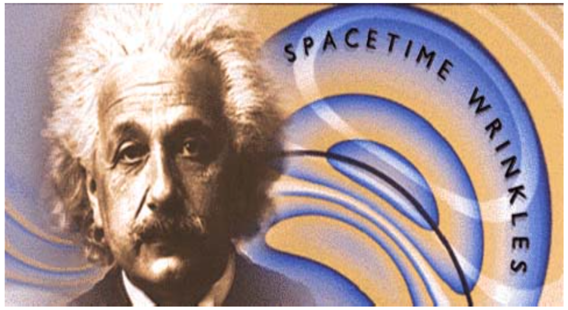
The Einstein equation in vacuum becomes

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})h_{\mu\nu} = 0$$

Having solutions

$$h_{\mu\nu}(t - x/c)$$

Spacetime perturbations, propagating in vacuum like waves, at the speed of light : gravitational waves



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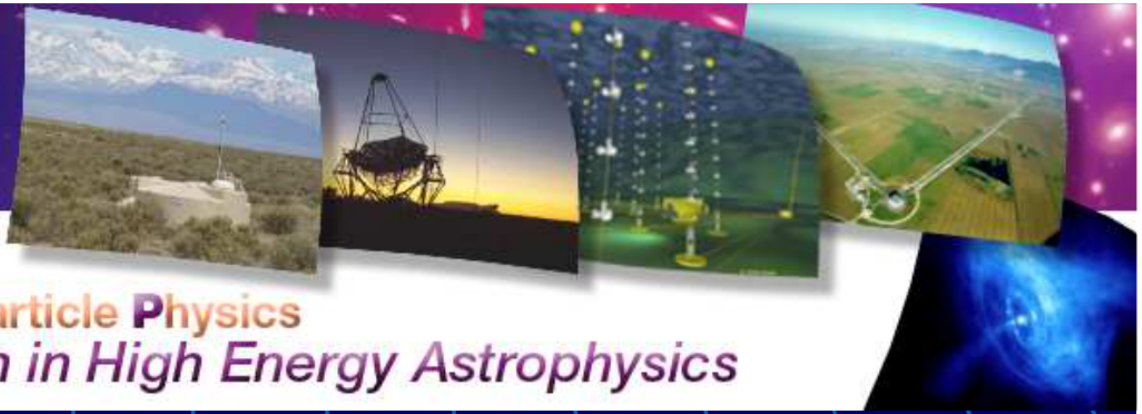
European Doctorate School



International School of AstroParticle Physics
Multi-Messenger Approach in High Energy Astrophysics

Gravitational Waves Detection And Fourier Methods

ISAPP2012
Paris, France, July 2012

Patrice Hello

Laboratoire de l'Accélérateur Linéaire
Orsay-France

VIRGO

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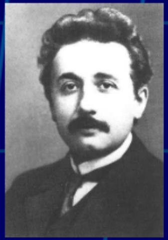
What are Gravitational Waves ?

Gravitational Waves (GW) are ripples of space-time

Theory of GW :

1. Einstein equations: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$
2. Far from sources: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$
3. Linearization: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
4. Gauge TT: $\nabla^2 h_{\mu\nu}^{TT} = 0$

Propagation of some tensor field – *h* - on flat space-time



Prediction in 1916 !

Gravitational Wave general properties

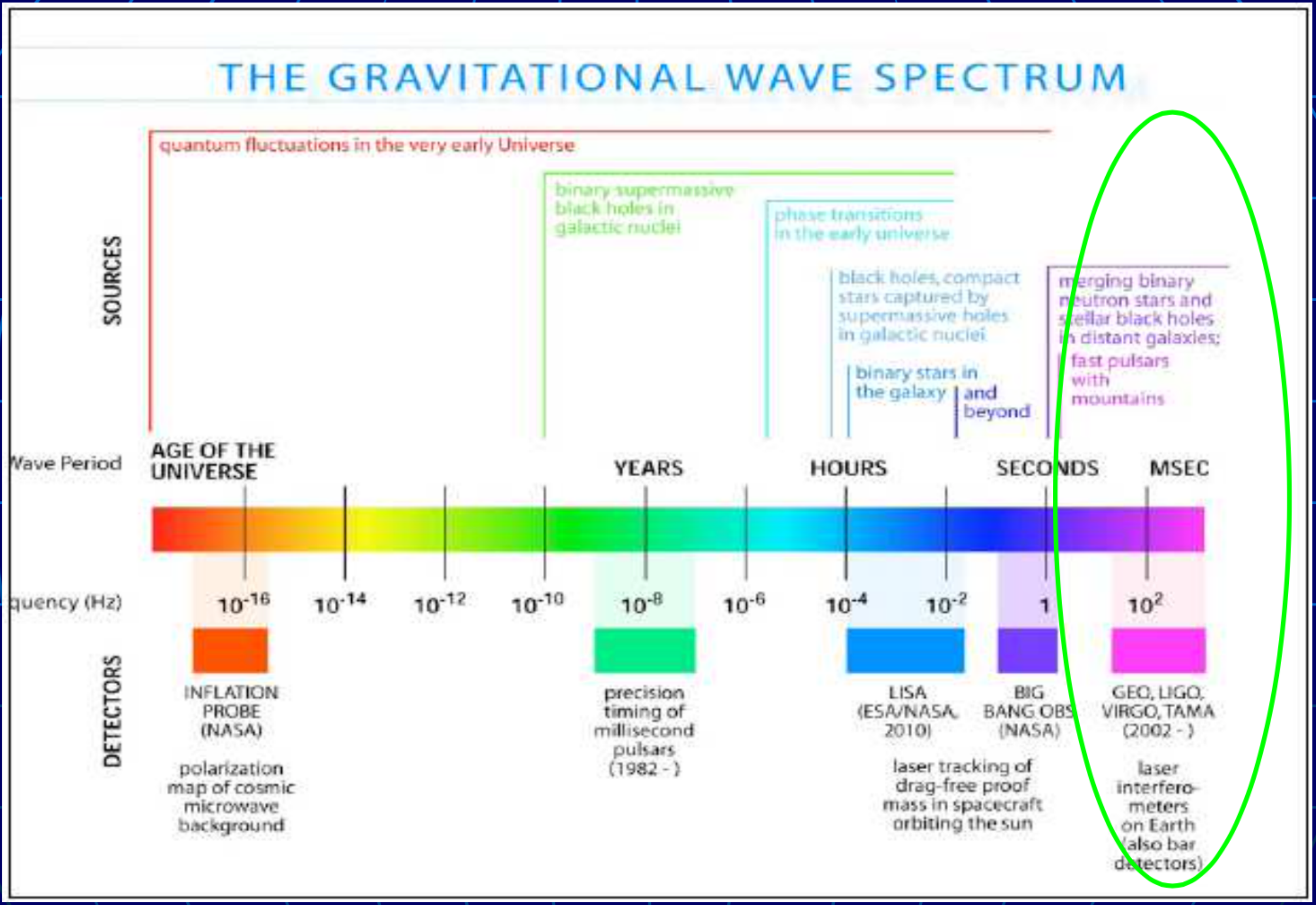
- GW propagate at speed of light
- GW have two polarizations "+" and "x"
- GW emission is quadrupolar at lowest order

Example: plane wave propagating along z axis with 2 polarization amplitudes h_+ and $h_\times$:

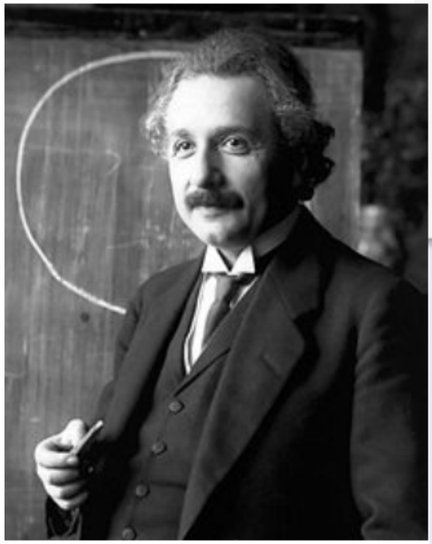
$$h_{\mu\nu}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Corresponding *Graviton* properties:

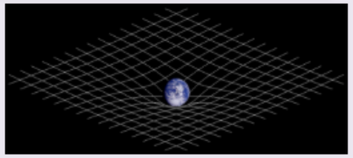
- Graviton has null mass
- Graviton has spin 2


THE GRAVITATIONAL WAVE SPECTRUM
SOURCES
quantum fluctuations in the very early Universe
binary supermassive black holes in galactic nuclei
phase transitions in the early universe
black holes, compact stars captured by supermassive holes in galactic nuclei
binary stars in the galaxy
and beyond
merging binary neutron stars and stellar black holes in distant galaxies;
fast pulsars with mountains
Wave Period
AGE OF THE UNIVERSE
YEARS
HOURS
SECONDS
MSEC
quency (Hz)
10-16
10-14
10-12
10-10
10-8
10-6
10-4
10-2
1
102
DETECTORS
INFLATION PROBE (NASA)
polarization map of cosmic microwave background
precision timing of millisecond pulsars (1982 -)
LISA (ESA/NASA, 2010)
laser tracking of drag-free proof mass in spacecraft orbiting the sun
BIG BANG OBS (NASA)
GEO, LIGO, VIRGO, TAMA (2002 -)
laser interferometers on Earth (also bar detectors)

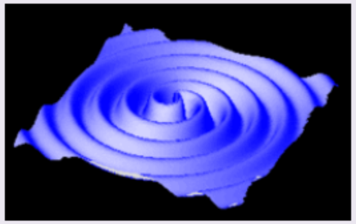
To reiterate:



1916 → Einstein's theory of relativity predicts the existence of a new type of wave: the gravitational waves



GWs are perturbations of the space-time metric:



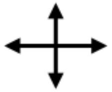
- ➢ Generated by mass distributions with time-varying quadrupole moments
- ➢ Propagating at the speed of light
- ➢ Change in the distance between stationary (inertial) masses

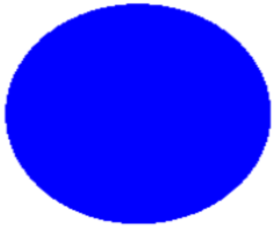
According to GR, GWs have two independent polarization states

Each GW signal can be described as a linear combination of them:

$$h = A_+ h_+(t) + A_x h_x(t)$$

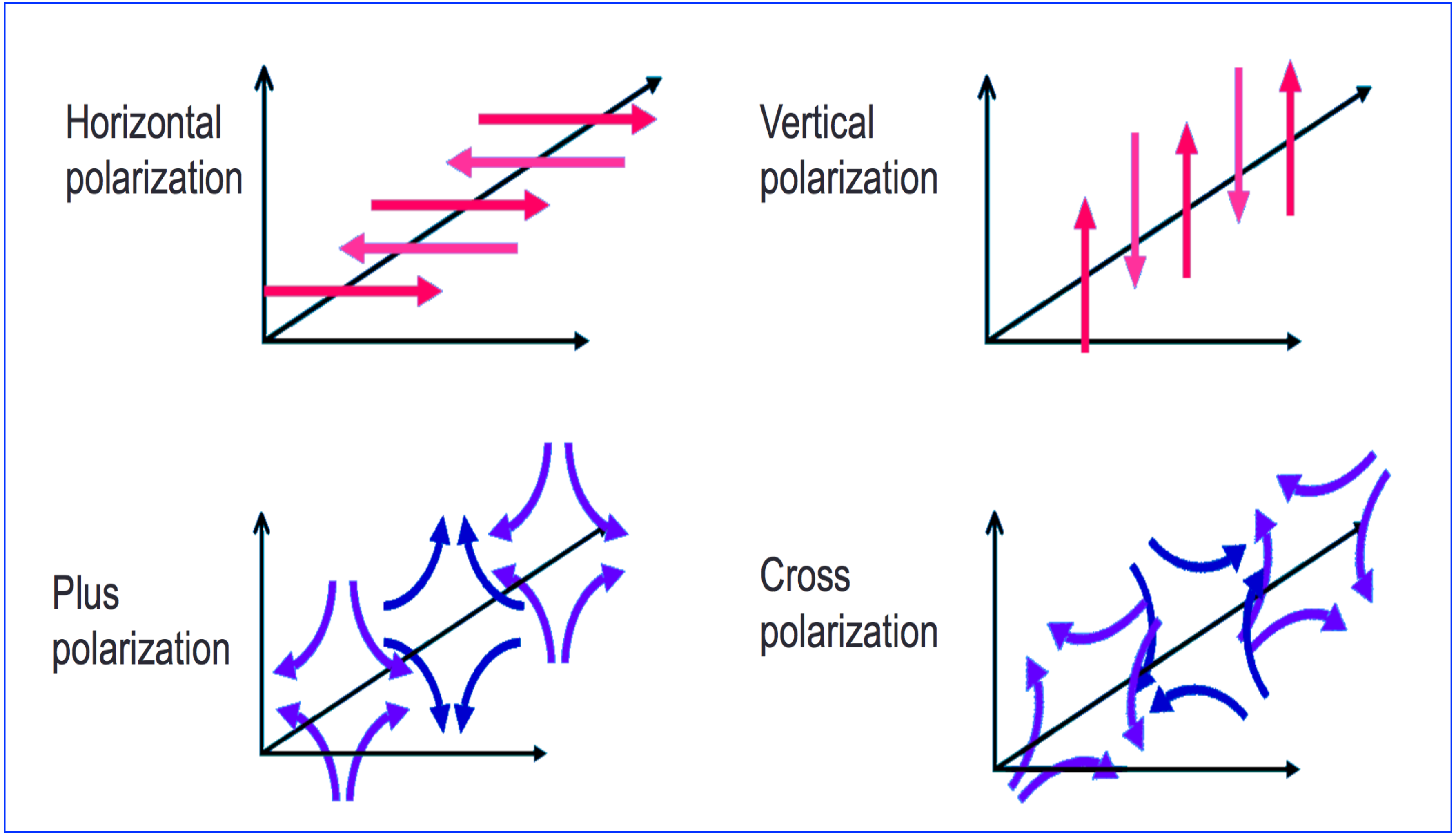
Polarization "Plus" h_+



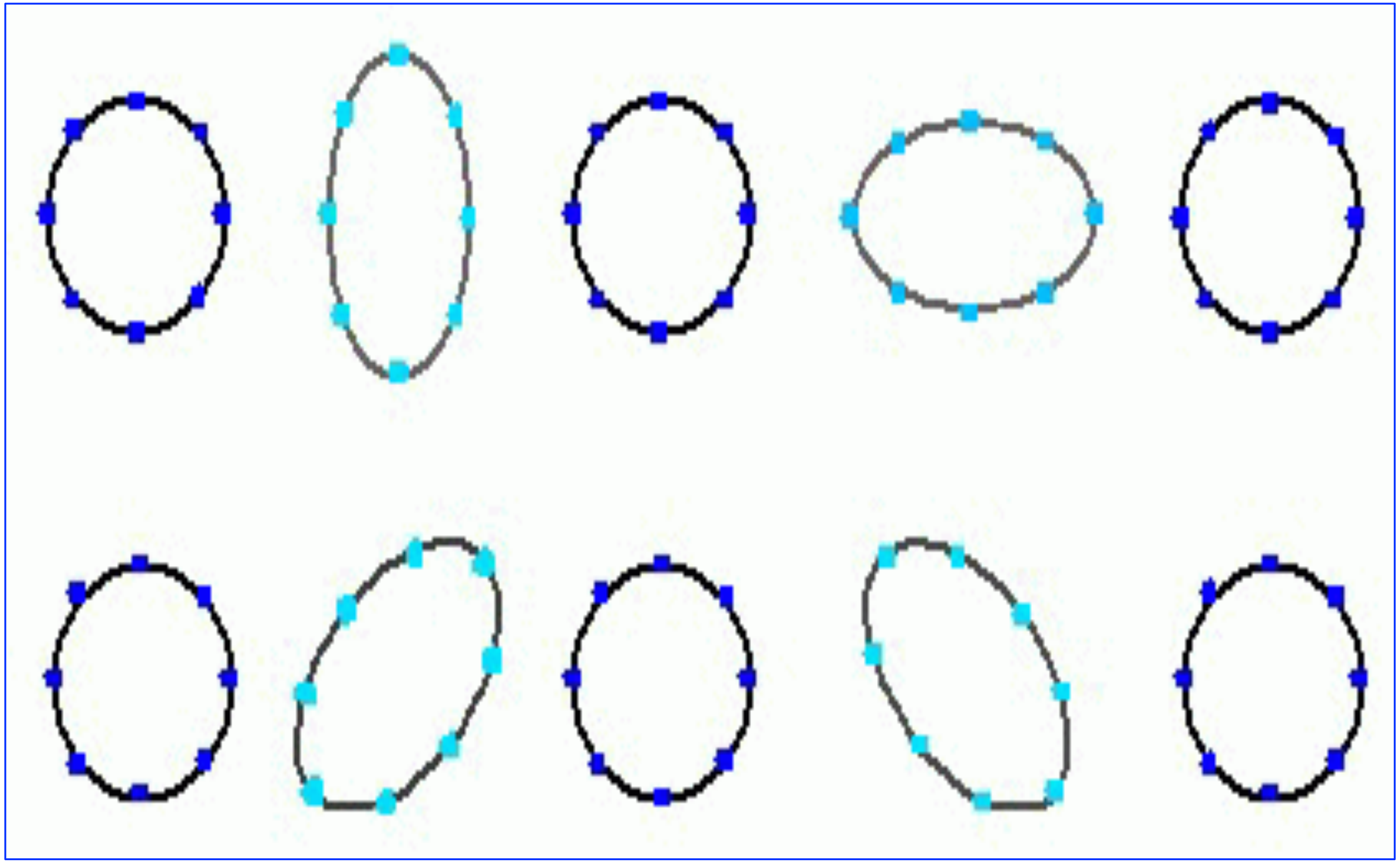


"Cross" h_x

Onde gravitazionali



Onde gravitazionali



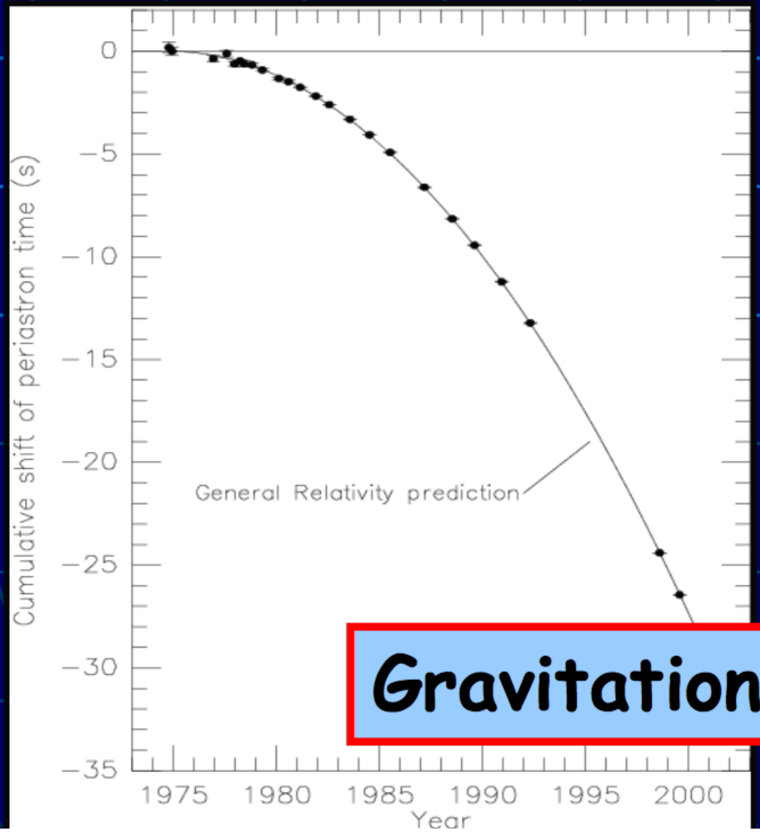
GW (indirect) discovery PSR 1913+16

(Hulse & Taylor, Nobel'93)

PSR 1913+16 : binary pulsar (system of 2 neutron stars, one being a radio pulsar seen by radiotelescopes) at ~ 7 kpc from Earth.

⇒ tests of Gravitation theory in strong field and dynamical regime

Loss of energy by GW emission : orbital period decreases

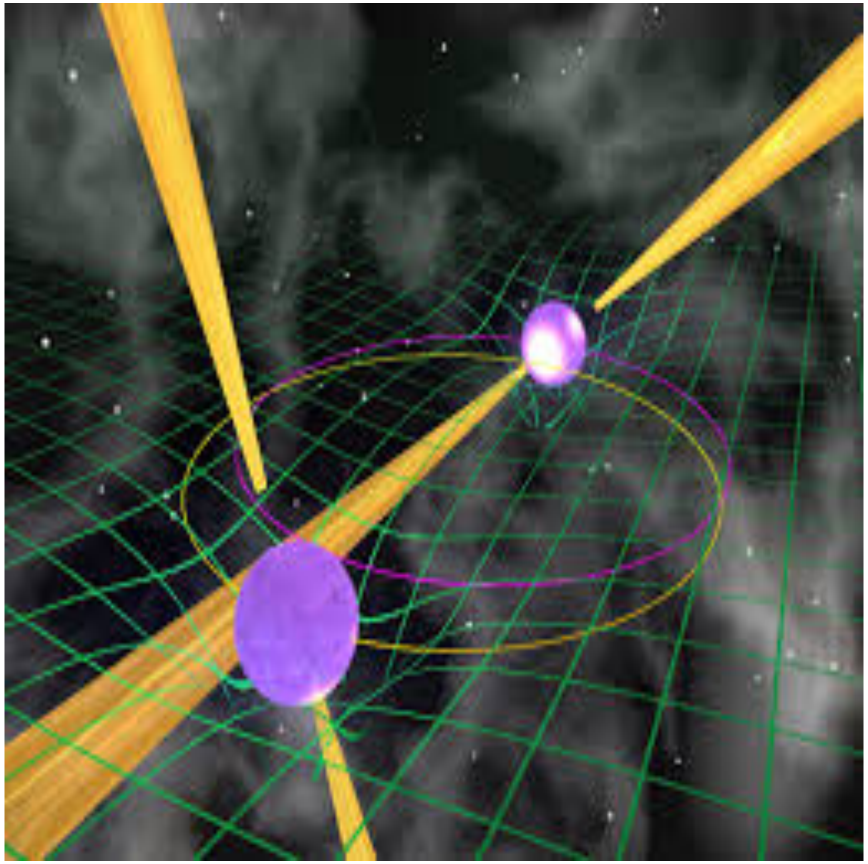


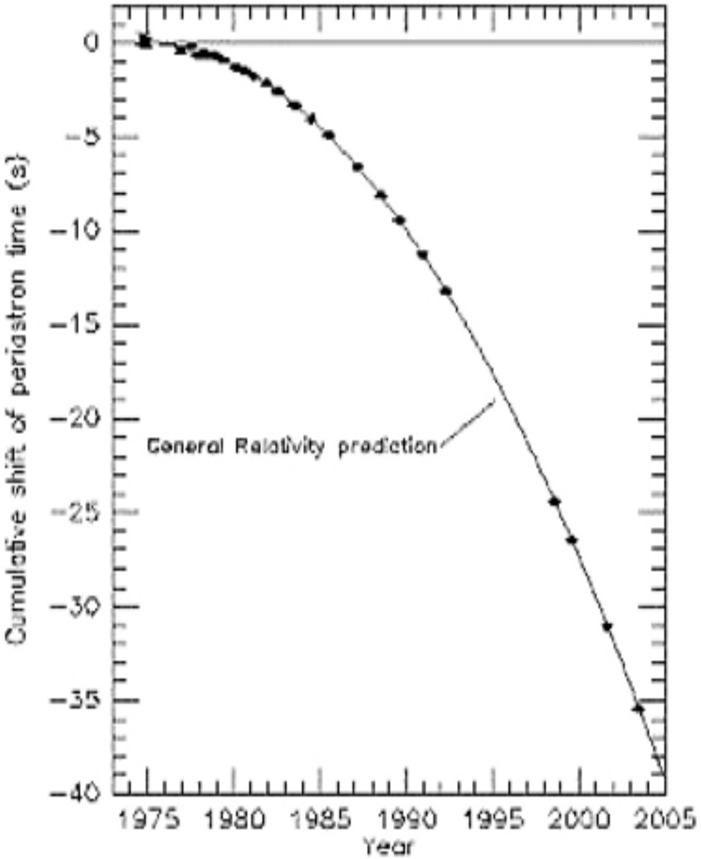
(merge in 300 billions years)

P (s)	**27906.9807807(9)**
dP/dt	**$-2.425(10)\cdot10^{-12}$**
*d*ω/dt (°/yr)	**4.226628(18)**
M_p	**$1.442 \pm 0.003\ M_{\odot}$**
M_c	**$1.386 \pm 0.003\ M_{\odot}$**

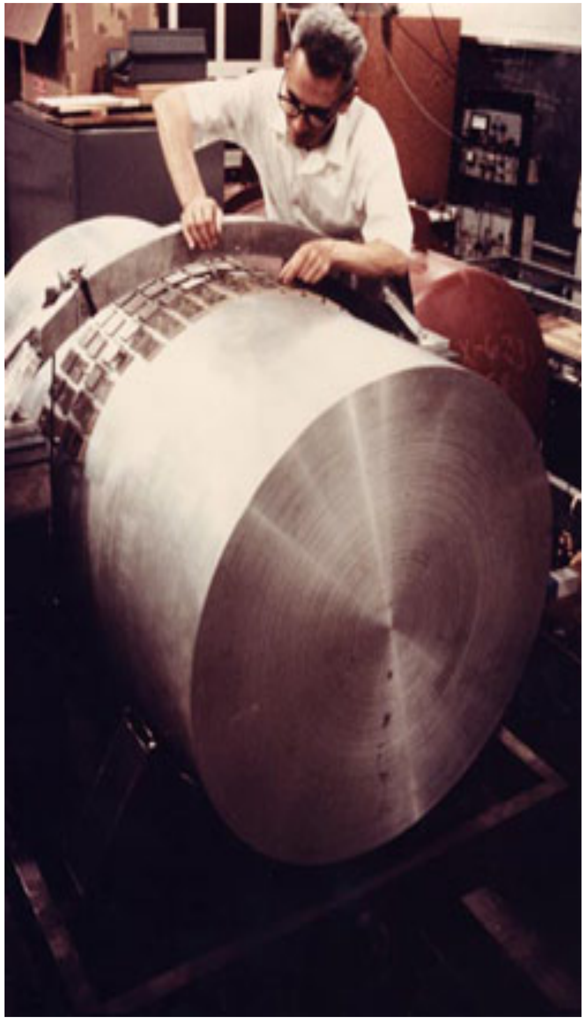
Gravitational Waves do exist !

Le onde gravitazionali

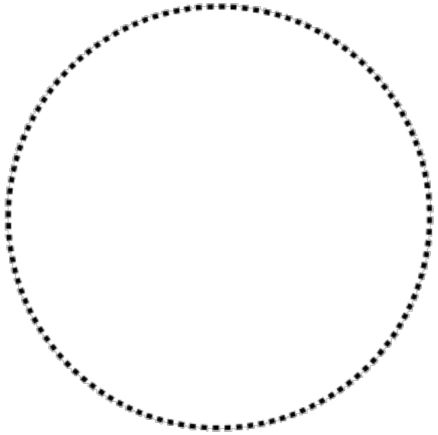


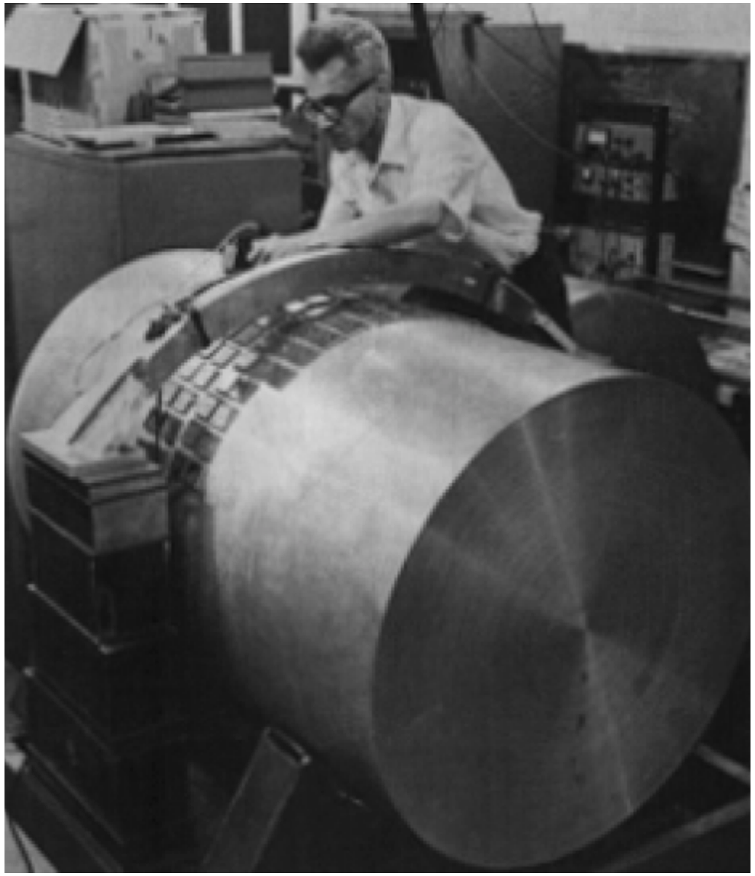


Le onde gravitazionali?

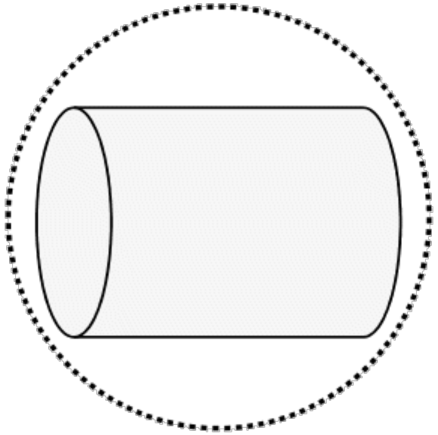


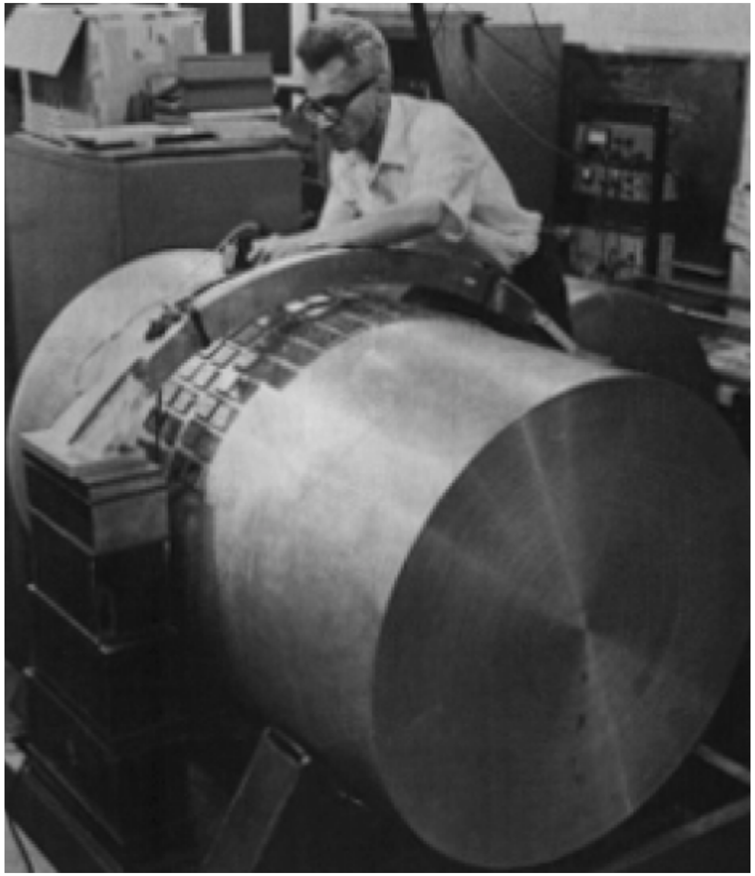
Design of gravitational wave detectors



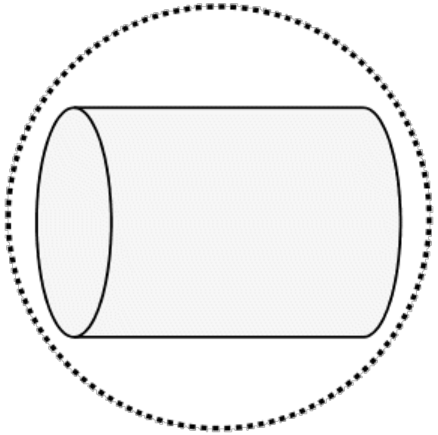


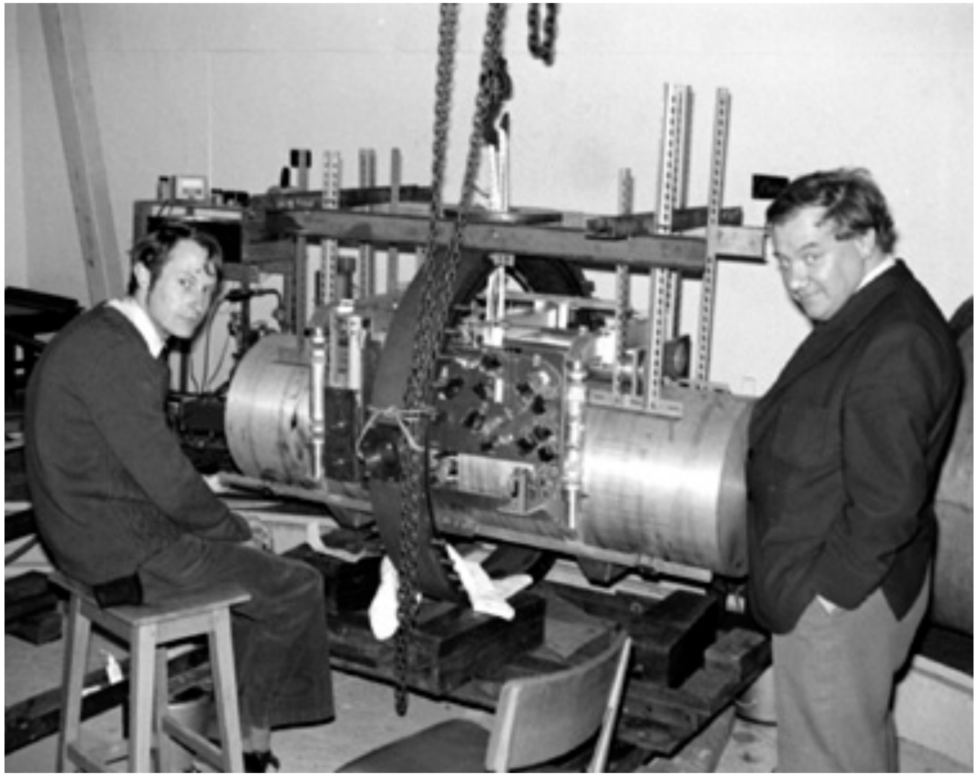
Design of gravitational wave detectors





Design of gravitational wave detectors





Prime allerte Onde Gravitazionali

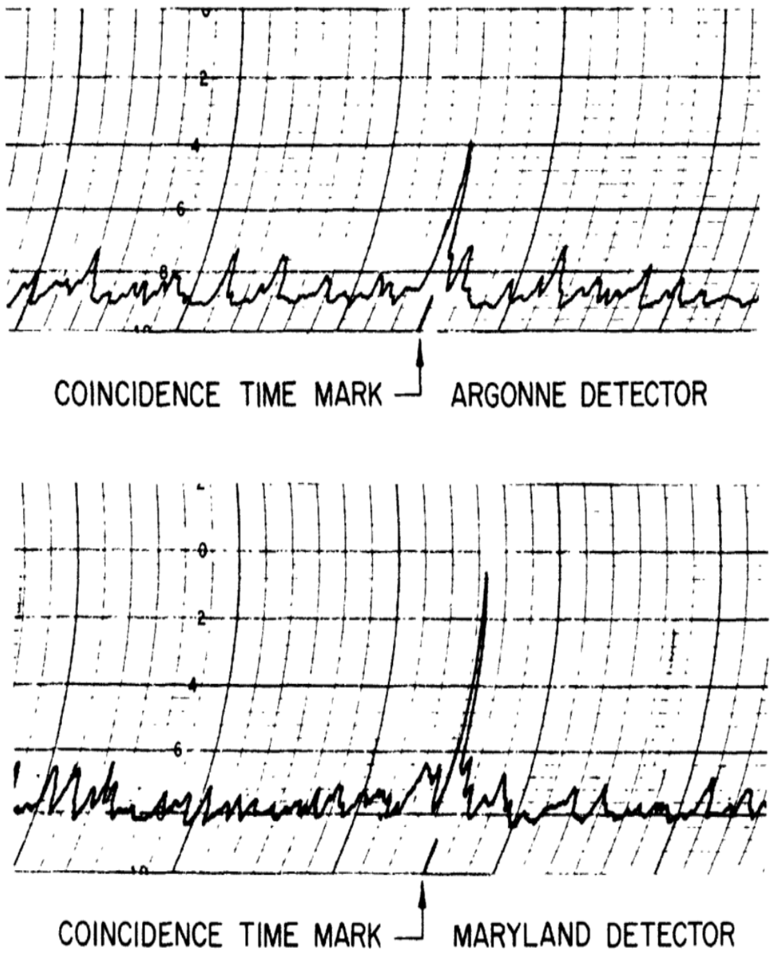
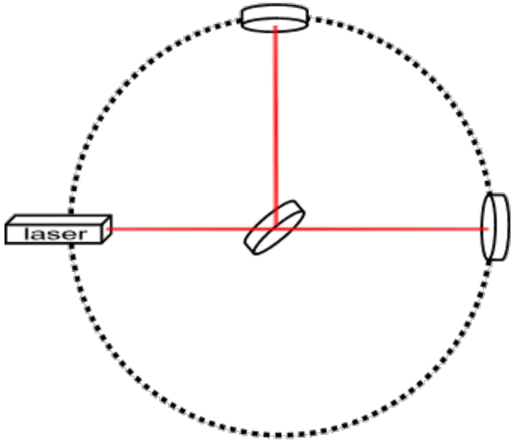
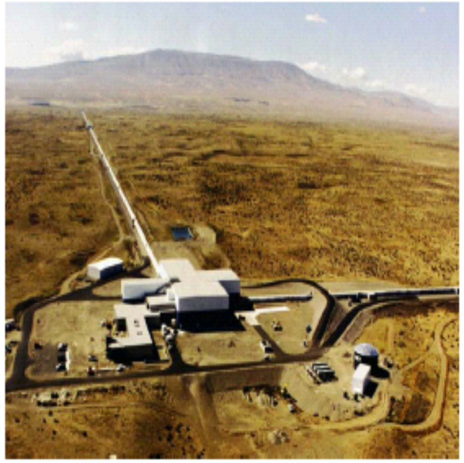
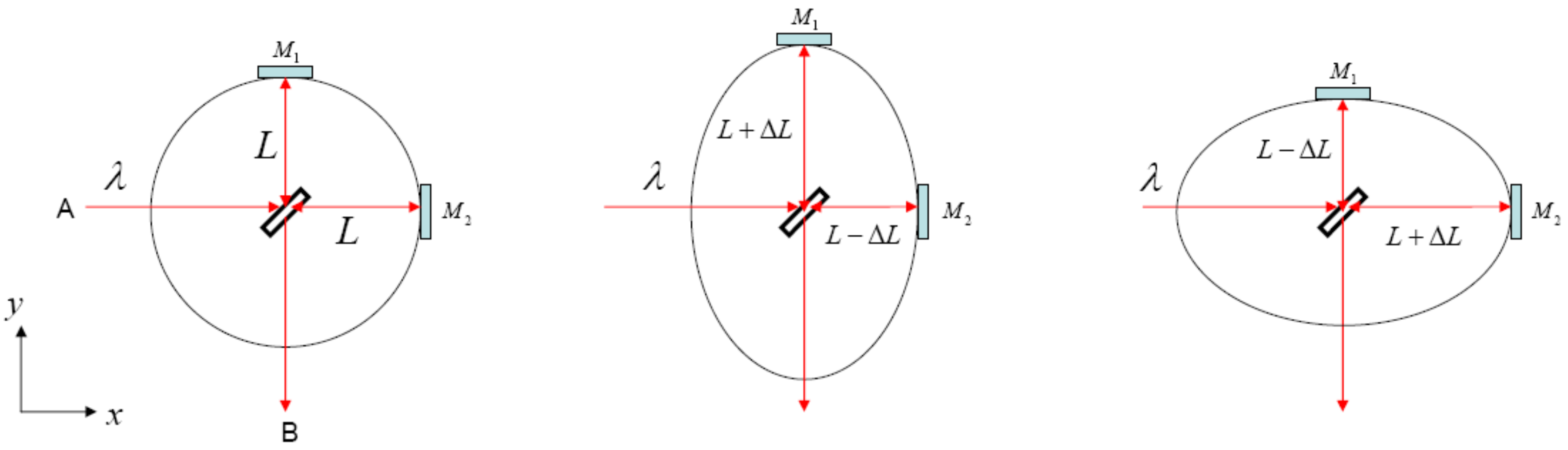


FIG. 2. Argonne National Laboratory and University of Maryland detector coincidence.

34 yrs on - Interferometric ground-based detectors







Gravitational wave $\mathbf{h} = h\mathbf{e}_+$ propagating along z axis.

Fractional change in proper separation

$$\boxed{\frac{\Delta L}{L} = \frac{h}{2}}$$

Astrofisica Nucleare e Subnucleare

Sources of GW

Gravitational Wave emission

(*quadrupole formalism*)

Emission equation in the TT Gauge: $\nabla^2 h_{\mu\nu}^{TT} = -\frac{16\pi G}{c^4} T_{\mu\nu}$

Retarded solution: $h_{\mu\nu}^{TT}(\vec{x},t) = \frac{2G}{Rc^4}\,\ddot{Q}_{\mu\nu}^{TT}(t-R/c)$

Hence: $h_{+}^{TT}(\vec{x},t) = \frac{G}{Rc^4}\left[\ddot{Q}_{11}^{TT} - \ddot{Q}_{22}^{TT}\right](t-R/c)$ $h_{\times}^{TT}(\vec{x},t) = \frac{2G}{Rc^4}\left[\ddot{Q}_{12}^{TT}\right](t-R/c)$

Where the ***reduced quadrupole*** moment:

$$Q_{\mu\nu}^{TT} = \iiint d^3x\, \rho\, (x_\mu\, x_\nu - \tfrac{1}{3}\delta_{\mu\nu} r^2)$$

Regular quadrupole (inertia) moment: $q_{\mu\nu} = \iiint d^3x\, \rho\, x_\mu x_\nu$

$\rho \sim T_{00}/c^2$: density of the source

9. The Production of Gravitational Waves (pgs 76 – 80)

We can understand something important about the nature of gravitational radiation by drawing analogies with the formulae that describe electromagnetic radiation. This approach is crude at best since the electromagnetic field is a vector field while the gravitational field is a tensor field, but it is good enough for our present purposes. Essentially, we will take familiar electromagnetic radiation formulae and simply replace the terms which involve the Coulomb force by their gravitational analogues from Newtonian theory.

Net electric dipole moment

$$L_{\text{electric dipole}} \propto e^2 \, \ddot{\mathrm{d}}^2$$

$$L_{\text{magnetic dipole}} \propto \ddot{\mu}$$

$$\mu = \sum_{q_i} (\text{position of } q_i) \times (\text{current due to } q_i)$$

Gravitational analogues?...

Mass dipole moment: $$\mathbf{d} = \sum_{A_i} m_i \mathbf{x}_i$$

But $$\dot{\mathbf{d}} = \sum_{A_i} m_i \dot{\mathbf{x}}_i \equiv \mathbf{p}$$

Conservation of linear momentum implies no mass dipole radiation

$$L_{\text{magnetic dipole}} \propto \ddot{\mu}$$

$$\mu \;=\; \sum_{q_i} \;(\text{position of } q_i) \times (\text{current due to } q_i)$$

Gravitational analogues?...

$$\mu \;=\; \sum_{A_i} \;(\mathbf{x}_i) \times (m_i \mathbf{v}_i) \quad \equiv \quad \mathbf{J}$$

Conservation of angular momentum implies no mass dipole radiation

Also, the quadrupole of a **spherically symmetric mass distribution** is zero.

Metric perturbations which are spherically symmetric don't produce gravitational radiation.

Example: binary neutron star system.

$$h_{\mu\nu} = \frac{2G}{c^4 r} \ddot{I}_{\mu\nu}$$

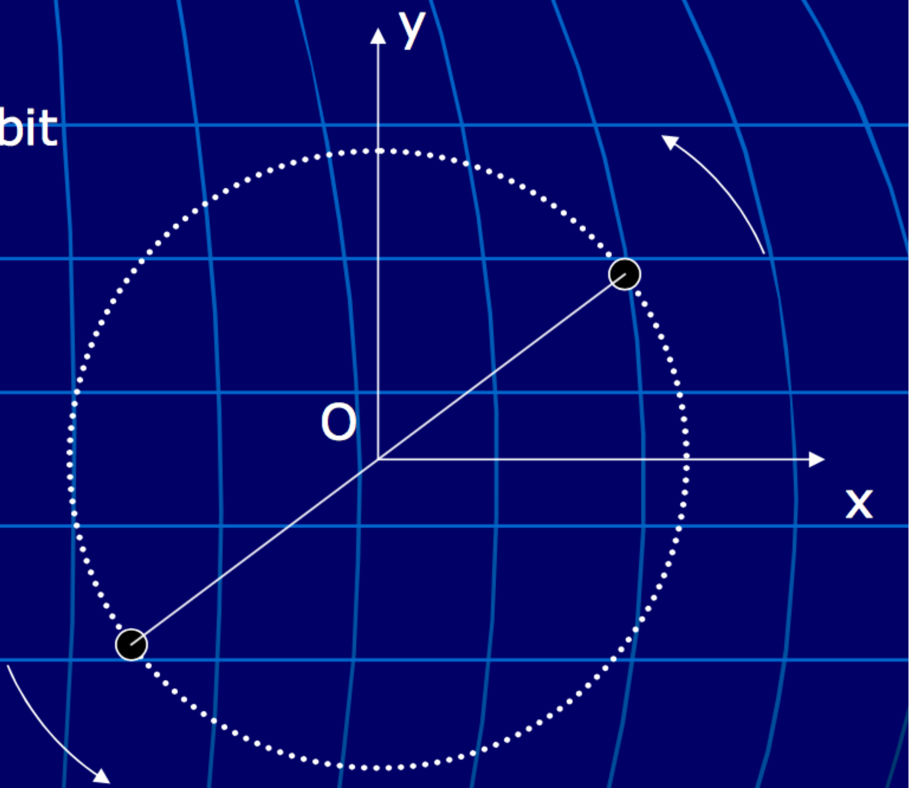
where $I_{\mu\nu}$ is the **reduced quadrupole moment** defined as

$$I_{\mu\nu} = \int \rho(\vec{r}) \left(x_\mu x_\nu - \frac{1}{3}\delta_{\mu\nu} r^2 \right) dV$$

Gravitational Wave emission: an example

2 identical point masses in circular orbit around their center of mass

- Orbital plane : xOy
- Mass : M
- Orbit radius : a
- Orbital frequency : $f_0 = 2\pi\omega_0$



Q: Compute the 2 amplitudes $h_+(t)$ and $h_x(t)$ at a distance r on the z axis (*without taking into account the radiation reaction !*)

Gravitational Wave emission: an example

Positions of the two masses:

$$x_1(\mathrm{t}) = a\cos(\omega_0 \mathrm{t}) \qquad x_2(\mathrm{t}) = -a\cos(\omega_0 \mathrm{t})$$

$$y_1(\mathrm{t}) = a sin(\omega_0 \mathrm{t}) \qquad y_2(\mathrm{t}) = -a sin(\omega_0 \mathrm{t})$$

So compute the reduced inertia tensor:

$$Q = \begin{pmatrix} ma^2\left(\tfrac{1}{3} + \cos(2\omega_0 t)\right) & ma^2 \sin(2\omega_0 t) & 0 \\ ma^2 \sin(2\omega_0 t) & ma^2\left(\tfrac{1}{3} - \cos(2\omega_0 t)\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

After projection on the z direction:

$$h_+(t) = -\frac{2G}{rc^4} ma^2\omega^2 \cos\left(\omega(t - r/c)\right)$$

$$h_\times(t) = -\frac{2G}{rc^4} ma^2\omega^2 \sin\left(\omega(t - r/c)\right)$$

Where $\omega = 2\omega_0$ is **TWICE** the orbital angular frequency

Note that if we look on the x direction:

$$h_+(t) = -\frac{G}{rc^4} ma^2\omega^2 \cos\left(\omega(t - r/c)\right)$$

$$h_\times(t) = 0$$

Face-on binary => **circular** polarization

Edge-on binary => **linear** polarization

Onde gravitazionali

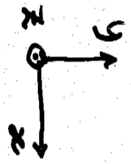
Main features

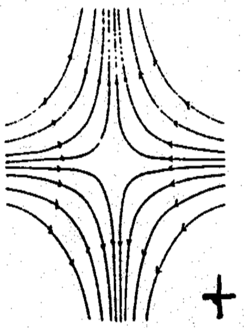
- 2 transversal polarization states
- Associated with massless, spin 2 particles (gravitons)
- Emitted by time-varying quadrupole mass moment
  no dipole radiation because of conservation laws

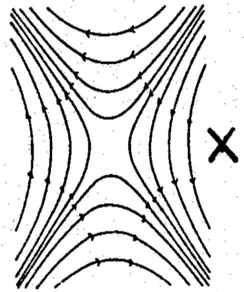
$$-\frac{dE}{dt} = \cancel{\frac{2G}{3c^3}\left(\ddot{d}\right)^2} + \frac{G}{45c^5}\left(\dddot{Q}\right)^2 + \dots$$

$$\dot{d} = \sum_i m_i \dot{x}_i \Rightarrow \ddot{d} \equiv 0 \qquad Q_{ij} = \int \rho x_i x_j d^3x$$

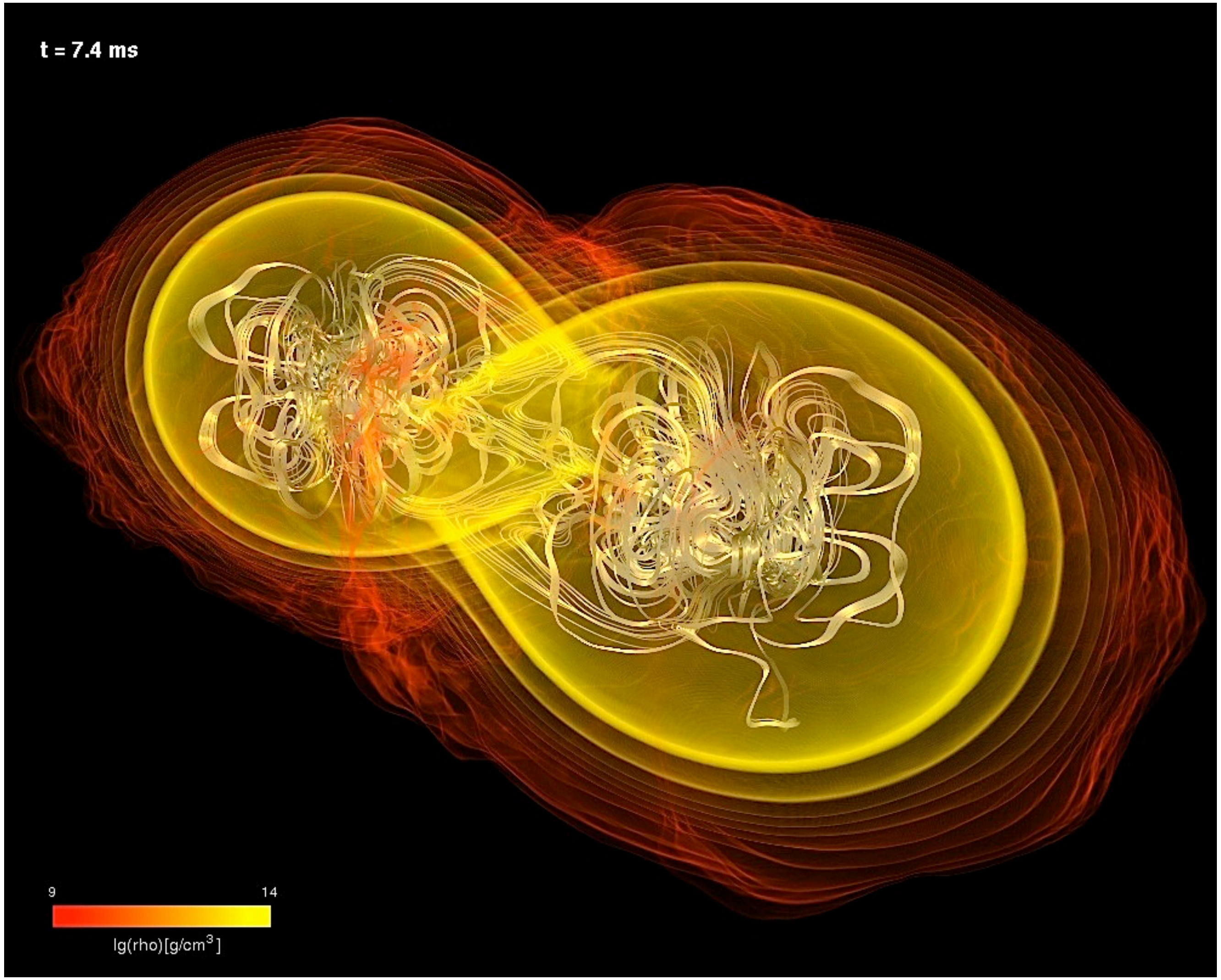
$$\boxed{h_{ij}(t) = \frac{2G}{rc^4}\ddot{Q}_{ij}(t - r/c)}$$



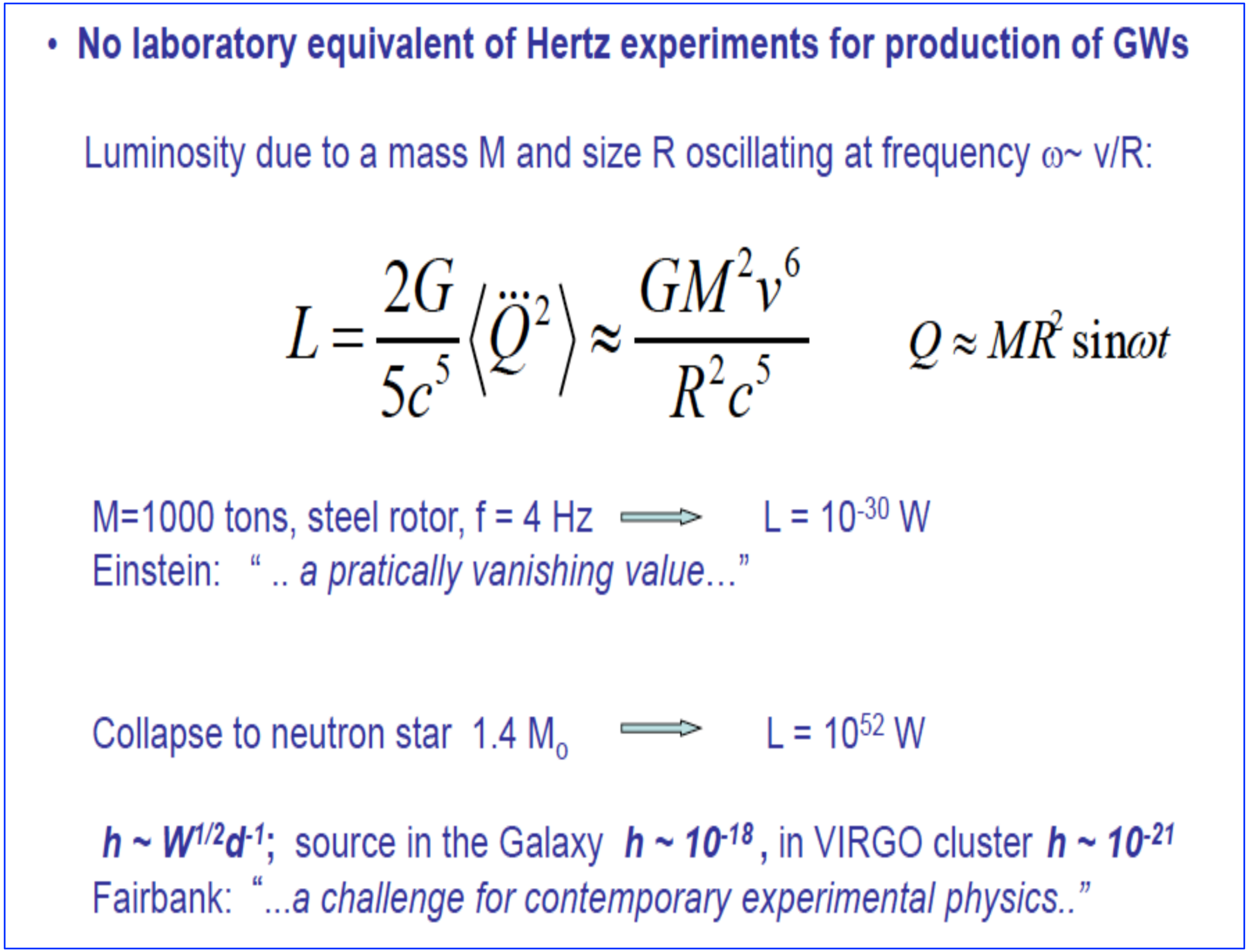




Sorgenti di onde gravitazionali



Onde gravitazionali



- **No laboratory equivalent of Hertz experiments for production of GWs**

Luminosity due to a mass M and size R oscillating at frequency $\omega \sim v/R$:

$$L = \frac{2G}{5c^5}\left\langle \dddot{Q}^2 \right\rangle \approx \frac{GM^2v^6}{R^2c^5} \qquad Q \approx MR^2 \sin\omega t$$

M=1000 tons, steel rotor, f = 4 Hz ⟹ $L = 10^{-30}$ W

Einstein: *" .. a pratically vanishing value…"*

Collapse to neutron star 1.4 M_0 ⟹ $L = 10^{52}$ W

$h \sim W^{1/2}d^{-1}$; source in the Galaxy **$h \sim 10^{-18}$**, in VIRGO cluster **$h \sim 10^{-21}$**

Fairbank: *"...a challenge for contemporary experimental physics.."*

Gravitational Wave emission:
Orders of magnitude

Luminosity (Einstein quadrupole formula):

$$P = \frac{G}{5c^5}\left\langle \dddot{Q}_{\mu\nu}\,\dddot{Q}^{\mu\nu} \right\rangle$$

$G/5c^5 \sim 10^{-53}$ W^{-1}

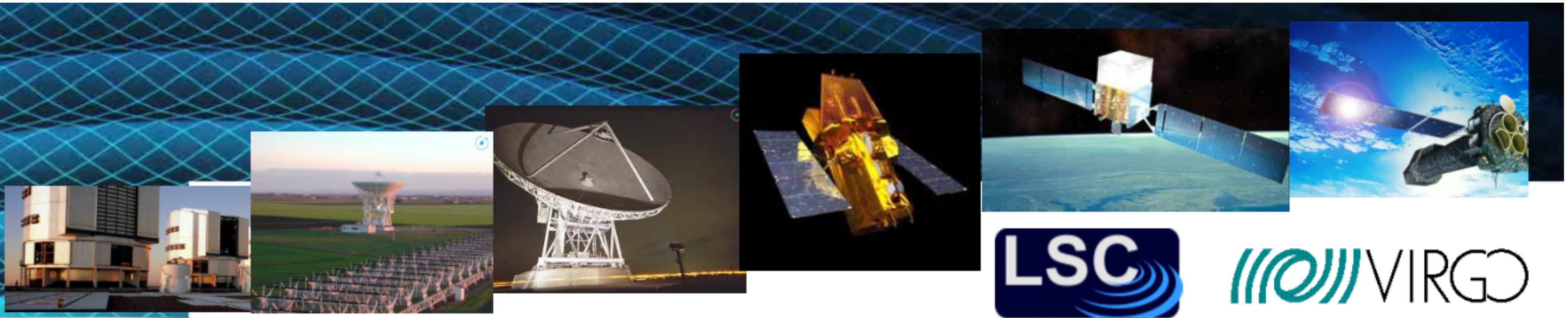
Factor ridiculously « small » !

source	distance	*h*	*P* (W)
Steel bar, 500 T, ∅ = 2 m L = 20 m, 5 cycles/s	1 m	2×10^{-34}	10^{-29}
H bomb, 1 megatonne Asymmetry 10%	10 km	2×10^{-39}	10^{-11}
Supernova 10 $M_\odot$ asymmetry 3%	10 Mpc	10^{-21}	10^{44}
Coalescence 2 black holes 10 $M_\odot$	10 Mpc	10^{-20}	10^{50}

Hertz experiment is impossible for GWs ...

Astrofisica Nucleare e Subnucleare

Ground Detectors for GW



Gravitational waves – Experiments and sources

I PART

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FUTURO IN RICERCA

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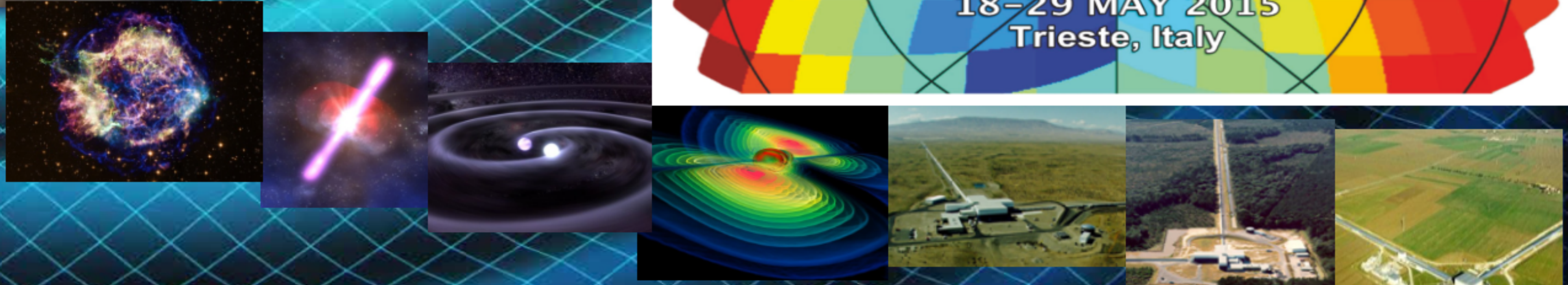
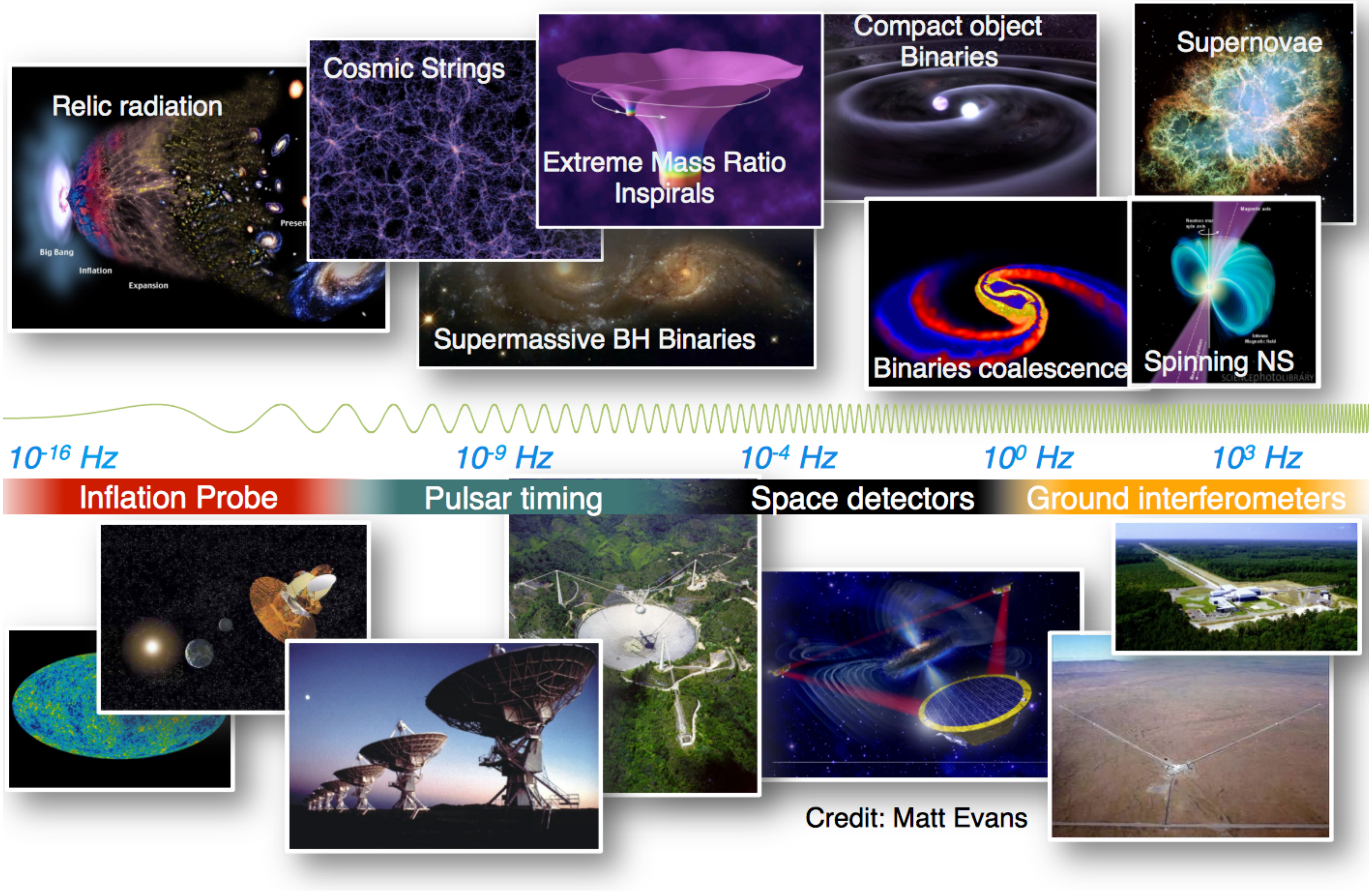


The GW Spectrum



Relic radiation

Cosmic Strings

Extreme Mass Ratio Inspirals

Compact object Binaries

Supernovae

Supermassive BH Binaries

Binaries coalescence

Spinning NS

10^{-16} Hz

10^{-9} Hz

10^{-4} Hz

10^{0} Hz

10^{3} Hz

Inflation Probe

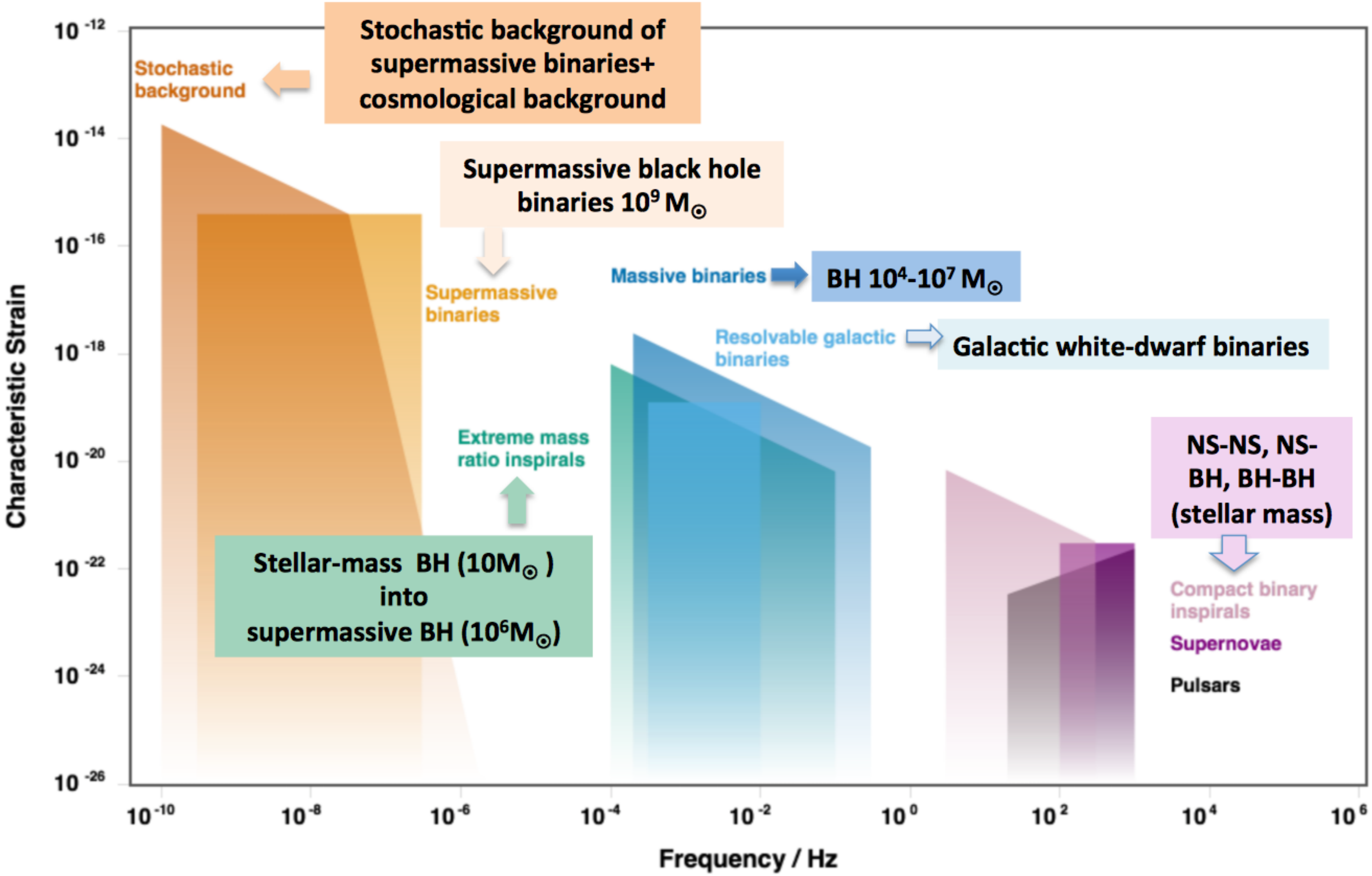
Pulsar timing

Space detectors

Ground interferometers

Credit: Matt Evans

Astrophysical Sources



Stochastic background of supermassive binaries+ cosmological background

Supermassive black hole binaries $10^9 M_\odot$

BH 10^4-$10^7 M_\odot$

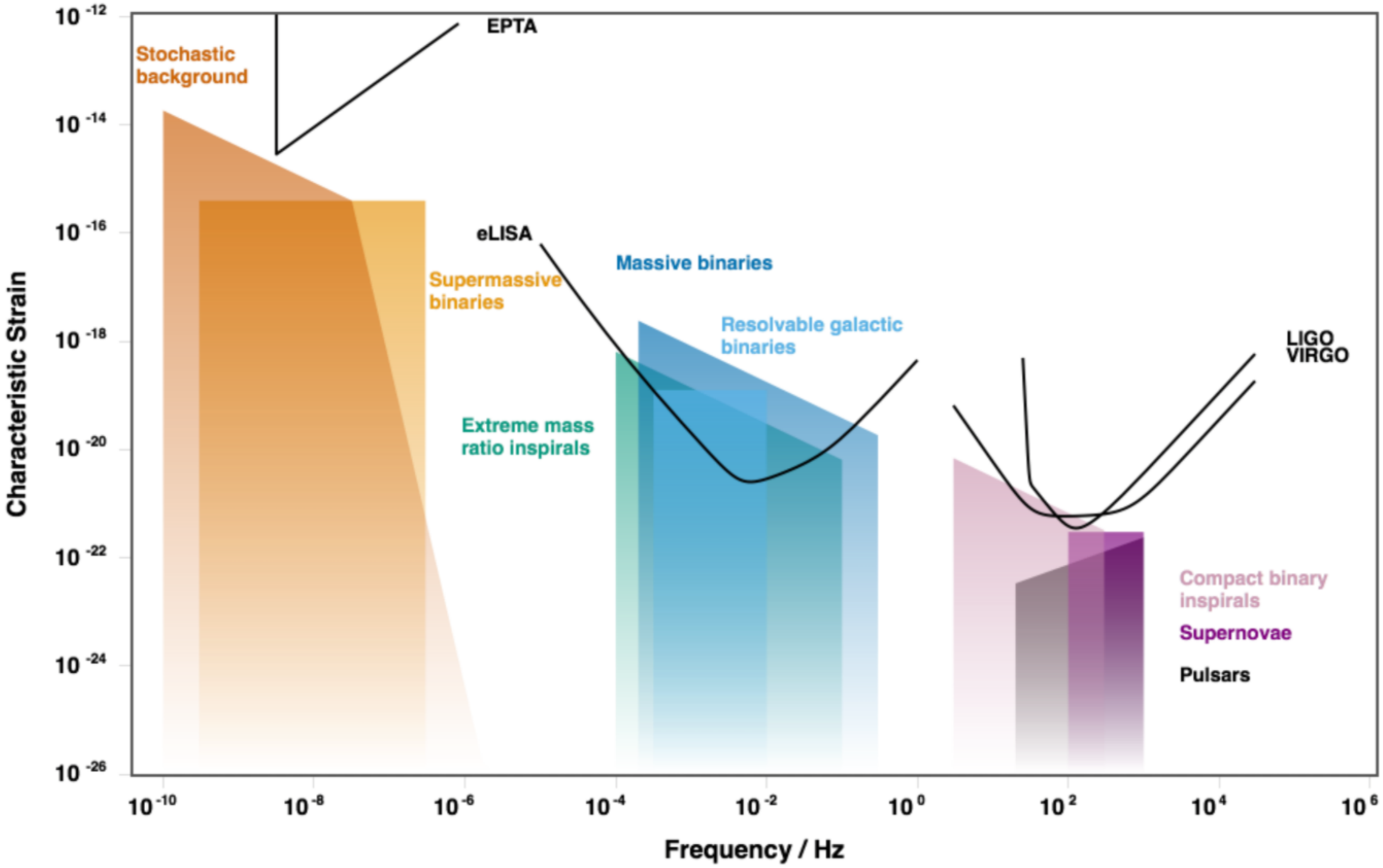
Galactic white-dwarf binaries

NS-NS, NS-BH, BH-BH (stellar mass)

Stellar-mass BH ($10M_\odot$) into supermassive BH ($10^6 M_\odot$)

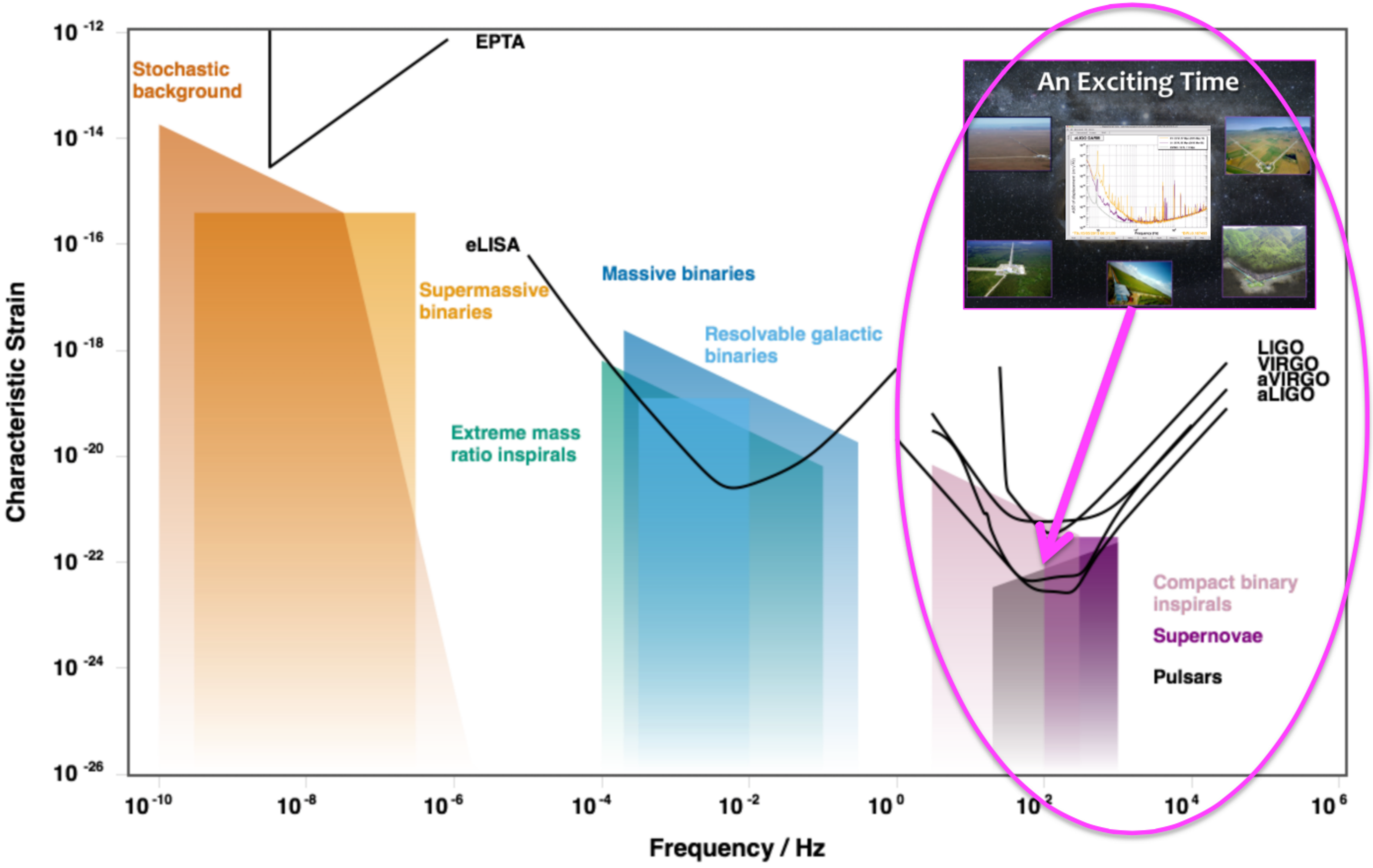
http://rhcole.com/apps/GWplotter/

Astrophysical Sources



http://rhcole.com/apps/GWplotter/

Astrophysical Sources



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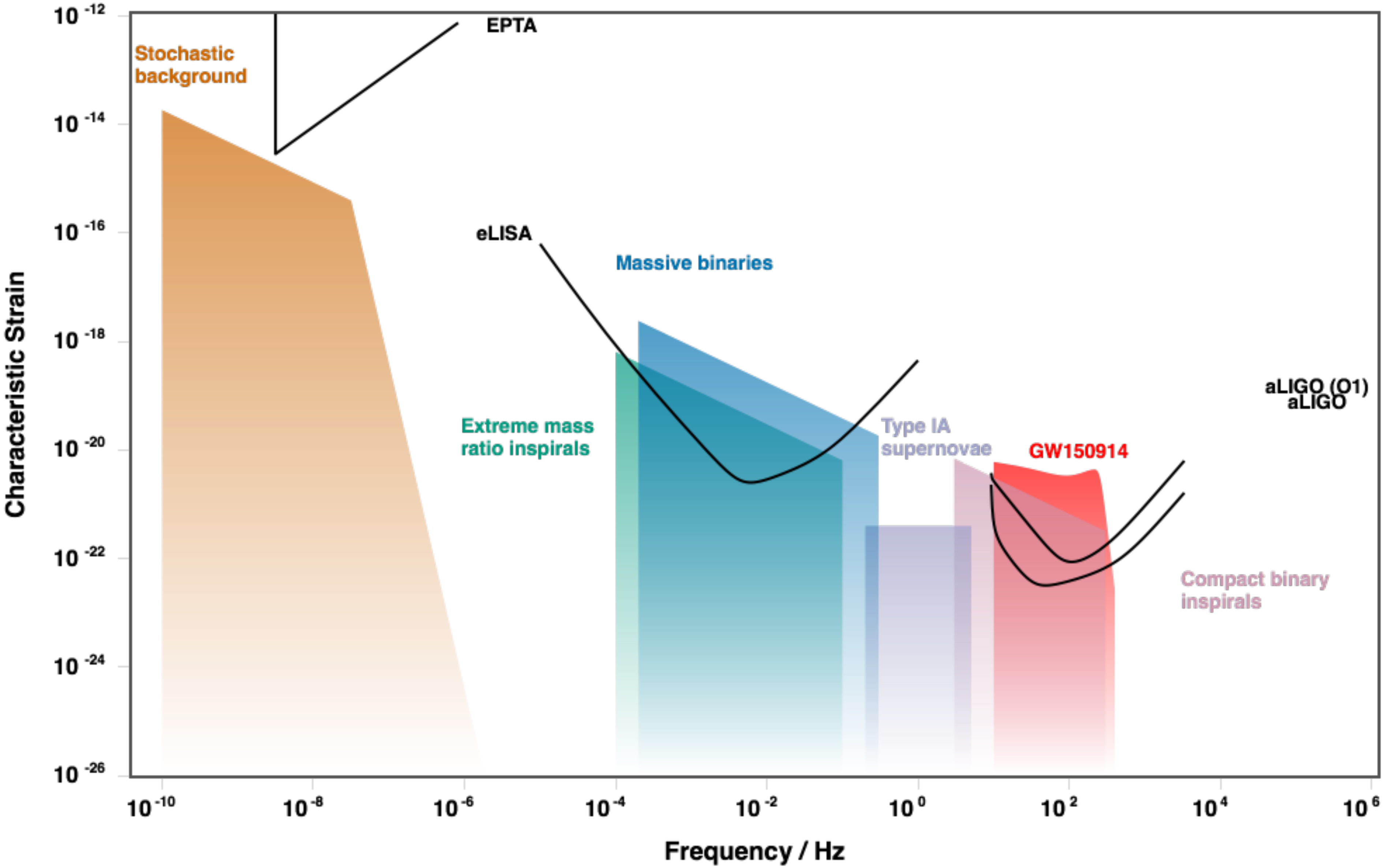
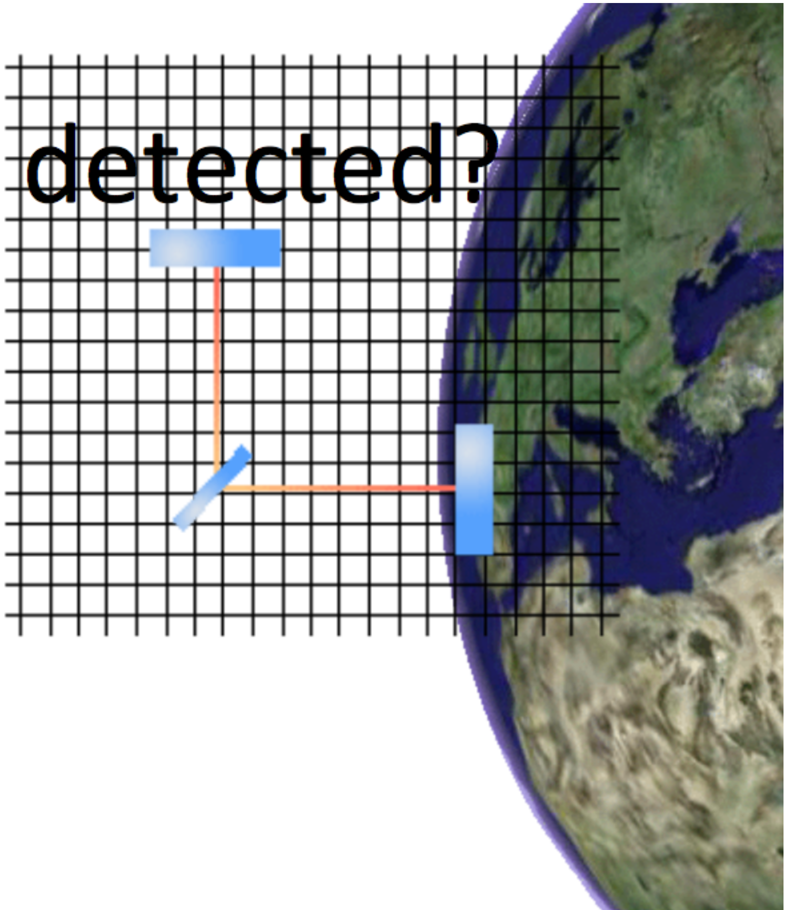

Characteristic Strain
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-26}
Stochastic background
EPTA
eLISA
Massive binaries
Extreme mass ratio inspirals
Type IA supernovae
GW150914
aLIGO (O1)
aLIGO
Compact binary inspirals
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
Frequency / Hz

How can GWs be detected?

A GW deforms space

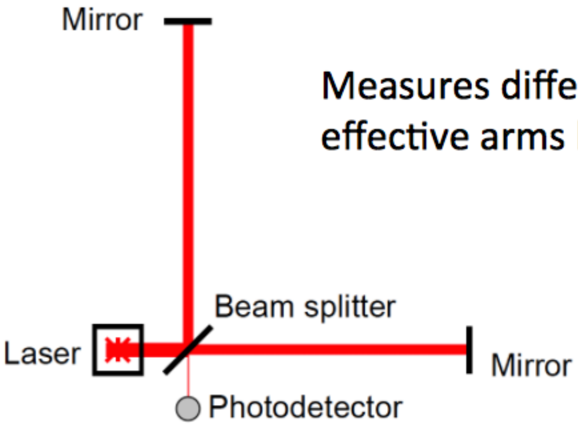
$$\Delta L = \frac{1}{2} L h(t)$$

*The displacement is proportional to length: it is a **strain**!*

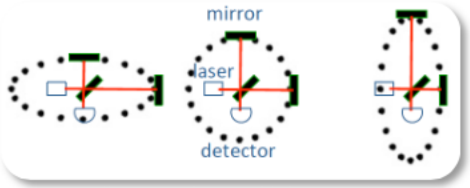


Do we know how to detect displacement?

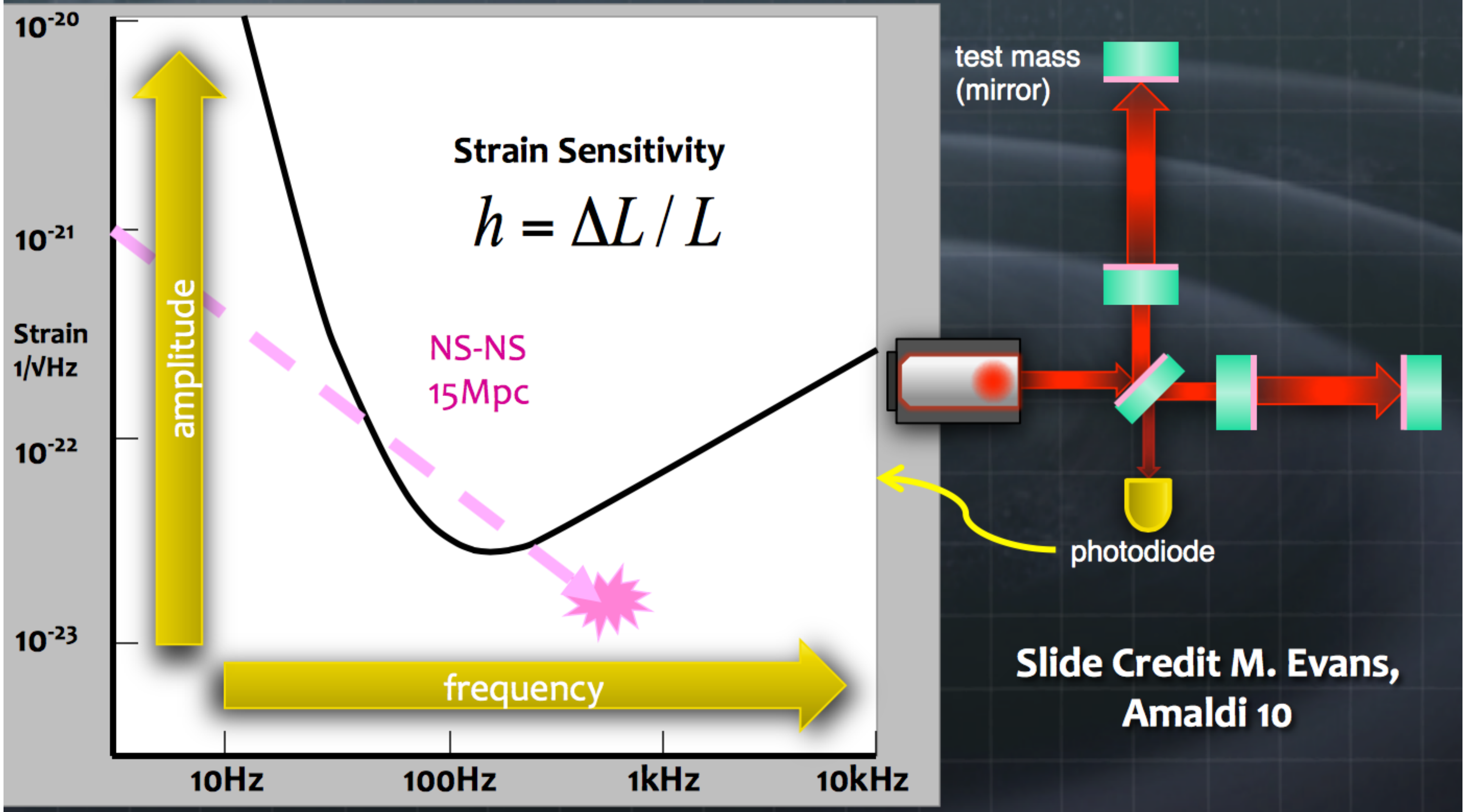
*The length change is measured interferometrically by using a **laser light beam***



Measures differences in effective arms lengths!

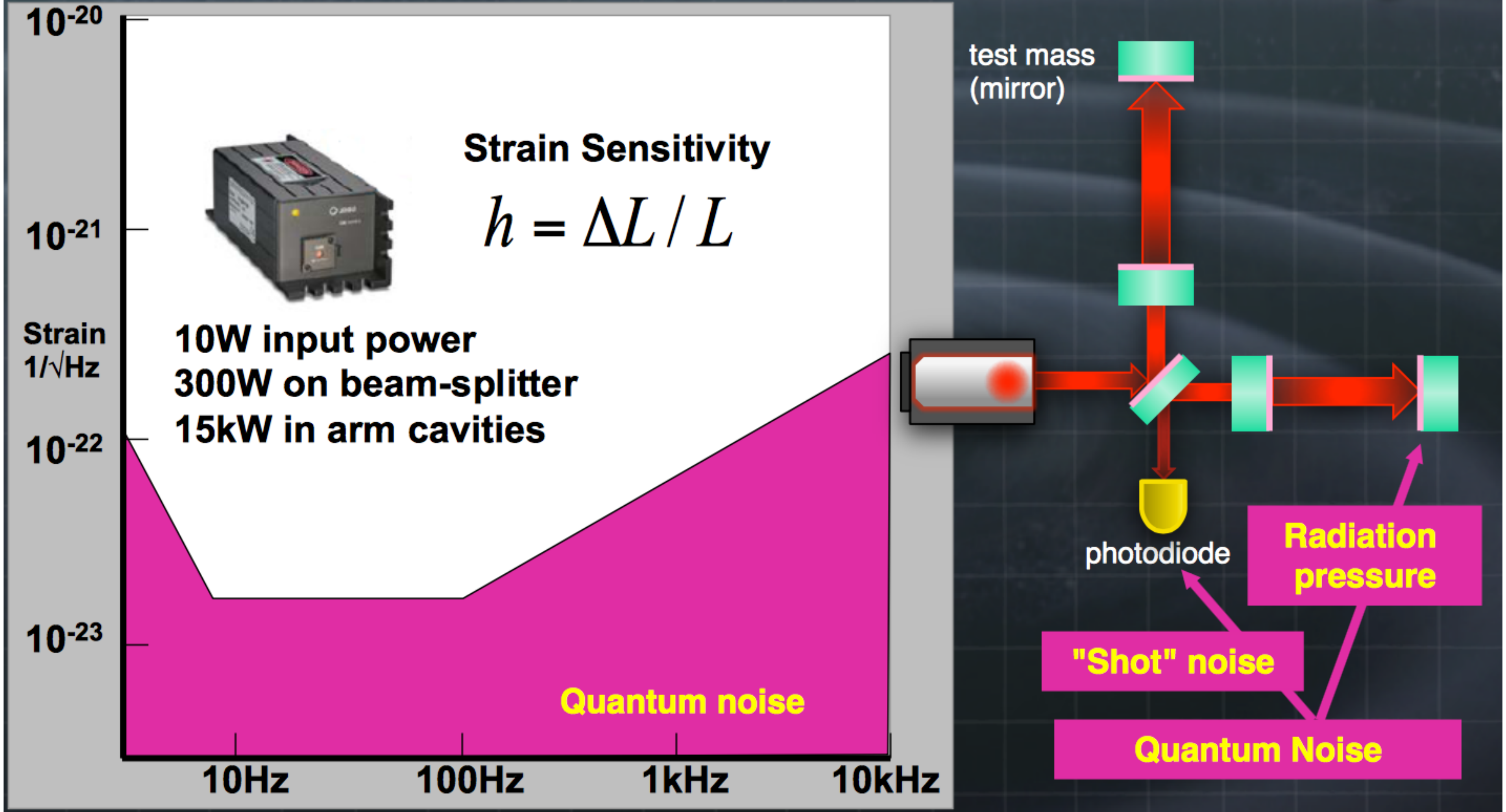


First Generation Sensitivity



Slide Credit M. Evans, Amaldi 10

First Generation Sensitivity



Strain Sensitivity

$$h = \Delta L / L$$

10W input power
300W on beam-splitter
15kW in arm cavities

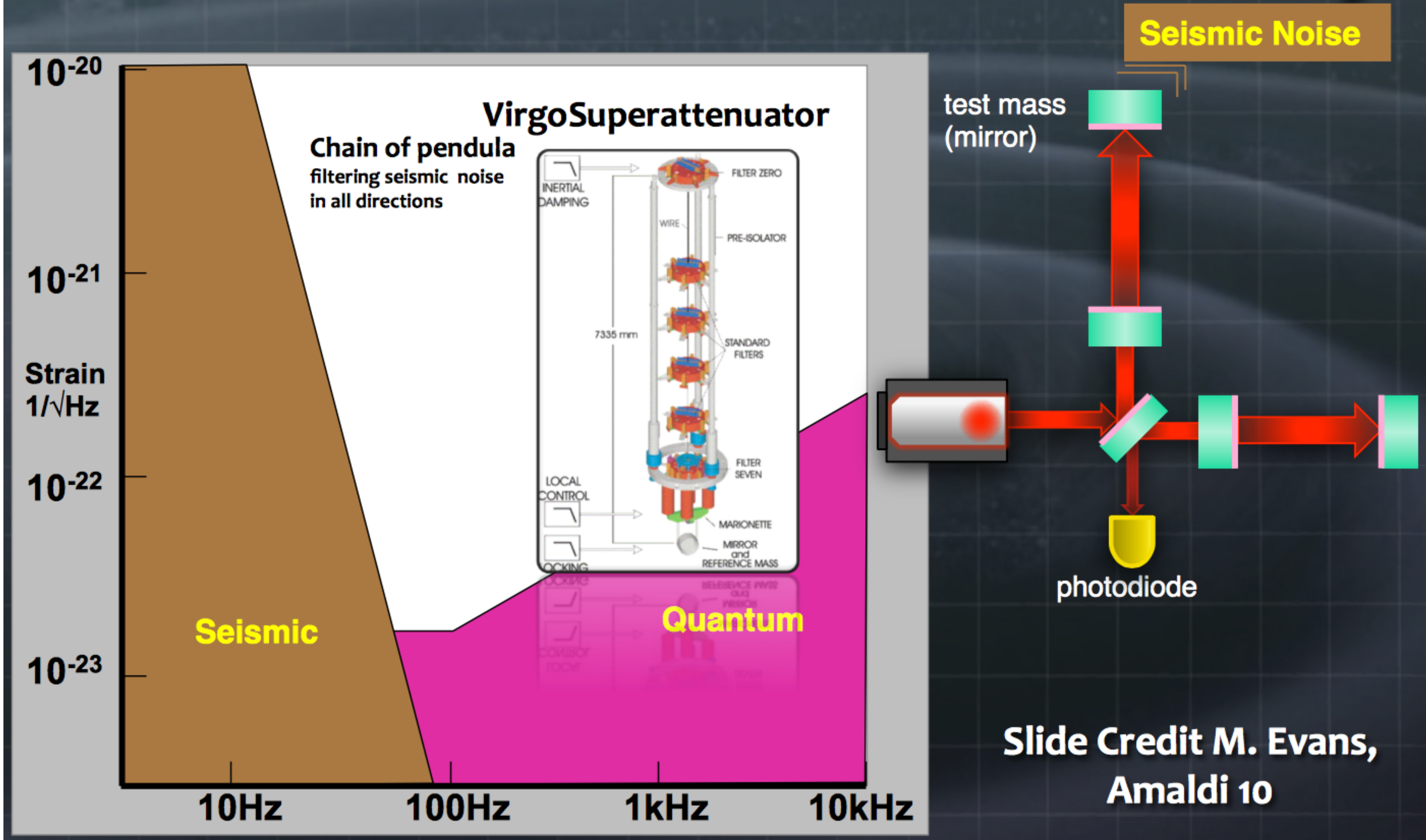
Radiation pressure

"Shot" noise

Quantum Noise

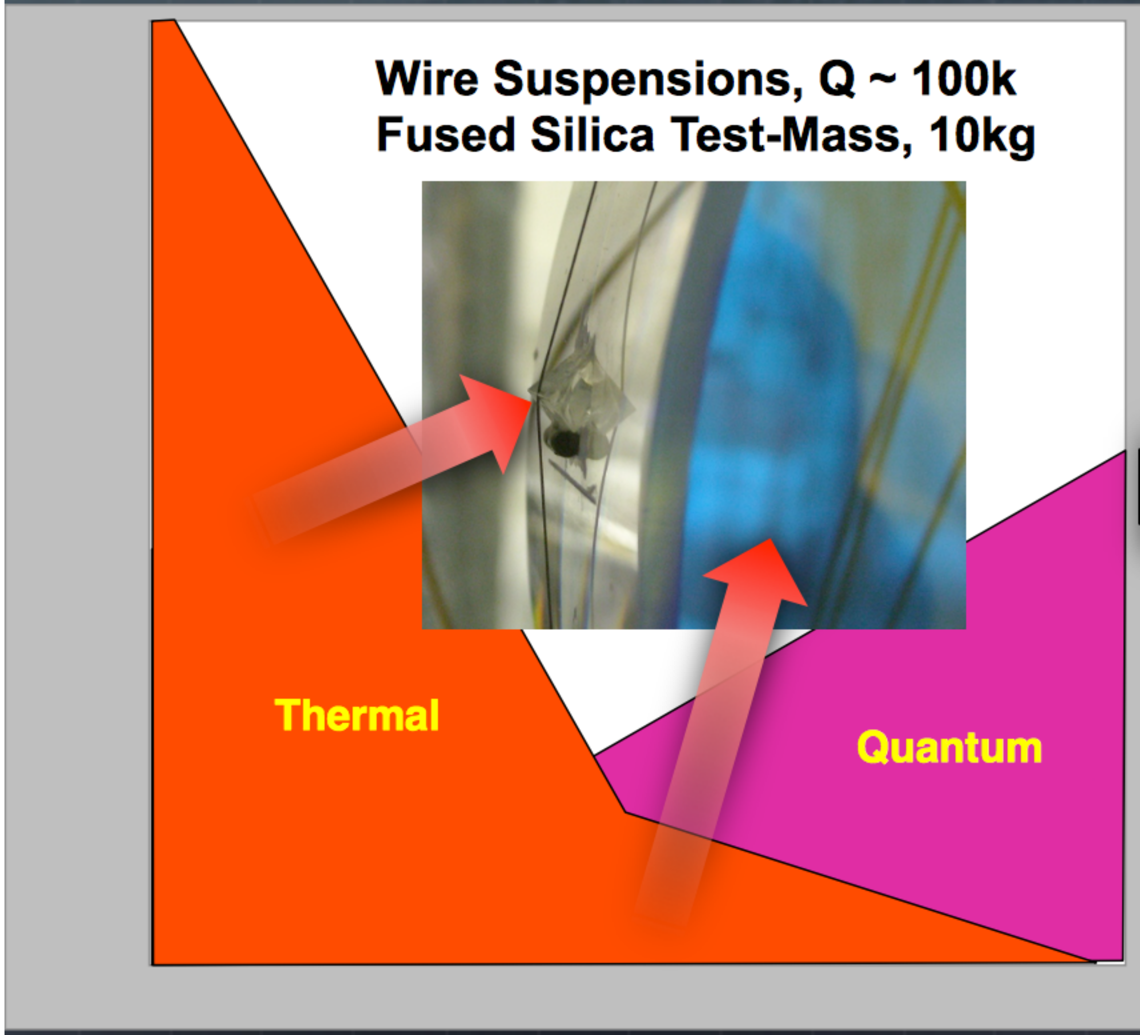
Slide Credit M. Evans, Amaldi 10

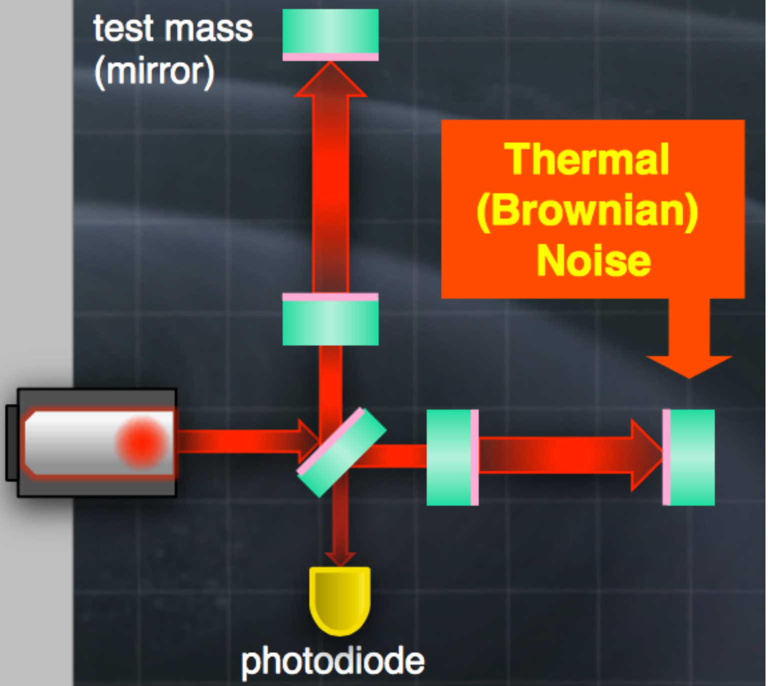
First Generation Sensitivity



Slide Credit M. Evans, Amaldi 10

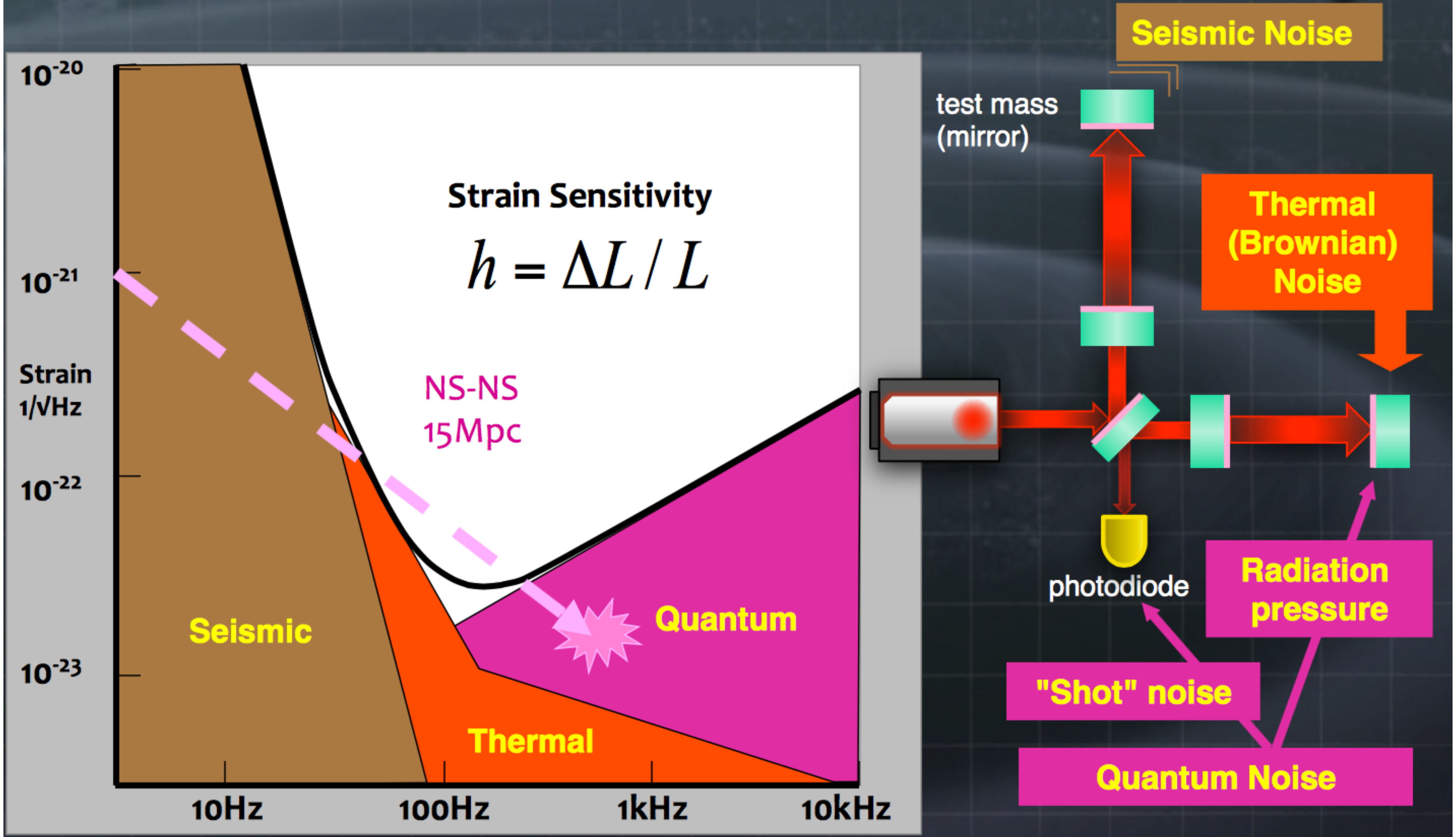
First Generation Sensitivity





Slide Credit M. Evans, Amaldi 10

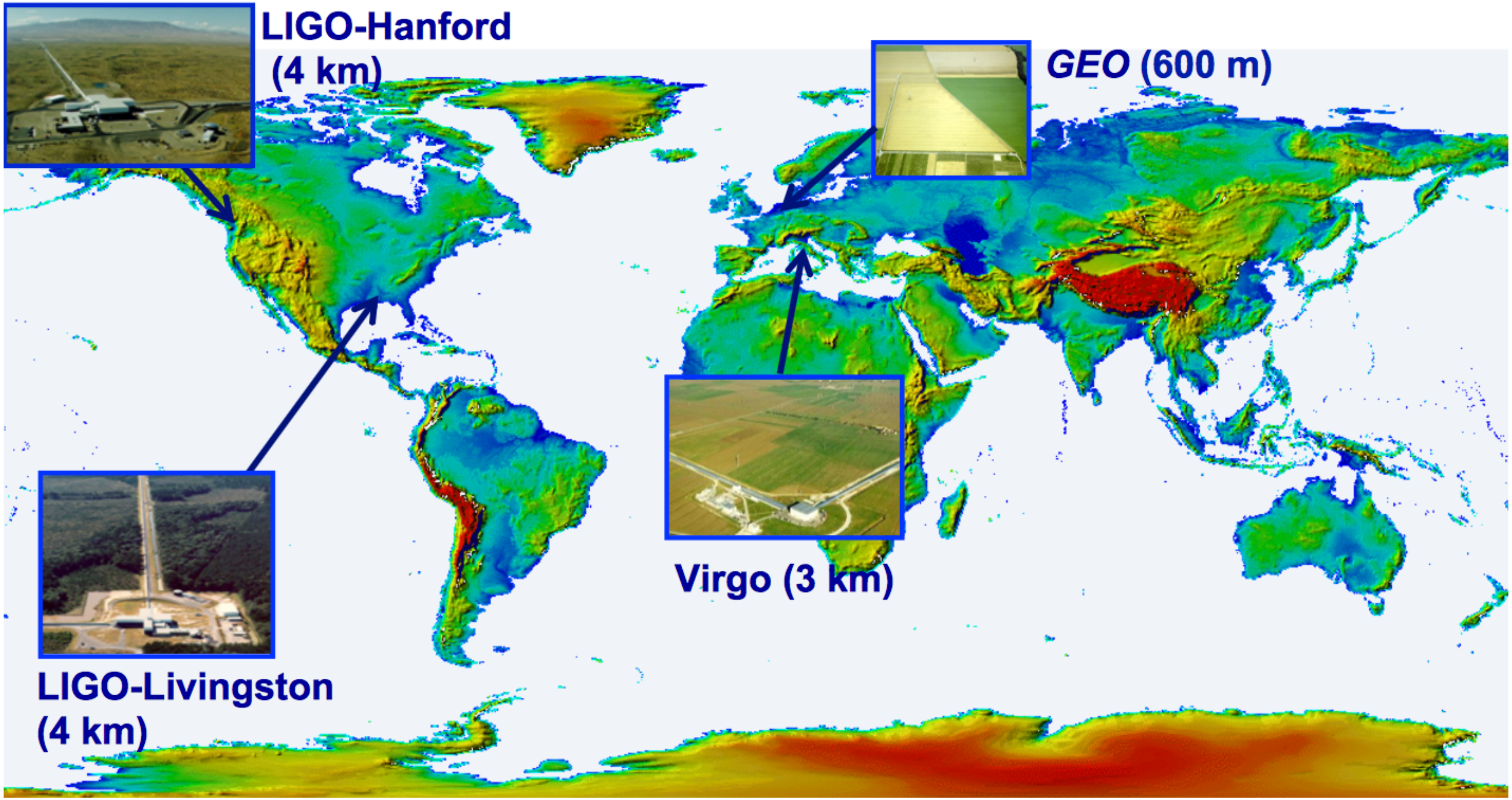
First Generation Sensitivity



Strain Sensitivity

$$h = \Delta L / L$$

Ground-based Gravitational Wave Detectors



LIGO-Hanford (4 km)

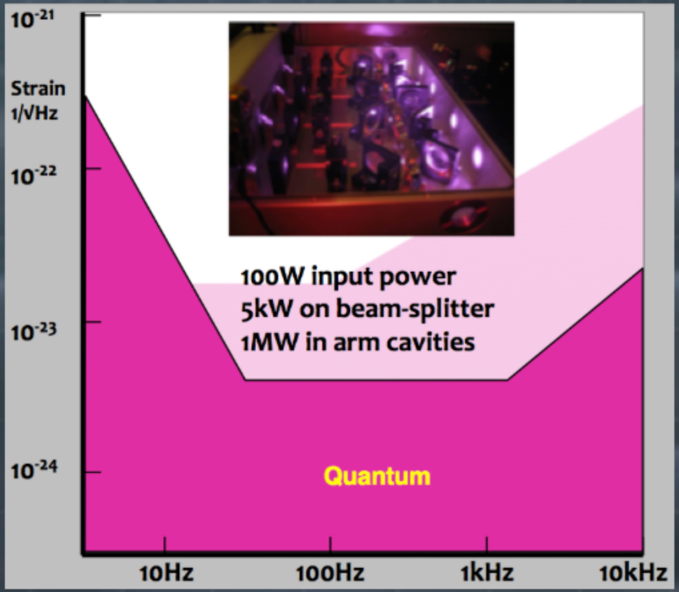
GEO (600 m)

Virgo (3 km)

LIGO-Livingston (4 km)

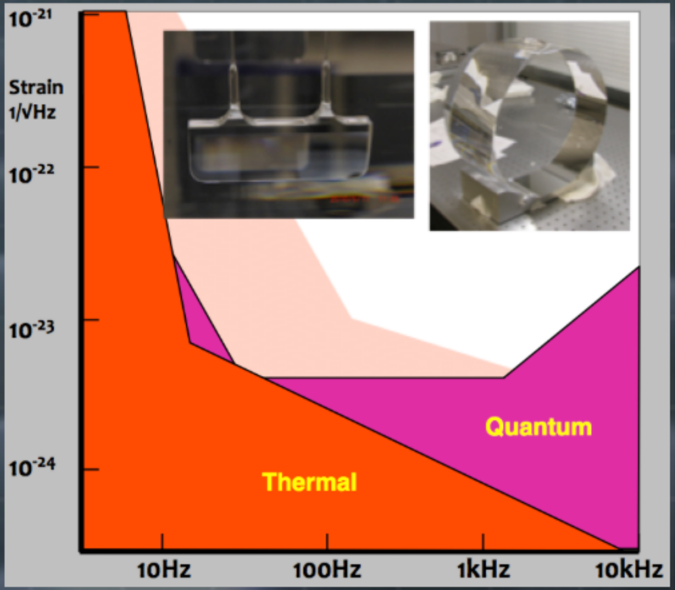
Advanced GW detectors

More Laser Power…



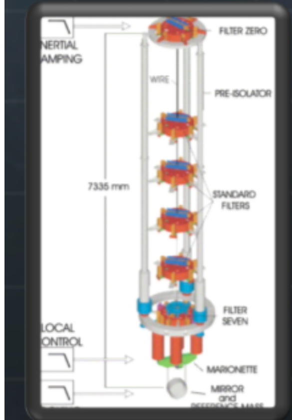
100W input power
5kW on beam-splitter
1MW in arm cavities

Better mechanical quality…

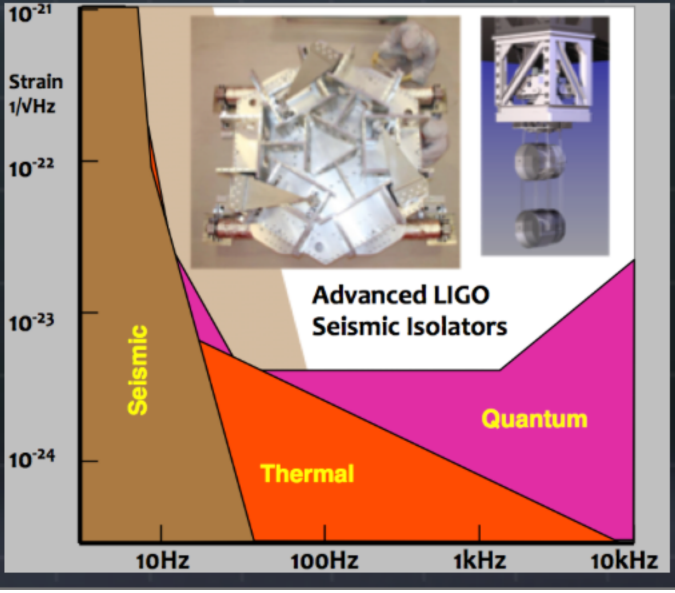


Test mass of 40kg suspended by 400 micron glass fibers…

… and better isolation



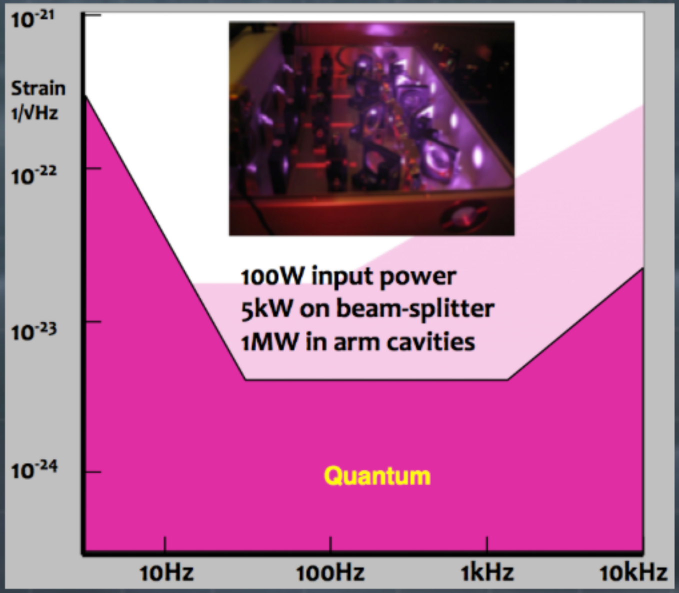
Virgo Super Attenuator



Advanced LIGO Seismic Isolators

Advanced GW detectors

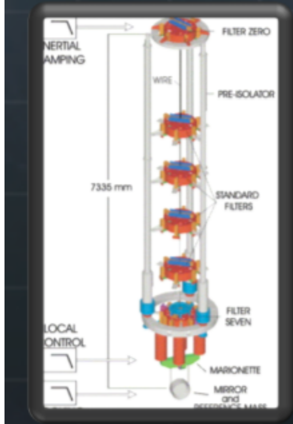
More Laser Power…



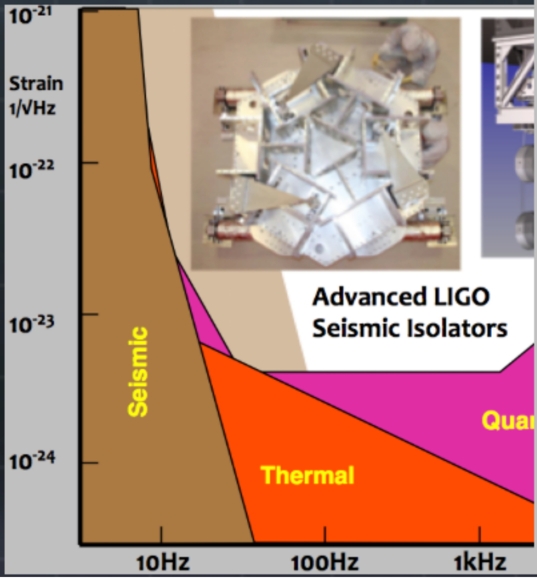
Better mechanical quality…

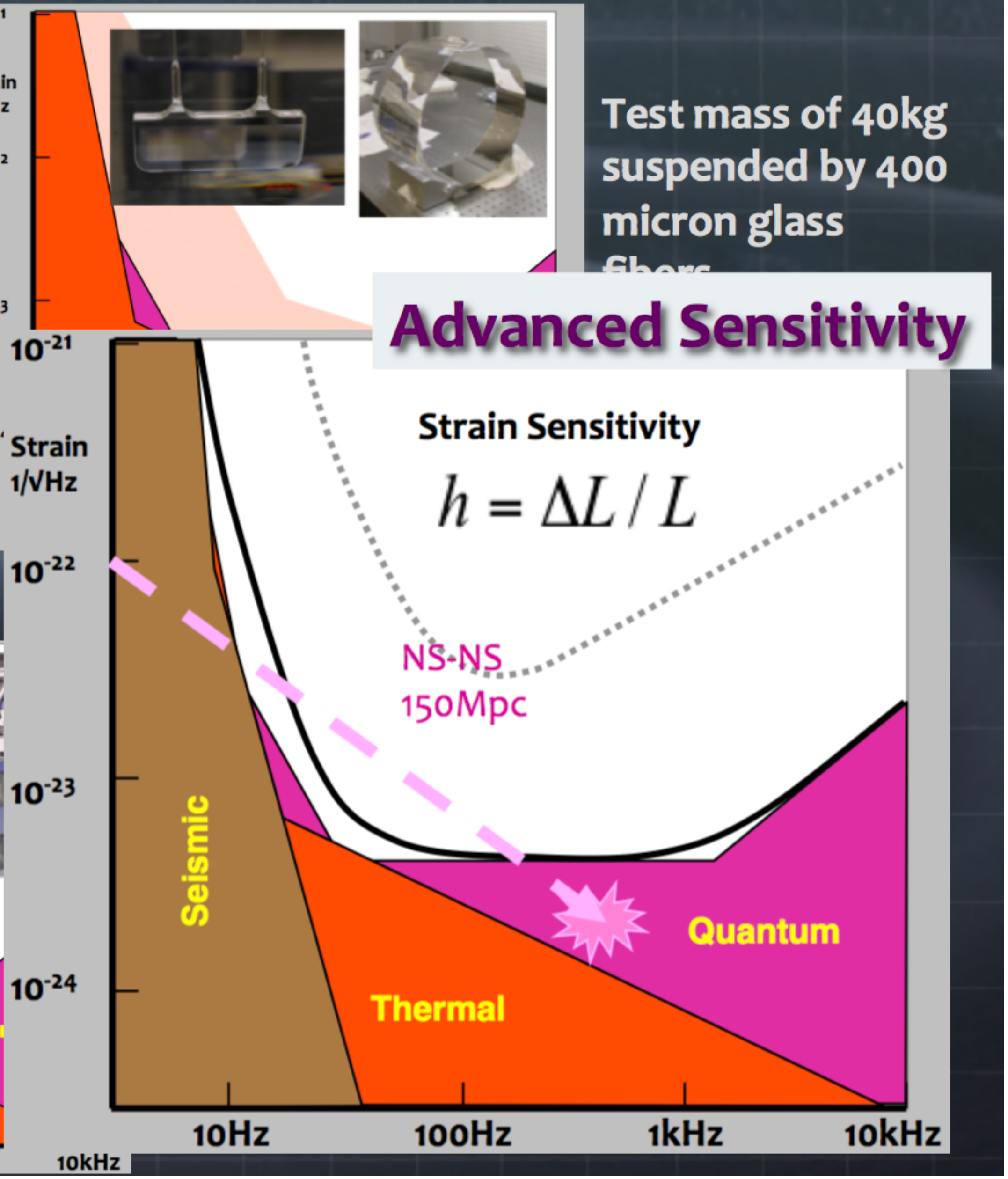
Test mass of 40kg suspended by 400 micron glass fibers

… and better isolation



Virgo Super Attenuator

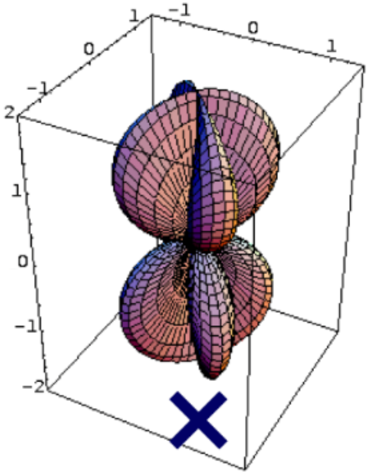


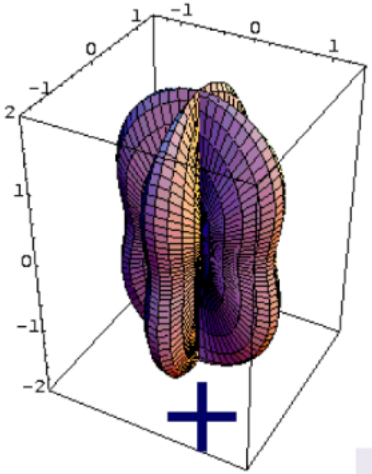


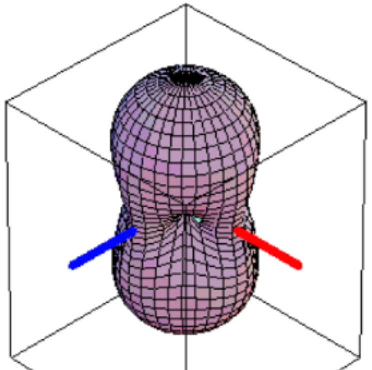
Single GW detector directional sensitivity

$$\frac{\Delta L}{L} = h_{\mathrm{det}}(t) = F_+ h_+(t) + F_x h_x(t)$$

The **antenna pattern** depends on the polarization in a certain (x,+) basis.

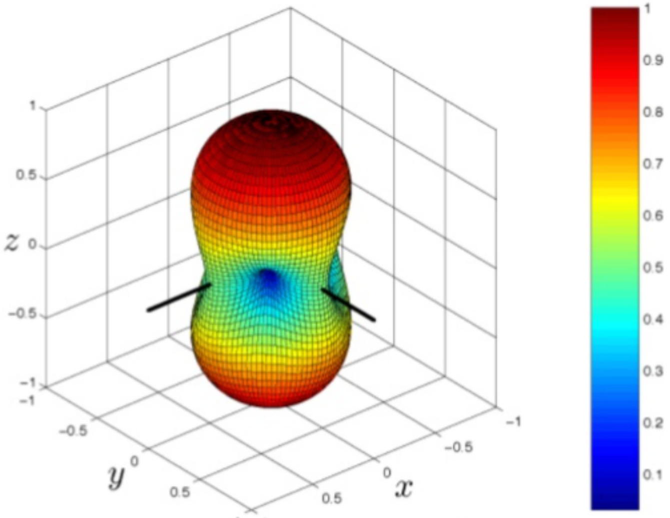






RMS sensitivity

$$\sqrt{F_+(\theta,\phi)^2 + F_x(\theta,\phi)^2}$$

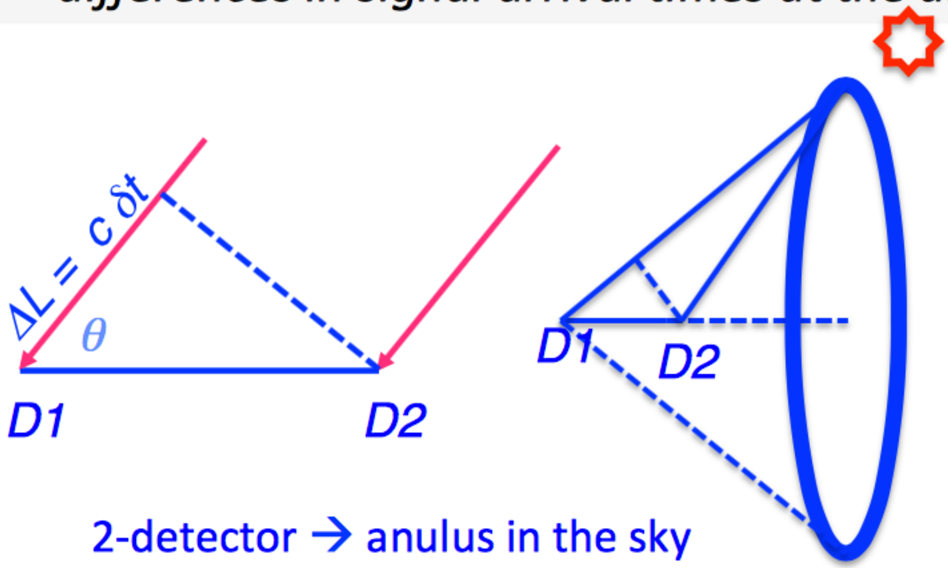


$\sqrt{F_+^2(\theta,\phi,\psi=0)+F_\times^2(\theta,\phi,\psi=0)}$

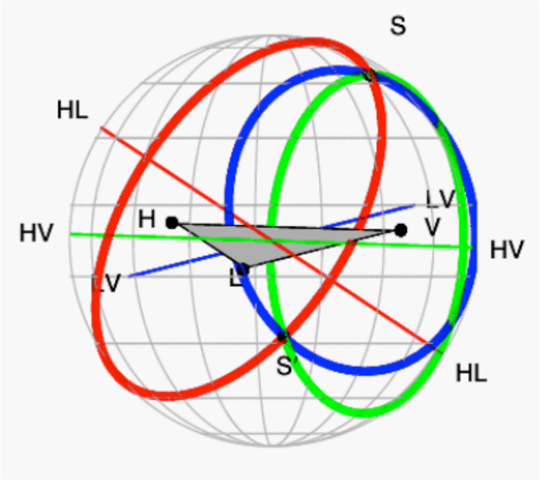
- Single GW detector is a **good all-sky monitor**, nearly omni-directional
  (the transparency of Earth to GWs)
- But does not have good directional sensitivity, **not a pointing instrument!**
  It has a very poor angular resolution (about 100 degrees)

The source localization requires a network of GW detectors

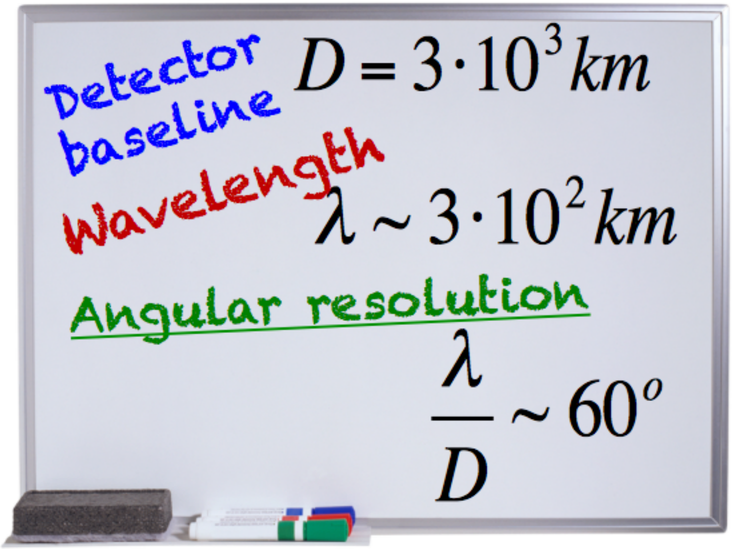
The ***sky position*** *of a GW source is mainly* ***evaluated by triangulation****, measuring the differences in signal arrival times at the different network detector sites*



2-detector → anulus in the sky



3-detectors → localize



Detector baseline $D = 3 \cdot 10^3\,km$

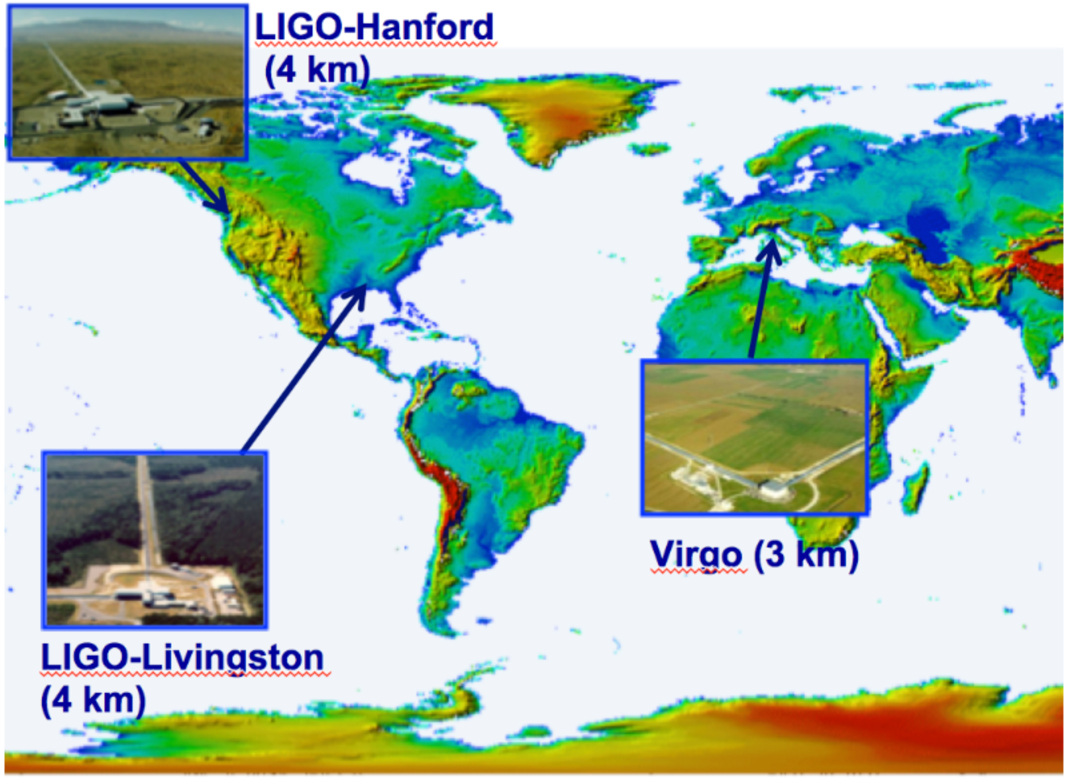
Wavelength $\lambda \sim 3 \cdot 10^2\,km$

Angular resolution $\dfrac{\lambda}{D} \sim 60^o$

The GW lengths are comparable to Earth diameter

→ *longer baseline and greater number of the sites* *distributed* *worldwide* *significantly improve the sky-localization capabilities!*

Other benefits of a network of GW detectors



Improvements:

- ❖ Sensitivity
- ❖ Observation time, and sky coverage
- ❖ In determining the polarization
- ❖ Ability to reconstruct the GW source parameters
- ❖ False alarm rejection thanks to coincidence

Virgo and the LIGO Scientific Collaborations have signed a MoA for full data exchange and joint data analysis and publication policy

Advanced LIGOs and **Virgo** will observe the sky (10-1000 Hz) as a **single network** aiming at the **first direct detection of GWs**

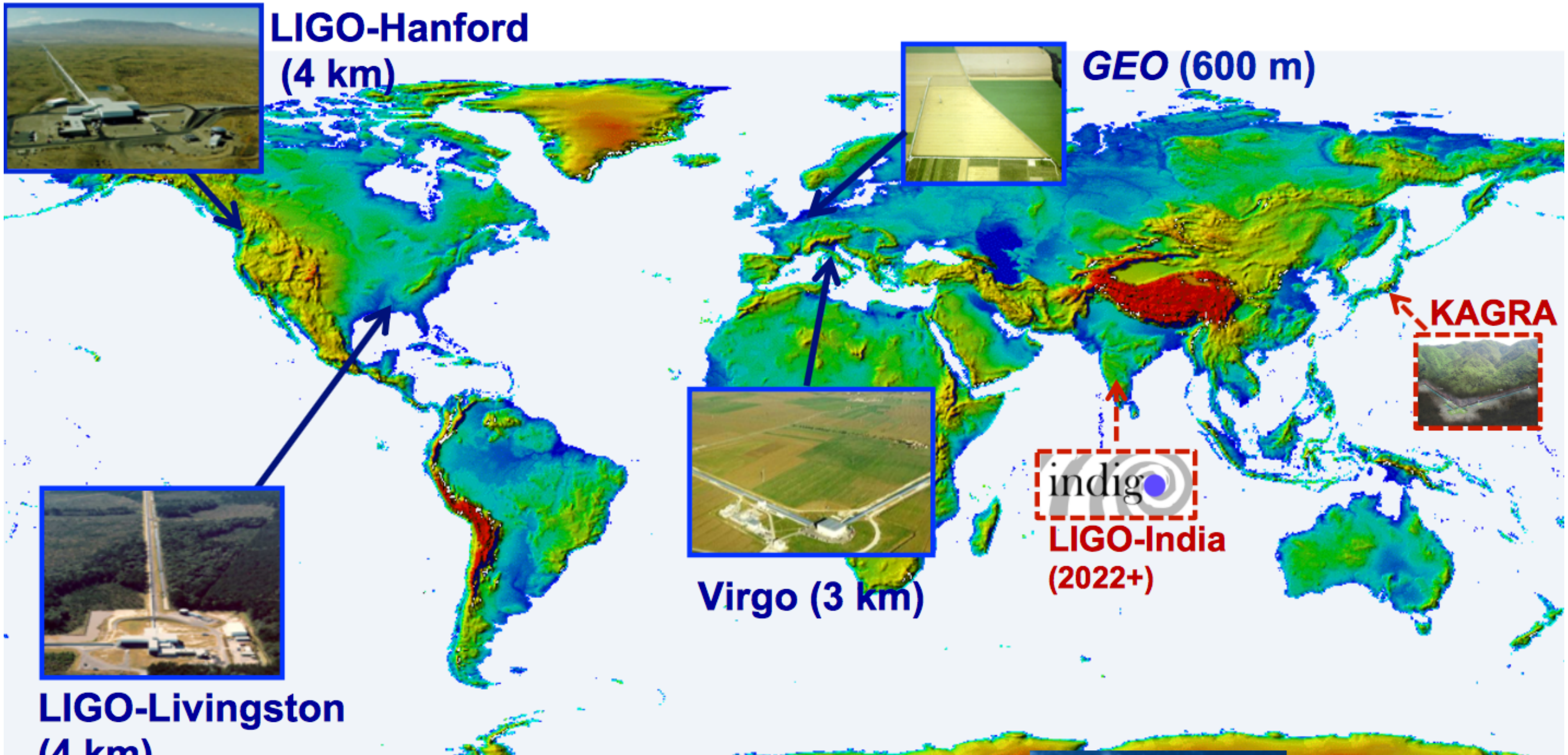
➡

Example of sky-localization capabilties

NS-NS with SNR=7 in each of the LIGOs and Virgo:

- best case localization of **20 deg^2**
  (signal is directly over the plane of network)
- median of **40 deg^2** **(Fairhurst 2009)**

Near Future Gravitational Wave Detectors



LIGO-Hanford (4 km)

GEO **(600 m)**

KAGRA

Virgo (3 km)

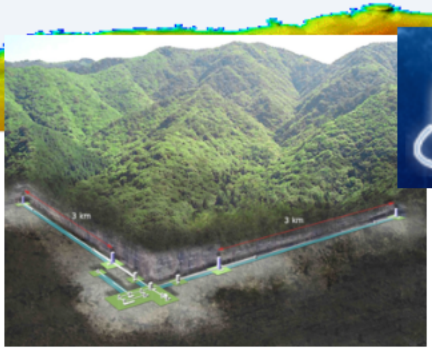
indigo

LIGO-India (2022+)

LIGO-Livingston (4 km)

indigo

Move one of the two LIGO detectors in Hanford to India



KAGRA

Underground detector in the Kamioka mine: 3km length,
– Cryogenic mirrors (2nd phase)

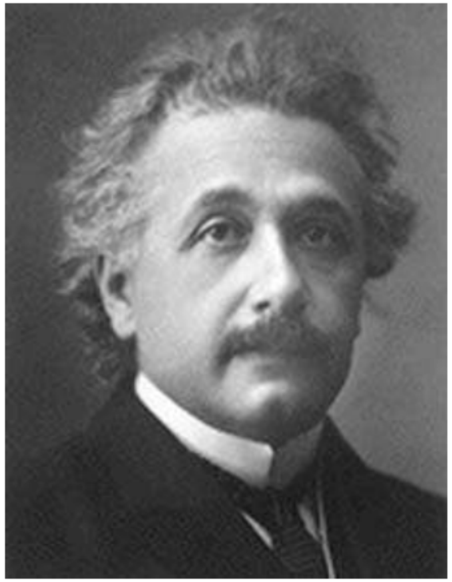
SUPAGWD

An Introduction to General Relativity, Gravitational Waves and Detection Principles

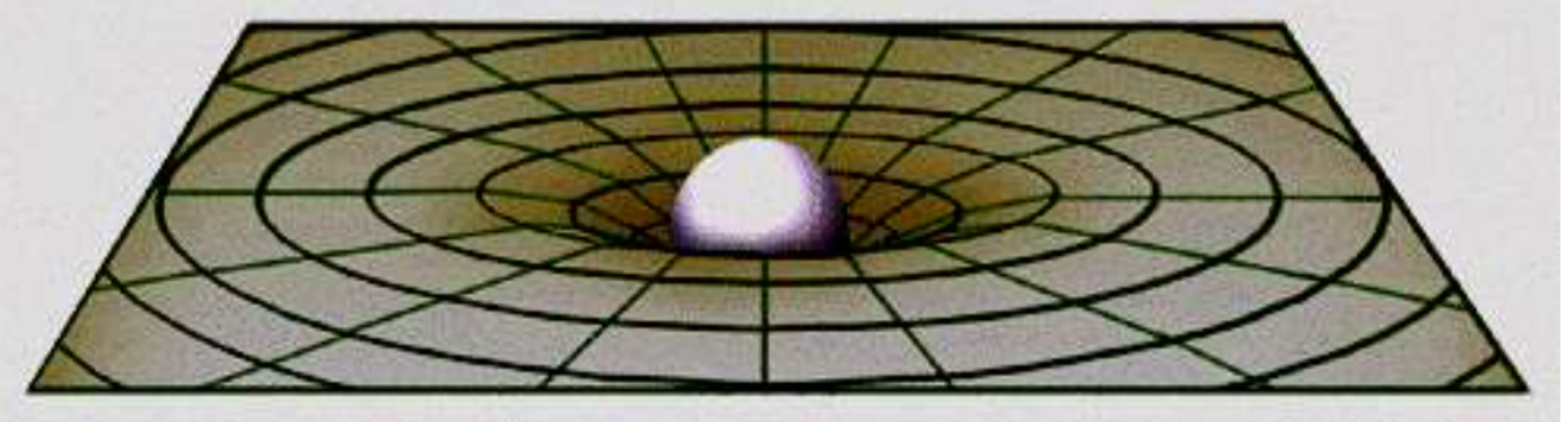
Prof Martin Hendry
University of Glasgow
Dept of Physics and Astronomy

October 2012

Gravity in Einstein's Universe



Spacetime tells matter how to move, and matter tells spacetime how to curve



"...joy and amazement at the beauty and grandeur of this world of which man can just form a faint notion."

$$\boxed{G_{\mu\nu} = \kappa T_{\mu\nu}}$$

Spacetime curvature

Matter (and energy)

5. Einstein's Equations (pgs.38 - 45)

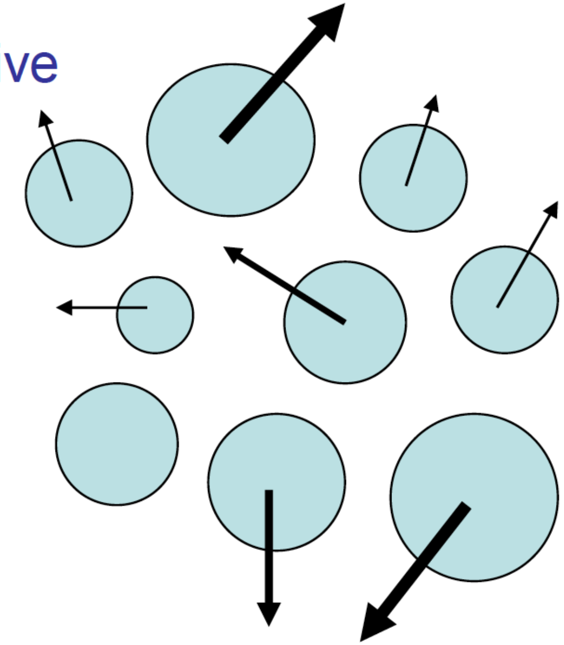
What about "matter tells spacetime how to curve"?...

The source of spacetime curvature is the **Energy-momentum tensor** which describes the presence and motion of gravitating matter (and energy).

We define the E-M tensor for a **perfect fluid**

In a fluid description we treat our physical system as a smooth continuum, and describe its behaviour in terms of locally averaged properties in each ***fluid element****.*

Each fluid element may possess a **bulk motion** with respect to the rest of the fluid, and this relative motion may be non-uniform.



At any instant we can define

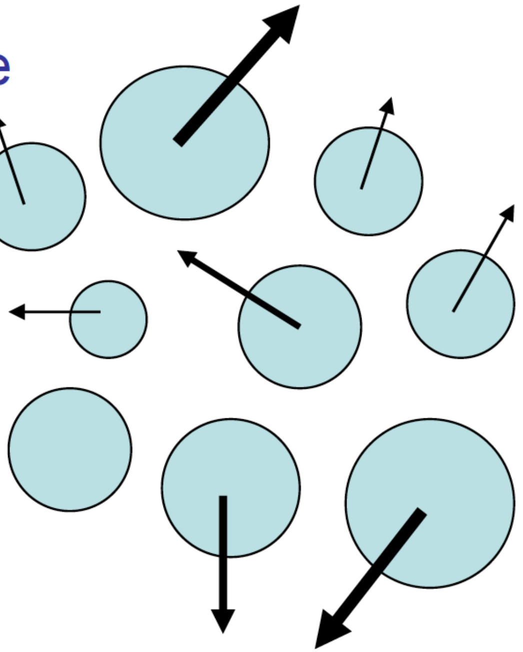
Momentarily comoving rest frame (MCRF)

of the fluid element – Lorentz Frame in which the fluid element as a whole is instantaneously at rest.

Particles in the fluid element will not be at rest:

1. Pressure (c.f. molecules in an ideal gas)
2. Heat conduction (energy exchange with neighbours)
3. Viscous forces (shearing of fluid)

Each fluid element may possess a **bulk motion** with respect to the rest of the fluid, and this relative motion may be non-uniform.



Perfect Fluid if, in MCRF, each fluid element has no heat conduction or viscous forces, only pressure.

Dust = special case of pressure-free perfect fluid.

Definition of E-M tensor

We can define the energy momentum tensor, **T**, in terms of its components in some coordinate system, $\{x^1, x^2, ..., x^n\}$, for each fluid element. Thus we define $T^{\alpha\beta}$ for a fluid element to be equal to the **flux of the α component of four momentum of all gravitating matter**[2] **across a surface of constant** x^β.

---

[2] By 'gravitating matter' we mean here all material particles, plus (from the equivalence of matter and energy) any electromagnetic fields and particle fields which may be present

Components of T in the MCRF for dust

only non-zero component is $\underline{T^{00} = \rho}$, the energy density of the fluid element.

Components of T in the MCRF for a general perfect fluid

$$\mathbf{T} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Pressure due to random motion of particles in fluid element

Components of T in a general Lorentz frame

Extending our expression for $T^{\alpha\beta}$ from the MCRF to a general Lorentz frame is fairly straightforward, but the interested reader is referred e.g. to Schutz for the details and here we just state the result. If $\vec{u} = \{u^\alpha\}$ is the *four* velocity of a fluid element in some Lorentz frame, then

$$\boxed{T^{\alpha\beta} = (\rho + P)u^\alpha u^\beta + P\eta^{\alpha\beta},}$$

where $\eta^{\alpha\beta}$ is the Minkowski metric of SR.

Conservation of energy and momentum requires that

$$\boxed{T^{\alpha\beta}_{\ \ ,\beta} = 0.}$$

Extending to GR

In Section 1 we introduced the strong principle of equivalence which stated that, in a LIF, all physical phenomena are in agreement with special relativity. In the light of our discussion of tensors, we can write down an immediate consequence of the strong principle of equivalence as follows

Any physical law which can be expressed as a tensor equation in SR has exactly the same form in a local inertial frame of a curved spacetime

How is this extension justified? From the principle of covariance a tensorial description of physical laws must be equally valid in any reference frame. Thus, if a tensor equation holds in one frame it must hold in any frame. In particular, a tensor equation derived in a LIF (i.e. assuming SR) remains valid in an arbitrary reference frame (i.e. assuming GR).

Hence

$$\boxed{T^{\mu\nu} = (\rho + P)u^{\mu}u^{\nu} + Pg^{\mu\nu}}$$

and

$$\boxed{T^{\mu\nu}_{;\nu} = 0}$$

Covariant expression of energy conservation in a curved spacetime.

So how does "matter tell spacetime how to curve"?...

Einstein's Equations

BUT the E-M tensor is of rank 2, whereas the R-C tensor is of rank 4.

Einstein's equations involve contractions of the R-C tensor.

Define the **Ricci tensor** by $R_{\alpha\gamma} = R^{\mu}_{\alpha\mu\gamma}$

and the **curvature scalar** by $R = g^{\alpha\beta} R_{\alpha\beta}$

We can raise indices via $R^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} R_{\alpha\beta}$

and define the Einstein tensor

$$\boxed{G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R}$$

We can show that $G^{\mu\nu}{}_{;\nu} = 0$

so that

$$\boxed{T^{\mu\nu}{}_{;\nu} = G^{\mu\nu}{}_{;\nu}}$$

Einstein took as solution the form

$$\boxed{G^{\mu\nu} = kT^{\mu\nu}}$$

where we can determine the constant k by requiring that we should recover the laws of Newtonian gravity and dynamics in the limit of a weak gravitational field and non-relativistic motion. In fact k turns out to equal $8\pi G/c^4$.

Solving Einstein's equations

Given the metric, we can compute the Christoffel symbols, then the geodesics of 'test' particles.

We can also compute the R-C tensor, Einstein tensor and E-M tensor.

6. Wave Equation for Gravitational Radiation (pgs.46 – 57)

Weak gravitational fields

In the absence of a gravitational field, spacetime is flat. We define a weak gravitational field as one is which spacetime is 'nearly flat'

i.e. we can find a coord system such that

$$\boxed{g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}}$$

where

$$\eta_{\alpha\beta} = \mathrm{diag}\ (-1, 1, 1, 1)$$

$$|h_{\alpha\beta}| << 1 \ \text{ for all } \ \alpha \ \text{ and } \ \beta$$

This is known as a Nearly Lorentz coordinate system.

If we find a coordinate system in which spacetime looks nearly flat, we can carry out certain coordinate transformations after which spacetime will *still* look nearly flat:

1) Background Lorentz transformations

$$(t', x', y', z')^T = \begin{pmatrix} \gamma & -v\gamma & 0 & 0 \\ -v\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} (t, x, y, z)^T$$

i.e. **Lorentz boost** of speed v

If we find a coordinate system in which spacetime looks nearly flat, we can carry out certain coordinate transformations after which spacetime will *still* look nearly flat:

1) Background Lorentz transformations

Under this transformation

$$\boxed{g'_{\alpha\beta} = \eta'_{\alpha\beta} + \frac{\partial x^\mu}{\partial x'^\alpha}\frac{\partial x^\nu}{\partial x'^\beta} h_{\mu\nu} = \eta_{\alpha\beta} + h'_{\alpha\beta}}$$

provided $v << 1$, then if $|h_{\alpha\beta}| << 1$

for all α and β, then $|h'_{\alpha\beta}| << 1$ also.

If we find a coordinate system in which spacetime looks nearly flat, we can carry out certain coordinate transformations after which spacetime will *still* look nearly flat:

1) Background Lorentz transformations

Hence, our original nearly Lorentz coordinate system remains nearly Lorentz in the new coordinate system. In other words, a spacetime which looks nearly flat to one observer still looks nearly flat to any other observer in uniform relative motion with respect to the first observer.