MECCANICA RAZIONALE

- LEZIONE DEL 20 MARZO 2020
- PRIMA PARTE

STATICA → PLV

→ EQUAZIONI CARDINALI DELLA STATICA

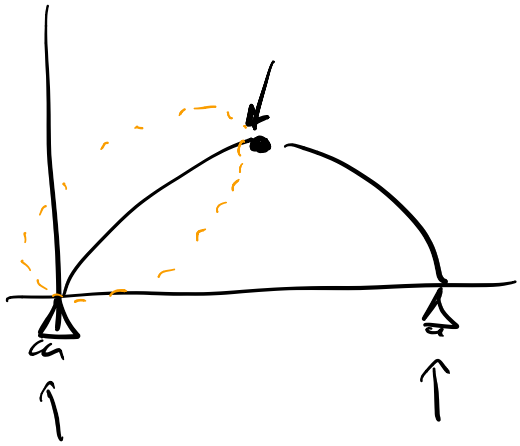
→ bilancio

$$\begin{cases} \underline{R}^e = \underline{0} \\ \underline{M}^e(O) = \underline{0} \end{cases}$$

- CORPO RIGIDO ECS = PLV

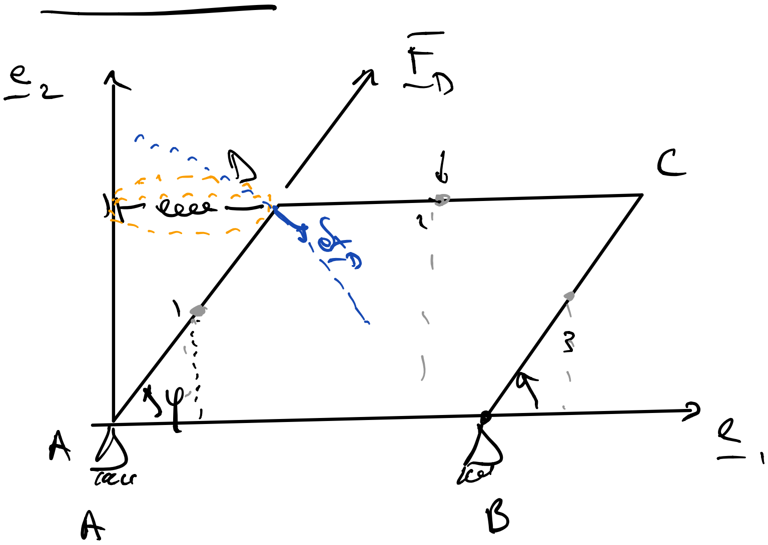
- IN GENERALE: se abbiamo una struttura articolata possiamo applicare le ECS a
  - tutta la struttura
  - tutte le sue componenti separatamente

(le azioni di una componente sull'altra)



arco a 3 cerniere

Esercizio



$\downarrow \underline{g}$ piano verticale

- determinare le configurazioni di eq.
- stabilità
- reazioni in A e B nelle config di eq.

1° PLV , 2 ECS

- forze peso $V_{peso} = m_1 g\, y_{G_1} + m_2 g\, y_{G_2} + m_3 g\, y_{G_3}$

$$= g(m_1 + m_3)\frac{l}{2}\sin\varphi + m_2 g\, l \sin\varphi$$

$$= g\, l \sin\varphi \left(\frac{m_1 + m_3}{2} + m_2 \right)$$

- forza elastica : $V_{el} = \frac{c}{2} x_D^2 = \frac{c}{2}(l\cos\varphi)^2$

• forza follower in D $\rightarrow$ $\underline{F}_D \cdot \delta \underline{x}_D = 0$

$$V_{tot} = g\ell \sin\varphi \left(\frac{m_1 + m_3}{2} + m_2 \right) + \frac{c}{2} \ell^2 \cos^2\varphi$$

$$\frac{dV}{d\varphi} = g\ell \left(\frac{m_1 + m_3}{2} + m_2 \right) \cos\varphi - c\ell^2 \sin\varphi \cos\varphi$$

$$= c\ell^2 (\mu - \sin\varphi) \cos\varphi = 0$$

$$\mu := g \left(\frac{m_1 + m_3}{2} + m_2 \right) \frac{1}{c\ell}$$

Equilibrio per $\varphi \in [-\pi, \pi]$

• $\mu > 1$: solo due conf. $\varphi = -\frac{\pi}{2}, \frac{\pi}{2}$

• $\mu < 1$: $\varphi = -\frac{\pi}{2}, \frac{\pi}{2}$

φ_1 $\sin\varphi_1 = \mu$ $0 < \varphi_1 < \frac{\pi}{2}$

φ_2 $\sin\varphi_2 = \mu$ $\frac{\pi}{2} < \varphi_2 < \pi$

Derivata seconda $\rightarrow$ stabilità

$$V'' = c\ell^2 (-\mu \sin\varphi + \sin^2\varphi - \cos^2\varphi)$$

$$= c\ell^2 (-\mu \sin\varphi + 2\sin^2\varphi - 1)$$

• $\underline{\mu > 1}$ $\quad \varphi = -\frac{\pi}{2} \quad \rightarrow$ stabile

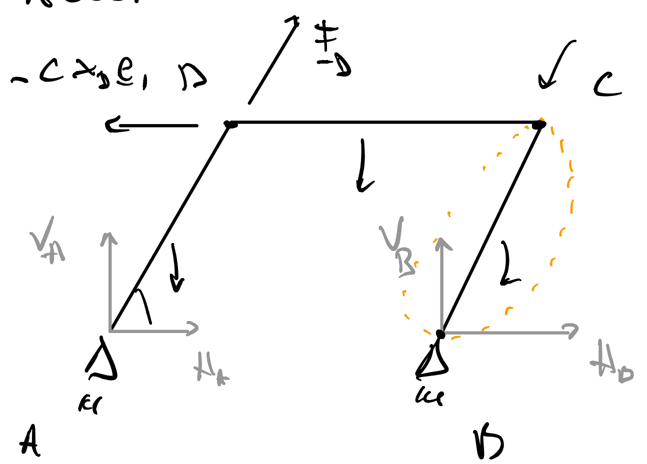
$\varphi = \frac{\pi}{2} \quad \rightarrow$ instabile

• $\underline{\mu < 1}$ $\quad \varphi = -\frac{\pi}{2}, \frac{\pi}{2}$ stabili

$\varphi = \varphi_1, \varphi_2$ instabile

$(\sin\varphi_1 = \sin\varphi_2 = \mu)$

Adesso reazioni vincolari : ECS



$$\underline{F}_D = F \operatorname{vers}(\underline{x}_D - \underline{x}_A)$$
$$= F(\cos\varphi\, \underline{e}_1 + \sin\varphi\, \underline{e}_2)$$

ECS a tutto il sistema

$\underline{R}^e = \underline{0}$ $\quad \underline{e}_1$: $H_A + H_B - cx_D + F\cos\varphi =$

$= H_A + H_B - c\, l\cos\varphi + F\cos\varphi = 0$

$\underline{e}_2$: $V_A + V_B + F\sin\varphi$

$- (m_1 + m_2 + m_3)g = 0$

$$\underline{M}^e(A) = l V_B - m_1 g \frac{l}{2}\cos\varphi - m_2 g \left(l\cos\varphi + \frac{l}{2}\right)$$
$$- m_3 g \left(l + \frac{l}{2}\cos\varphi\right) + c(l\cos\varphi)\, l \sin\varphi = 0$$

$\rightarrow$ ricaviamo $\underline{V_B}$

Consideriamo l'asta BC

$$\underline{M}^e_{BC}(C) = \underline{0}$$

$$l \sin\varphi\, H_B - l\cos\varphi\, V_B + m_3 g \frac{l}{2}\cos\varphi = 0$$

$$\underbrace{(\underline{x}_B - \underline{x}_C)\wedge(H_B \underline{e}_1 + V_B \underline{e}_2)}_{\text{momento reazioni in B (con polo C)}} = \Big[\, l\underline{e}_1 - \underbrace{(l + l\cos\varphi)\underline{e}_1 +}_{\underline{x}_B}$$

$$- l\sin\varphi\, \underline{e}_2 \Big] \wedge (H_B \underline{e}_1 + V_B \underline{e}_2)$$

$$= \left[- l\cos\varphi\, \underline{e}_1 - l\sin\varphi\, \underline{e}_2 \right] \wedge (H_B \underline{e}_1 + V_B \underline{e}_2)$$

$$= - l\cos\varphi\, V_B\, \underline{e}_3 + l\sin\varphi\, H_B\, \underline{e}_3$$

$$\underline{e}_1 \wedge \underline{e}_2 = \underline{e}_3$$

Mettiamo tutto insieme

$$V_B = g\left(\frac{m_2}{2} + m_3\right) + cl\cos\varphi\,(\varphi - \sin\varphi)$$

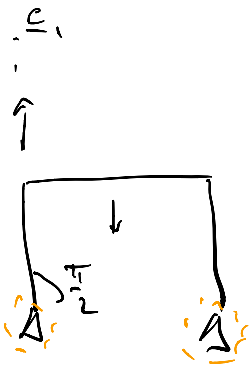
$$V_A = (m_1 + m_2 + m_3)\,g - F\sin\varphi - V_B$$

$$H_B = \left(V_B - \frac{m_3 g}{2}\right)\cot\varphi$$

$$H_A = -H_B + (cl - F)\cos\varphi$$

calcoliamole nelle configurazioni di eq.

- $\varphi = \frac{\pi}{2}$



$$V_B = g\left(\frac{m_2}{2} + m_3\right)$$

$$V_A = (m_1 + m_2 + m_3)g - F - V_B$$
$$= \left(m_1 + \frac{m_2}{2}\right)g - F$$

$$H_B = 0 \quad , \quad H_A = 0$$

- $\varphi = -\frac{\pi}{2}$ ......

- $\varphi = \varphi_1$

$$V_B = g\left(\frac{m_2}{2} + m_3\right)$$

$$V_A = g\left(m_1 + \frac{m_2}{2}\right) - F\mu$$

$$H_B = \frac{\sqrt{1-\mu^2}}{\mu}\left(V_B - m_3\frac{g}{2}\right)$$

$$H_A = -H_B + (cl - F)\sqrt{1-\mu^2}$$

$\sin\varphi_1 = \mu$

$\Downarrow$

$\cos\varphi$

$\sin^2 + \cos^2 = 1$

$\cos = \sqrt{1-\sin^2}$

- $\varphi = \varphi_2$ .....

MECCANICA RAZIONALE

— LEZIONE DEL 20 MARZO 2020

— SECONDA PARTE

Esercizio

$\overline{OB} = R$

$\overline{AB} = L \geq R$

Piano verticale

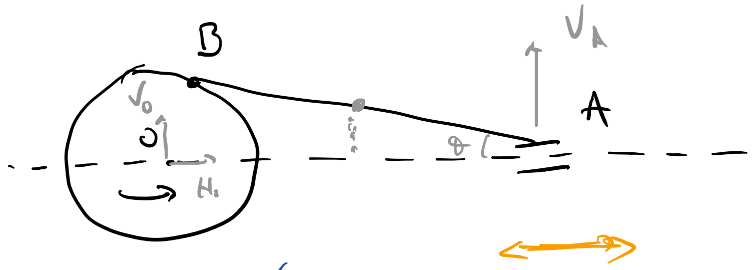
coppia $\underline{\mu}$ applicata al disco

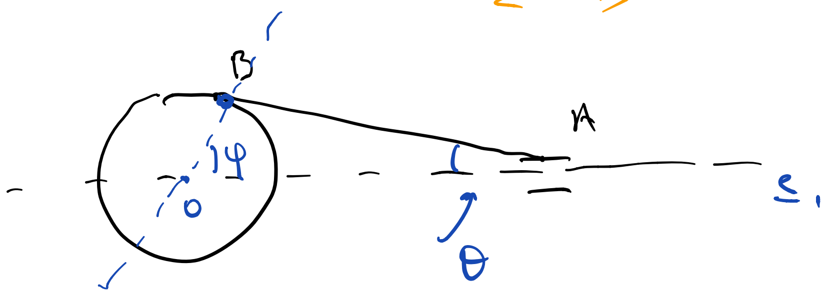
Determinare le reazioni in A e in O all'eq.

disco M

asta m

$g \downarrow$





1 grado di libertà : $\theta = \theta(\varphi)$

PLV per l'eq.

$$LV = \mu\, \delta\varphi - \frac{d}{d\varphi} V_{peso}\, \delta\varphi$$

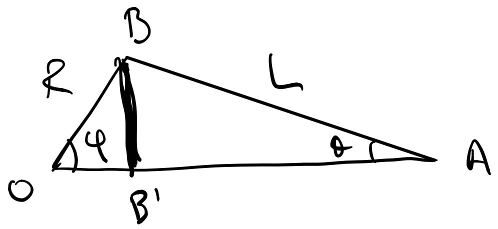
$$LV = \underbrace{\delta x_O}_{\substack{=0 \\ \text{perché} \\ \text{vincolo in O}}} \cdot \underline{R} + \underline{\chi} \cdot \underbrace{\underline{M}(O)}_{\underline{\mu}}$$

$$V_{peso} = mg\, y_G + \cancel{Mg\, y_o} = mg\,\frac{L}{2}\sin\theta(\varphi)$$

$$LV = \delta\varphi\left(\mu - \frac{d}{d\varphi}\left(mg\,\frac{L}{2}\sin\theta(\varphi)\right)\right)$$

$$= \delta\varphi\left(\mu - mg\,\frac{L}{2}\cos\theta\,\frac{d\theta}{d\varphi}\right)$$

Bisogna trovare una relazione $\theta = \theta(\varphi)$



$$L\sin\theta(\varphi) = R\sin\varphi$$

prendiamone la derivata

$$\frac{d}{d\varphi}: \quad R\cos\varphi = L\cos\theta\,\frac{d\theta}{d\varphi} \qquad \delta\theta = \frac{d\theta}{d\varphi}\,\delta\varphi$$

$$\hookrightarrow \cos\theta\,\frac{d\theta}{d\varphi} = \frac{R}{L}\cos\varphi$$

$$LV = \delta\varphi\left(\mu - mg\,\frac{\cancel{L}}{2}\cancel{\cos\theta}\,\frac{R\cos\varphi}{\cancel{L}\,\cancel{\cos\theta}}\right)$$

$$= \delta\varphi\left(\mu - mg\,\frac{R}{2}\cos\varphi\right) = 0 \qquad \forall\,\delta\varphi$$

$$\boxed{\mu - mg\,\frac{R}{2}\cos\varphi = 0} \qquad \cos\varphi = \frac{2\mu}{mgR}$$

Quindi

- se $\mu > mg\frac{R}{2}$: no sol $\Rightarrow$ non c'è equilibrio

- se $\mu < mg\frac{R}{2}$ $\Rightarrow$ $\cos\varphi = \frac{2\mu}{mgR} = \lambda$

$\rightarrow$ 2 soluzioni $-\frac{\pi}{2} < \varphi_1 < 0$

$0 < \varphi_2 < \frac{\pi}{2}$

- $\mu = mg\frac{R}{2}$ $\Rightarrow$ $\cos\varphi = 1$

$\Rightarrow$ $\varphi = 0$

<u>Nota</u> potevamo introdurre un'energia potenziale generalizzata

$$V = -\mu\varphi + V_{peso}$$

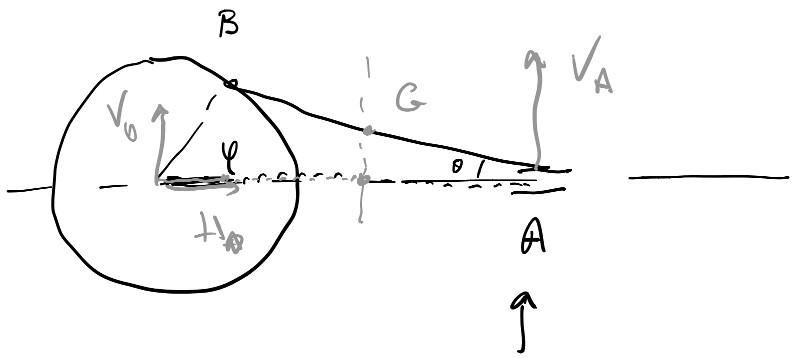
$$V' = -\mu + mg\frac{R}{2}\cos\varphi$$

$$V'' = -mg\frac{R}{2}\sin\varphi$$

φ_1 stabile

φ_2 instabile

Adesso $\rightarrow$ le forze di reazione



ECS applicate a tutto il sistema

$$\underline{R}^e = \underline{0} \quad \Rightarrow \quad H_0 = 0$$

$$V_0 + V_A - Mg - mg = 0$$

$$\underline{M}^e(O) = \underline{0} \qquad \left(R\cos\varphi + L\cos\theta(\varphi)\right) V_A$$

$$(\underline{M}\cdot\underline{e}_3 = 0) \qquad - mg\left(R\cos\varphi + \frac{L}{2}\cos\theta(\varphi)\right) + \mu = 0$$

Vogliamo $\cos\theta(\varphi_1)$, $\cos\theta(\varphi_2)$ nelle conf. di equilibrio

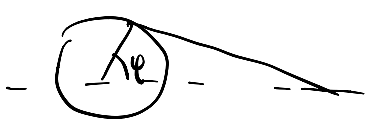
- $\mu < mg\frac{R}{2} \Rightarrow \cos\varphi_1 = \cos\varphi_2 = \frac{2\mu}{mgR} = \lambda > 0$

$$|\sin\varphi_1| = |\sin\varphi_2| = \sqrt{1-\lambda^2}$$

$$\sin\theta(\varphi) = \frac{R}{L}\sin\varphi$$

$$\sin^2\theta(\varphi_1) = \sin^2\theta(\varphi_2) = (1-\lambda^2)\frac{R^2}{L^2}$$

$$\cos\theta(\varphi_1) = \cos\theta(\varphi_2) = \underbrace{\sqrt{1 - \frac{R^2}{L^2}(1-\lambda^2)}}_{\sqrt{1-\sin^2\theta(\varphi_{1,2})}}$$

- $\mu = mg\frac{R}{2}$ $\quad \varphi = 0$  $\Rightarrow \theta(\varphi) = 0$