

## Complements to Lessons 1 - 2.

### 1. $k$ -varieties in $\mathbb{A}_K^n$ or $\mathbb{P}_K^n$ .

We start from two fields  $k \subset K$ ,  $k$  subfield of  $K$ .

A  $k$ -variety in  $\mathbb{A}_K^n$  is a subset  $V \subset \mathbb{A}_K^n$  of the form  $V(S)$ , where  $S$  is a subset of  $k[x_1, x_2, \dots, x_n]$ . A similar definition can be given in  $\mathbb{P}_K^n$ .

Also  $k$ -varieties satisfy the axioms of the closed subsets of a (different) topology in  $\mathbb{A}_K^n$ . This more general point of view is adopted when interested in arithmetic problems. For instance, if  $k = \mathbb{Q}$  and  $K = \mathbb{C}$ , we consider only equations with rational, or integer coefficients. This point of view is adopted in the book of E. Kunz: Introduction to Commutative Algebra and Algebraic Geometry, Birkhauser.

### 2. Graded rings.

In the definition of graded ring, we could replace  $\mathbb{Z}$  with any group or simply monoid  $G$ . This leads to the definition of a  $G$ -graduation.

On the polynomial ring  $K[x_0, x_1, \dots, x_n]$  there are many different  $\mathbb{Z}$ -graduations. We can declare that  $\deg x_0 = a_0, \dots, \deg x_n = a_n$  where  $a_0, \dots, a_n$  are arbitrary positive integers. This leads to the definition of weighted projective space.

### 3. Historical note.

The Zariski topology takes its name from the Russian mathematician Oscar Zariski (1899 - 1986). Zariski, who studied and lived many years in Rome before emigrating to the USA, re-founded classical algebraic geometry on solid algebraic bases.

A nice biography of Zariski can be found in the web site

<http://mathshistory.st-andrews.ac.uk/Biographies/Zariski.html>

of the University of Saint Andrews. Also the wikipedia page on Zariski in English gives interesting informations.

Among the students of Zariski I would like to recall, for instance, Heisuke Hironaka, Robin Hartshorne, Emil Artin, David Mumford, Steve Kleiman, and several others.

A book on Zariski, containing both a personal and a mathematical biography, is due to Carol Parikh (2008): The Unreal Life of Oscar Zariski. Springer. It is in the library of the DMG.