Homework 01

Exercise 1

Let Y be a random variable taking values in $\{1, \ldots, N\}$ and suppose that any value is equally likely, i.e. that $P(Y = j) = \frac{1}{N}$.

Knowing that $\sum_{j=1}^{N} j = \frac{N(N+1)}{2}$, show that $\mathbb{E}(Y) = \frac{N+1}{2}$.

Exercise 2

Prove that $\mathbb{E}(X) = \mu$ for $X \sim \mathcal{N}(\mu, \sigma^2)$.

Exercise 3

Let X and Y have discrete joint distribution

$$p(x, y) = \begin{cases} \frac{1}{30}(x + y) & \text{for } x = 0, 1, 2 \text{ and } y = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

Are X and Y independent?

Exercise 4

Let X and Y be two continuos random variables with joint probability density

$$f(x, y) = \begin{cases} 24xy & \text{for } 0 < x < 1, 0 < y < 1, x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find

1. the marginal density of Y;
2. the conditional density of X given $Y = 1/2$.

Exercise 5

Let the joint distribution of X and Y be

$$f(X, Y) = \frac{\Gamma(\alpha_1 + \alpha_2 + \alpha_3)}{\Gamma(\alpha_1)\Gamma(\alpha_2)\Gamma(\alpha_3)} x^{\alpha_1 - 1} y^{\alpha_2 - 1} (1 - x - y)^{\alpha_3 - 1}.$$

Suppose that $0 < x < 1, 0 < y < 1, x + y < 1$ and prove that the marginal distribution of X is a $\text{Beta}(\alpha_1, \alpha_2 + \alpha_3)$.