MECCANICA RAZIONALE

- LEZIONE DEL 10 MARZO 2020
- PRIMA PARTE

Problema: il corpo rigido R.

Coordinate libere:

corpo rigido in 3D → 6

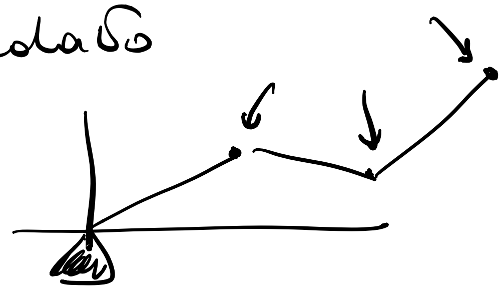
corpo rigido in 2D → 3

Analogamente: corpo rigido libero

→ corpo rigido vincolato

→ corpo articolato

Corpo articolato



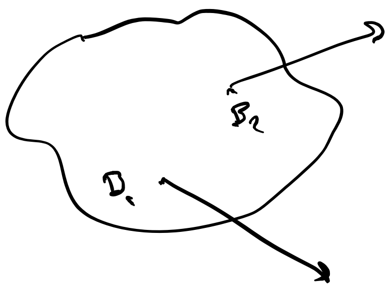
 cerniera fissa

 carrello mobile

Principio dei lavori virtuali

Spostamenti virtuali (invertibili)

R : insieme di forze applicate su R

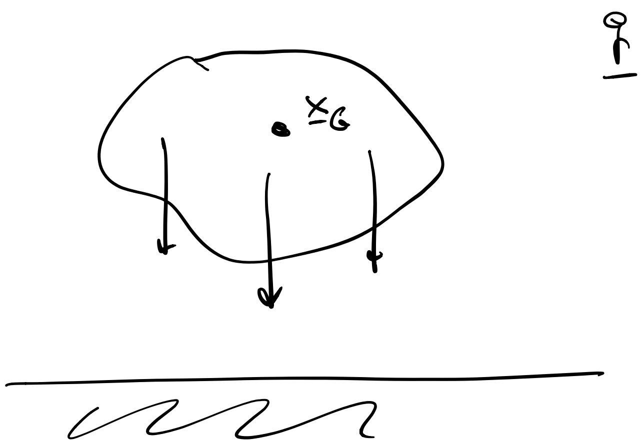


$$LV = \sum_{B \in R} \underline{F}_B \cdot \delta \underline{x}_B$$

All'equilibrio $\quad LV \leq 0 \quad$ (= 0 spostamenti invertibili)

Sistemi notevoli

sistemi materiali soggetti al solo peso



$$LV = M \underline{g} \cdot \delta \underline{x}_G$$

PLV per sistemi a 1 grado di libertà

$$LV = \sum_{B \in R}' \underline{F}_B \cdot \delta \underline{x}_B \quad \longrightarrow \quad \text{coordinate libere}$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\text{Tutti i punti}}$$

$$LV = \sum_{B \in R}' \underline{F}_B \cdot \delta \underline{x}_B = \sum_{B \in R} \underline{F}_B \frac{d\underline{x}_B}{dq} \delta q =$$

$$\longrightarrow \delta \underline{x}_B = \frac{d\underline{x}_B}{dq} \delta q$$

$$= \underbrace{\left(\sum_{B \in R} \underline{F}_B \cdot \frac{d\underline{x}_B}{dq} \right)}_{Q} \underline{\underline{\delta q}}$$

Q forza generalizzata

$$LV = Q \, \delta q \xrightarrow{PLV} Q = 0$$

Tutti gli spostamenti virtuali $\delta \underline{x}_B$, ora dipendono da δq

<u>Principio dei lavori virtuali per sistemi a ℓ gradi di libertà</u>

ℓ gradi di libertà $\Rightarrow$ $\underline{q} = (q_1, \dots, q_\ell)$

$\delta \underline{x}_B (q_1, \dots, q_\ell) \longrightarrow \delta q_1, \dots, \delta q_\ell$

Espandere in serie $\underline{x}_B(\underline{q})$

$$\underline{x}_B(\underline{q} + \delta\underline{q}) = \underline{x}_B(\underline{q}) + \sum_{i=1}^{\ell} \left(\frac{\partial}{\partial q_i} \underline{x}_B(\underline{q}) \right) \delta q_i + \dots$$

spostamenti virtuali $\longleftrightarrow$ lavoriamo al primo ordine

$$LV = \sum_{B \in R} \underline{F}_B \cdot \delta \underline{x}_B = \sum_{B \in R} \underline{F}_B \cdot \left(\sum_{i=1}^{\ell} \frac{\partial \underline{x}_B}{\partial q_i} \delta q_i \right)$$

$$\left(\delta \underline{x}_B(\underline{q}) = \sum_{i=1}^{\ell} \left(\frac{\partial}{\partial q_i} \underline{x}_B(\underline{q}) \right) \delta q_i \right)$$

$$LV = \sum_{i=1}^{\ell} \left(\sum_{B \in R} \underline{F}_B \cdot \frac{\partial \underline{x}_B}{\partial q_i} \right) \delta q_i$$

Forze generalizzate : $Q_i = \sum'_{B \in R} \underline{F}_B \cdot \frac{\partial \underline{x}_B}{\partial q_i}$

relativa a q_i

Tutti spostamenti virtuali invertibili

$\Rightarrow$ PLV : $LV = \sum'_{B \in R} \underline{F}_B \cdot \delta \underline{x}_B =$

$= \sum_i Q_i \, \delta q_i = 0 \iff Q_i = 0 \quad \forall i = 1, \dots, l$

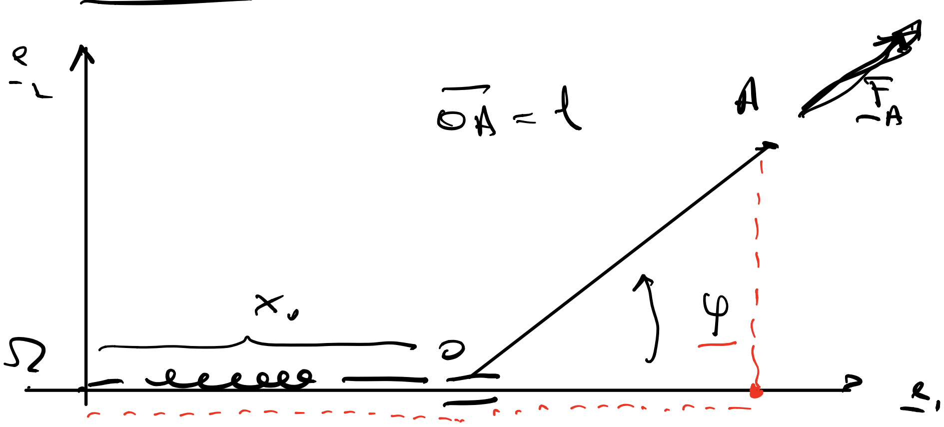
Importante : tutti δq_i sono <u>indipendenti</u>

Equilibrio : $LV = 0 \quad \forall \, \delta \underline{x}_B$

$$\Updownarrow$$

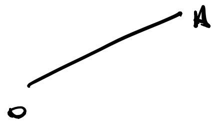
$$Q_i = 0$$

<u>Esempio</u> : Problema inverso in statica



date le forze agenti,

$\rightarrow$ determinare la configurazione di equilibrio



3 gradi di libertà + vincolo $\longrightarrow$ rimangono solamente 2

=

Forze attive

1. forza elastica in O : $\underline{F}_0 = -c\, x_0\, \underline{e}_1$
2. forza "follower" : $\underline{F}_A = f \,\text{vers}\,(\underline{x}_A - \underline{x}_0)$

$$= f \, \frac{\underline{x}_A - \underline{x}_0}{\| \underline{x}_A - \underline{x}_0 \|}$$

Problema : andiamo a trovare Q_{x_0} e Q_φ

$$LV = \underline{F}_0 \cdot \delta \underline{x}_0 + \underline{F}_A \cdot \delta \underline{x}_A$$

$\rightarrow$ $\underline{x}_0 = x_0\, \underline{e}_1 \implies \delta \underline{x}_0 = \delta x_0\, \underline{e}_1$

$\rightarrow$ $\underline{x}_A = \underbrace{(x_0 + l \cos\varphi)\, \underline{e}_1 + l \sin\varphi\, \underline{e}_2}$

$$\delta \underline{x}_A = \delta x_0\, \underline{e}_1 + (-l \sin\varphi\, \underline{e}_1 + l \cos\varphi\, \underline{e}_2)\, \delta\varphi$$

Allora

$$LV = \underline{F}_0 \cdot \delta \underline{x}_0 + \underline{F}_A \cdot \delta \underline{x}_A =$$

$$= (-c\, x_0\, \underline{e}_1) \cdot (\delta x_0 \cdot \underline{e}_1) +$$

$$+ f \, \frac{\underline{x}_A - \underline{x}_0}{\| \underline{x}_A - \underline{x}_0 \|} \left(\delta x_0\, \underline{e}_1 + \delta\varphi \left(-l \sin\varphi\, \underline{e}_1 + l \cos\varphi\, \underline{e}_2 \right) \right)$$

$$= \left(-c\, x_0\, \delta x_0\right) + f\, \frac{l \cos\varphi\, \underline{e}_1 + l \sin\varphi\, \underline{e}_2}{l} \times$$

$$\times \left(\delta x_0\, \underline{e}_1 + \delta\varphi \left(- l \sin\varphi\, \underline{e}_1 + l \cos\varphi\, \underline{e}_2 \right) \right)$$

$$= -c\, x_0\, \delta x_0 + f \cos\varphi \left(\delta x_0 - l\, \delta\varphi \sin\varphi \right) +$$

$$+ f \sin\varphi\, l \cos\varphi\, \delta\varphi =$$

$$= \underbrace{\left(-c\, x_0 + f \cos\varphi \right)}_{Q_{x_0}} \delta x_0 + \delta\varphi \underbrace{\left(-f l + f l \right) \cos\varphi \sin\varphi}_{Q_\varphi}$$

Equilibrio

$Q_{x_0} = 0 \implies x_0 = \frac{f}{c} \cos\varphi$

$Q_\varphi = 0$ sempre

$\rightarrow$ infinite configurazioni di equilibrio

$$\begin{cases} \varphi = \alpha \\ x_0 = \frac{f}{c} \cos\alpha \end{cases} \qquad \forall\, \alpha$$

Notiamo : dinamometro ($\alpha = 0$) : misurare f sapendo c

MECCANICA RAZIONALE

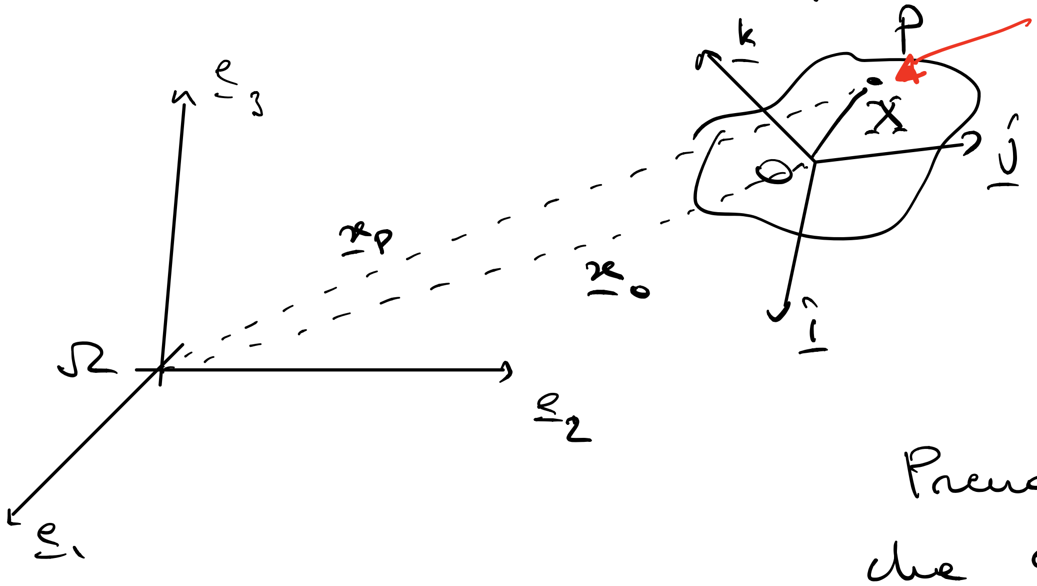
— LEZIONE DEL 10 MARZO 2020

— SECONDA PARTE

CINEMATICA DEI RIGIDI

Teorema di Poisson

Osservatore $\Sigma'(\Omega; \underline{e}_1, \underline{e}_2, \underline{e}_3)$



$S(O; \underline{i}, \underline{j}, \underline{k})$

Prendiamo τ, parametro che caratterizza uno spostamento

Teorema: fissato uno spostamento di $\mathcal{R}$ esiste un unico vettore $\underline{\omega}(\tau)$, tale che

$$\frac{d}{d\tau}\underline{x}_P(\tau) = \frac{d}{d\tau}\underline{x}_O(\tau) + \underline{\omega}_{(\tau)} \wedge (\underline{x}_{P(\tau)} - \underline{x}_{O(\tau)})$$

(formula di Poisson)

Dimostrazione

$$\underline{x}(\tau) = \underline{x}_0(\tau) + R_{(\tau)} \cdot \underline{X}$$

$\underline{x} = \underline{x}_p$

matrice rotazione $\underline{e}_1 \underline{e}_2 \underline{e}_3 \to \underline{i}\,\underline{j}\,\underline{k}$

$$\frac{d}{d\tau}\underline{x} = \frac{d}{d\tau}\underline{x}_0 + \frac{d}{d\tau}R_{(\tau)} \cdot \underline{X}$$

R : matrice ortogonale

$$\left(v^T v = (Qv)^T (Qv) = v^T \underbrace{(Q^T Q)}\, v = v^T v \right)$$

$$R R^T = I \quad \Rightarrow \quad R^{-1} = R^T$$

$$\underline{X} = R^T \cdot (\underline{x} - \underline{x}_0)$$

Sostituendo

$$\frac{d}{d\tau}\underline{x} = \frac{d}{d\tau}\underline{x}_0 + \left(\frac{d}{d\tau}R_{(\tau)}\right) R^T \cdot (\underline{x} - \underline{x}_0)$$

Definiamo $A = \frac{d}{d\tau}R_{(\tau)} \cdot R^T = \dot{R} R^T$

1) Risultato tecnico : A è antisimmetrica

$\rightarrow A_{(\tau)} + A^T_{(\tau)} = 0$

$$R(t)\,R^T(t) = I \implies \frac{d}{dt}\left(R(t)\,R^T(t)\right) = \underbrace{\frac{d}{dt}R \cdot R^T}_{A} + \underbrace{R\,\frac{d}{dt}R^T}_{A^T} = 0 = \frac{d}{dt}(\mathbb{1})$$

$$A^T = (\dot R\, R^T)^T = (R^T)^T \cdot \dot R^T = R\,\frac{d}{dt}R^T$$

$$\implies A + A^T = 0 \longrightarrow \text{antisimmetrica}$$

2) Risultato Tecnico

$A_{3\times 3}$ e antisimmetrica $\implies \exists!\ \underline{\omega} \in \mathbb{R}^3$

Tale che $A\underline{y} = \underline{\omega} \wedge \underline{y} \quad \forall\, \underline{y} \in \mathbb{R}^3$

Per calcolo diretto

$$A\underline{y} = \begin{pmatrix} 0 & A_{12} & A_{13} \\ -A_{12} & 0 & A_{23} \\ -A_{13} & -A_{23} & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\underline{\omega} \wedge \underline{y} = \left\| \begin{matrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_1 & \omega_2 & \omega_3 \\ y_1 & y_2 & y_3 \end{matrix} \right\| = \begin{pmatrix} \omega_2 y_3 - \omega_3 y_2 \\ \omega_3 y_1 - \omega_1 y_3 \\ -\omega_2 y_1 + \omega_1 y_2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$\rightarrow \begin{cases} \omega_3 = -A_{12} \\ \omega_2 = A_{13} \\ \omega_1 = -A_{23} \end{cases}$$

$$\frac{d}{dt}\underline{x} = \frac{d}{dt}\underline{x}_0 + \underbrace{\left(\frac{d}{dt}R\right)R^T}_{A} \cdot (\underline{x} - \underline{x}_0)$$

$$A\,(\underline{x} - \underline{x}_0) = \underline{\omega} \wedge (\underline{x} - \underline{x}_0)$$

CVD

$$\frac{d}{dt}x_p = \frac{d}{dt}x_0 + \underline{\omega} \wedge \left(x_p - x_0\right)$$

Significato fisico di $\underline{\omega}$

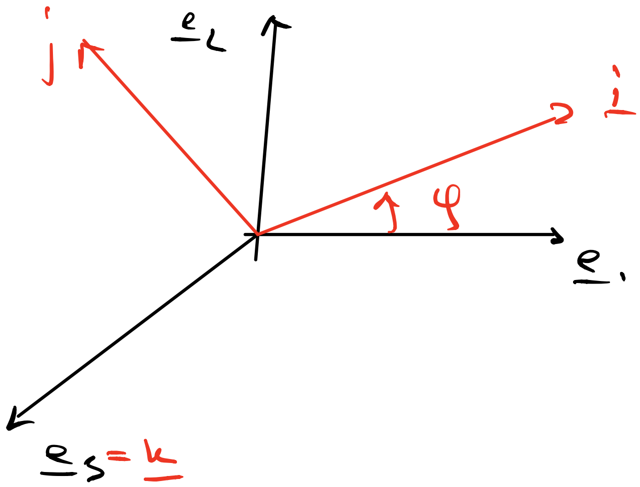
- Traslazioni: gli assi di S non cambiano orientazione rispetto agli assi di Σ'

$$\frac{d}{dt}R = 0 \Rightarrow A = \frac{dR}{dt} \cdot R^T = 0 \Rightarrow \underline{\omega} = \underline{0}$$

- Rotazione di un corpo rigido attorno ad un asse fisso

Scegliamo Σ, S

- $O \equiv \Omega$
- $\underline{e}_3 = \underline{k}$



$$\underline{x}(t) = R(\varphi(t))\,\underline{X}$$

(abbiamo scelto $\underline{x}_0 = 0$)

$$R(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \dot{R} R^T = \begin{pmatrix} -\dot\varphi \sin\varphi & -\dot\varphi\cos\varphi & 0 \\ \dot\varphi\cos\varphi & -\dot\varphi\sin\varphi & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \underbrace{-\dot\varphi \sin\varphi\cos\varphi + \dot\varphi\cos\varphi\sin\varphi}_{=0} & -\dot\varphi & 0 \\ \dot\varphi & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

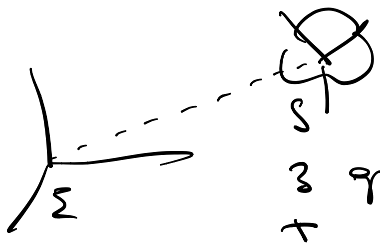
$$= \dot\varphi \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Longrightarrow \begin{matrix} \omega_1 = 0 \\ \omega_2 = 0 \\ \omega_3 = \dot\varphi \end{matrix}$$

$\Rightarrow \quad \underline{\omega} = \dot{\varphi}\, \underline{e}_3 = \dot{\varphi}\, \underline{k} \qquad (\underline{e}_3 = \underline{k})$

da cui $\boxed{\dfrac{d}{dt}\underline{x} = \dot{\varphi}\; \underline{e}_3 \wedge \underline{x}}$

•) **Precessione** (moto con punto fisso)

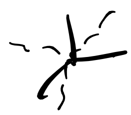
Σ, S $\qquad \Omega = O =$ punto fisso



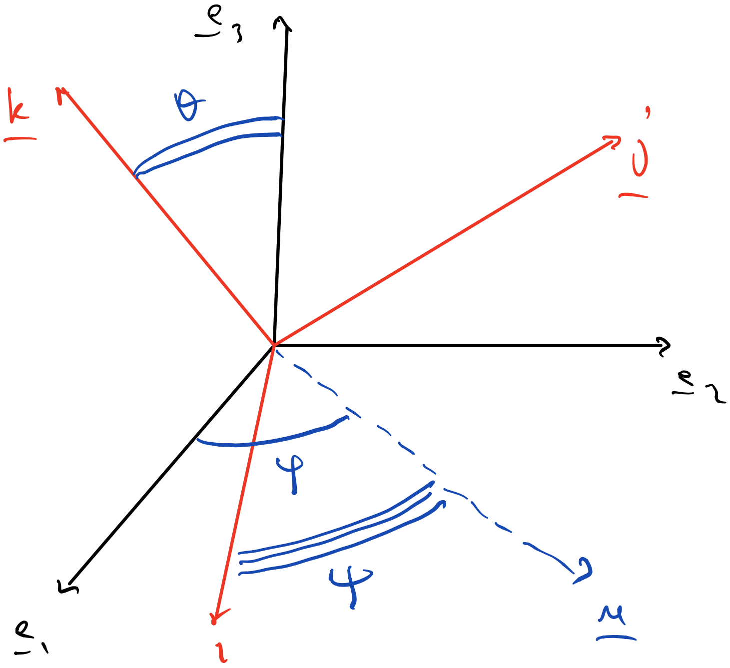
3 gradi di libertà ← rimossi

+

3



Angoli di Eulero



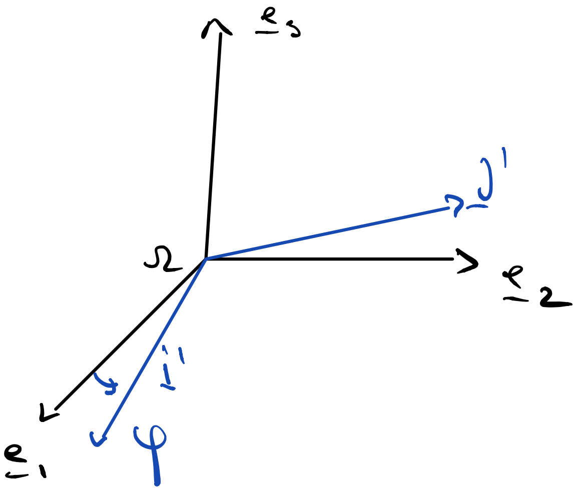
$\underline{u} = \underline{e}_3 \wedge \underline{k}$

asse dei nodi

- φ angolo di precessione (tra $\underline{e}_1$ e $\underline{u}$)
- θ angolo di nutazione (tra $\underline{k}$ e $\underline{e}_3$)
- ψ angolo di rotazione propria (tra $\underline{i}$ e $\underline{u}$)

Vogliamo costruire la matrice di rotazione fra S e Σ' come la successione di tre opportune rotazioni indipendenti

1 Rotazione intorno ad $\underline{e}_3$ di un angolo φ



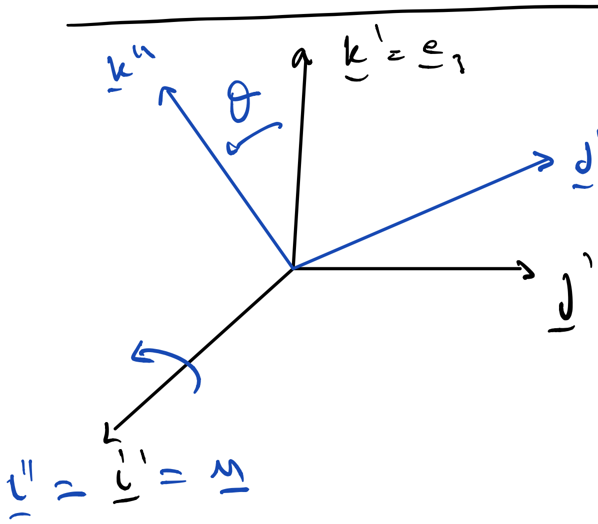
$$\Sigma' \longrightarrow S'(\Omega;\ \underline{i}',\ \underline{j}',\ \underline{k}'=\underline{e}_3)$$

$$(\underline{x})_{\Sigma} = R_1(\varphi)\cdot(\underline{x})_{S'}$$

$$R_1(\varphi)=\begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Poniamo $\underline{i}' = \underline{u}$

2 Rotazione intorno a $\underline{i}'=\underline{u}$ di un angolo θ

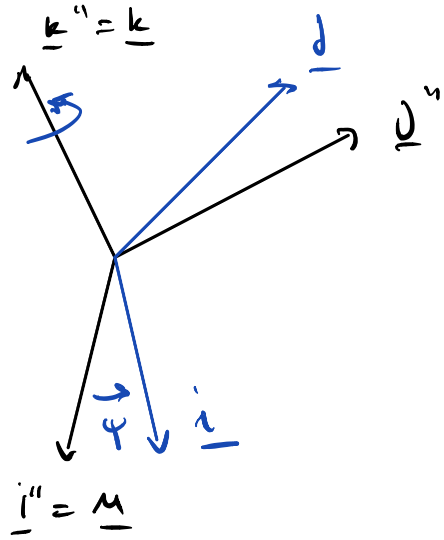


$$S' \longrightarrow S''(\Omega,\ \underline{i}''=\underline{i}'=\underline{u},\ \underline{j}'',\ \underline{k}'')$$

$$(\underline{x})_{S'} = R_2(\theta)\,(\underline{x})_{S''}$$

$$R_2(\theta)=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

3 Rotazione intorno a $\underline{k}''$ di un angolo ψ



$$S'' \longrightarrow S(R;\ \underline{i}, \underline{j}, \underline{k})$$

$$(\underline{x})_{S''} = R_3(\psi)(\underline{x})_S$$

$$R_3(\psi) = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(\underline{x})_\Sigma = R_1(\varphi)(\underline{x})_{S'} = R_1(\varphi)\, R_2(\theta)\, (\underline{x})_{S''}$$

$$= R_1(\varphi)\, R_2(\theta)\, R_3(\psi)\, (\underline{x})_S$$

$$R = \begin{pmatrix} \cos\varphi\cos\psi - \sin\varphi\cos\theta\sin\psi & -\cos\varphi\sin\psi - \sin\varphi\cos\theta\cos\psi & \sin\varphi\sin\theta \\ \sin\varphi\cos\psi + \cos\varphi\cos\theta\sin\psi & -\sin\varphi\sin\psi + \cos\varphi\cos\theta\cos\psi & -\cos\varphi\sin\theta \\ \sin\theta\sin\psi & \sin\theta\cos\psi & \cos\theta \end{pmatrix}$$

$$\longrightarrow \underline{\omega} \qquad \left(A = \dot{R} R^T \right)$$

$$\underline{x}(\tau) = \underline{x}(\varphi(\tau), \theta(\tau), \psi(\tau))$$

Allora $$\frac{d}{d\tau}\underline{x}(\tau) = \frac{\partial}{\partial\varphi}\underline{x}\,\dot{\varphi} + \frac{\partial}{\partial\theta}\underline{x}\,\dot{\theta} + \frac{\partial}{\partial\psi}\underline{x}\,\dot{\psi}$$

- $\frac{\partial}{\partial\varphi}\underline{x}\,\dot\varphi \longrightarrow$ (fissiamo $\underline{\theta}$ e $\underline{\psi}$)
rotazione attorno ad un asse fisso

$$\frac{\partial}{\partial\varphi}\underline{x}\,\dot\varphi = \dot\varphi\,\underline{e}_3\wedge\underline{x}$$

- consideriamo $\left(\frac{\partial}{\partial\theta}\underline{x}\right)\dot\theta$: se prendiamo $\underline{\varphi}$ e $\underline{\psi}$ fissati
rotazione di θ intorno ad asse $\underline{u}$

$$\frac{\partial\underline{x}}{\partial\theta}\dot\theta = \dot\theta\,\underline{u}\wedge\underline{x}$$

- φ e θ fissati $\rightarrow \left(\frac{\partial}{\partial\psi}\underline{x}\right)\dot\psi = \dot\psi\,\underline{k}\wedge\underline{x}$

Per concludere

$$\frac{d}{dt}\underline{x} = \left(\dot\varphi\,\underline{e}_3 + \dot\theta\,\underline{u} + \dot\psi\,\underline{k}\right)\wedge\underline{x}$$

Unicità di $\underline{\omega} \Rightarrow \underline{\omega} = \dot\varphi\,\underline{e}_3 + \dot\theta\,\underline{u} + \dot\psi\,\underline{k}$

= somma delle velocità angolari relative a 3 rotazioni indipendenti intorno agli assi $\underline{e}_3$, $\underline{u}$, $\underline{k}$

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– TERZA PARTE

Angoli di Eulero $\varphi, \theta, \psi \rightarrow \underline{e}_3, \underline{n}, \underline{k}$

<u>Spostamenti virtuali e lavoro virtuale di</u>

<u>un corpo rigido</u>

$$P \qquad \delta \underline{x}_P = \frac{d \underline{x}_P}{d\tau} d\tau = \frac{d}{d\tau} \underline{x}_O \, d\tau + \underline{\omega} \, d\tau \wedge (\underline{x}_P - \underline{x}_O)$$

Poisson   funzioni arbitrarie

Definiamo
$$\begin{cases} \delta \underline{x}_O = \dfrac{d}{d\tau} \underline{x}_O \, d\tau \\[2ex] \underline{\chi} = \underline{\omega}(\tau) \, d\tau \end{cases}$$

$$\underline{\chi} = \underline{\omega} \, d\tau = \dot{\varphi} \, d\tau \, \underline{e}_3 + \dot{\theta} \, d\tau \, \underline{n} + \dot{\psi} \, d\tau \, \underline{k}$$

$$= \delta\varphi \, \underline{e}_3 + \delta\theta \, \underline{n} + \delta\psi \, \underline{k}$$

$$\delta \underline{x}_P = \delta \underline{x}_O + \underline{\chi} \wedge (\underline{x}_P - \underline{x}_O) \qquad \forall \, P, O \in \mathcal{R}$$

$\rightarrow$ spostamento virtuale di $P \in \mathcal{R}$

LV per un rigido $= \sum'_{P \in R} \underline{F}_P \cdot \delta \underline{x}_P =$

sistema di forze agenti sul corpo rigido

$$= \sum'_{P \in R} \underline{F}_P \cdot \left(\delta \underline{x}_O + \underline{\chi} \wedge (\underline{x}_P - \underline{x}_O) \right) =$$

$$= \sum_{P \in R} \underline{F}_P \cdot \delta \underline{x}_O + \sum'_{P \in R} \left(\underline{\chi} \wedge (\underline{x}_P - \underline{x}_O) \right) \cdot \underline{F}_P$$

$$= \delta \underline{x}_O \cdot \left(\sum_{P \in R} \underline{F}_P \right) + \sum'_{P \in R} \underline{\chi} \cdot (\underline{x}_P - \underline{x}_O) \wedge \underline{F}_P$$

$$\underline{a} \wedge \underline{b} \cdot \underline{c} = \underline{a} \cdot \underline{b} \wedge \underline{c} \qquad \underline{a}, \underline{b}, \underline{c}$$

$$\left\| \begin{matrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix} \right\| \cdot \underline{c} = \underline{a} \cdot \left\| \begin{matrix} i & j & k \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{matrix} \right\|$$

Definiamo $\underline{R} = \sum'_{P \in R} \underline{F}_P$ risultante delle forze

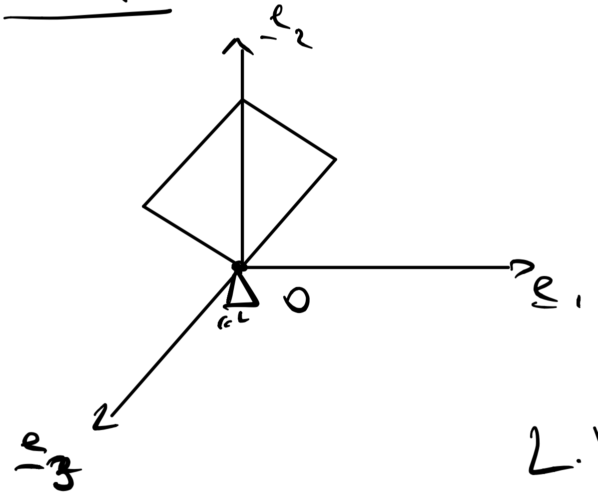
$$\underline{M}(O) = \sum'_{P \in R} (\underline{x}_P - \underline{x}_O) \wedge \underline{F}_P$$

momento risultante delle forze

$$\boxed{L.V. = \delta \underline{x}_O \cdot \underline{R} + \underline{\chi} \cdot \underline{M}(O)}$$

Lavoro virtuale di un rigido dipende solo da $\underline{R}$, $\underline{M}(O)$

<u>Esempio</u>



Sistema di forze

$$\underline{R} = f\,\underline{e}_1$$

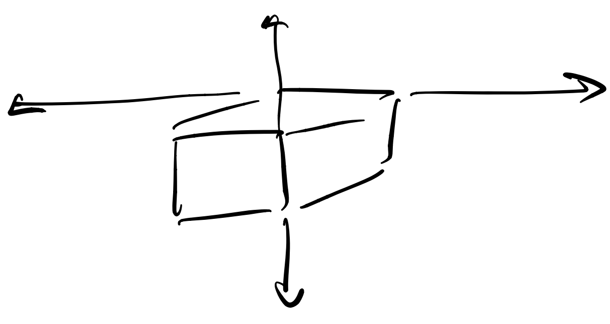
$$\underline{M}(O) = \mu\,\underline{e}_3$$

$$L.V. = \underbrace{\delta \underline{x}_O}_{0} \cdot \underline{R} + \underbrace{\underline{\chi}}_{\text{piano}} \cdot \underline{M}(O)$$

$$\underline{\chi} = \delta\varphi\,\underline{e}_3$$

$$LV = \mu\,\delta\varphi$$

<u>Esempio</u>: rigido R su cui agisce un sistema di forze con $\underline{R} = 0$ $\underline{M}(O) = 0$



$$L.V. = 0$$

<u>Caso di un corpo rigido piano</u>

in questo caso, scegliamo $\underline{e}_3 = \underline{k}$ (normale al piano)

φ angolo di rotazione

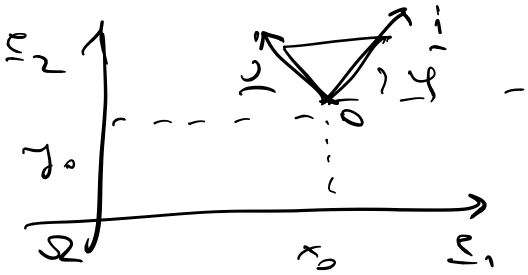
$\delta \underline{x}_0 = \delta x_0 \, \underline{e}_1 + \delta y_0 \, \underline{e}_2$

$\underline{\chi} = \delta\varphi \, \underline{e}_3$

(x, y, φ)
3 coordinate
libere
nel piano

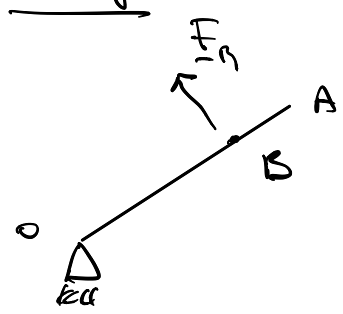
$\hookrightarrow \quad \delta \underline{x}_P = \delta \underline{x}_0 + \delta\varphi \, \underline{e}_3 \wedge (\underline{x}_P - \underline{x}_0)$

$P \in R$



$$LV_{\substack{\text{rigido}\\\text{piano}}} = \underline{R} \cdot \delta \underline{x}_0 + \underbrace{\left[\underline{M}(0) \cdot \underline{e}_3 \right]}_{\substack{\text{prodotto scalare}\\\text{proiezione } \underline{M}(0) \text{ lungo } \underline{e}_3}} \delta\varphi$$

Esempio



~~forza~~ $\underline{F}_B$ di modulo e verso costante

$\overline{OA} = l \, , \quad \overline{OB} = d$

Allora: $LV = \delta \underline{x}_0 \cdot \underline{F}_B + \delta\varphi \, \underline{e}_3 \cdot (\underline{x}_B - \underline{x}_0) \wedge \underline{F}_B$

$\delta \underline{x}_0 = 0$

$(\underline{x}_B - \underline{x}_0) \wedge \underline{F}_B = |F_B| \, d \, \underline{e}_3$

$LV = |F_B| \, d \, \delta\varphi$

Forze generalizzate per un corpo rigido

3D

$$LV = \delta \underline{x}_0 \cdot \underline{R} + \underline{\chi} \cdot \underline{M}(O)$$

$$\uparrow \qquad\qquad \uparrow$$

$$Q_{x_0} = \underline{R} \cdot \underline{e}_1 \qquad Q_{y_0} = \underline{R} \cdot \underline{e}_2 \qquad Q_{z_0} = \underline{R} \cdot \underline{e}_3$$

$$Q_{\psi} = \underline{M}(O) \cdot \underline{e}_3 \qquad Q_{\theta} = \underline{M}(O) \cdot \underline{n} \qquad Q_{\varphi} = \underline{M}(O) \cdot \underline{k}$$

2D

$$Q_{x_0} = \underline{R} \cdot \underline{e}_1 \quad , \quad Q_{y_0} = \underline{R} \cdot \underline{e}_2$$

$$Q_{\varphi} = \underline{M}(O) \cdot \underline{e}_3$$