Exercise 1. Two-particle phase space

Compute the Lorentz-invariant phase space for a scattering process of two-particle in two particles ($|q_1, q_2\rangle \to |p_1, p_2\rangle$)

$$\int f(s,\Omega) d\Pi_{\text{LIPS}} = \int f(s,\Omega)(2\pi)^2 \delta^4(q_1 + q_2 - p_1 - p_2) \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} . \qquad (1)$$

Notice that in the center of mass frame the amplitude for the process, $f(s,\Omega)$, can depend only on the center of mass energy $s = (p_1 + p_2)^2$ and an angle θ, defined above as part of the angular variable $d\Omega = d\phi\, d\sin\theta$.

Exercise 2.

Comoute the total cross section σ for the scattering $e^+(q_1)e^-(q_2) \to \mu^+(p_1)\mu^-(p_2)$, given the spin-avaraged amplitude square of the process

$$\sum_{\text{spin}} \frac{1}{4} |\mathscr{M}|^2 = \frac{2e^4}{s^2}(t^2 + u^2) , \qquad (2)$$

where s, t, u are the Mandelstam variables.

Hint. Assuming the energy is $s \gg m_\mu^2$, neglect electron and muon masses.