Homework 02

Exercise 1

One half percent of the population has a coronavirus and a test is being developed. This test gives a false positive 3% of the time and a false negative 2% of the time.

1. Find the probability that Luca is positive to the test.
2. Suppose Luca is positive to the test. What is the probability that he has contracted the disease?

Exercise 2

Implement the empirical cumulative distribution function $F_X(x) =$ `cdf(dist, x)` taking as inputs a `pyro.distributions` object `dist`, corresponding to the distribution of X, and integer value `x`.

Suppose that $X \sim \mathcal{N}(0, 1)$ and plot $F_X(x)$.

Exercise 3

Suppose the heights of male students are normally distributed with mean 180 and unknown variance σ^2. Suppose that σ^2 is in the range $[22, 41]$ with approximately 95% probability and assign to σ^2 an inverse-gamma $IG(38, 1110)$ prior distribution .

1. Empirically verify that the parameters of the inverse-gamma distribution lead to a prior probability of approximately 95% that $\sigma^2 \in [22, 41]$.
2. Derive the posterior density of σ^2 corresponding to the following data: $183, 173, 181, 170, 176, 180, 187, 176, 171, 190, 184, 173, 176, 179, 181, 186$. Then plot it together with the prior density.
3. Compute the posterior density of the standard deviation σ.

Exercise 4

Prove that the Gamma distribution is the conjugate prior distribution for the Exponential likelihood.