# Discrete-Time Markov Chains

Stochastic Modelling and Simulation

## Theoretical exercises

### Exercise 1

Let $\{X_n\}_{n\geq 0}$ be a time-homogeneous DTMC on state space $S = \{1, \dots, N\}$ with transition probability matrix $P = \{p_{i,j}\}_{i,j}$. Let $P^{(n)}$ be the matrix defined as $p^{(n)}_{i,j} = Pr(X_n = j \mid X_0 = i)$, $\forall i, j \in S$.

Show that $P^{(n)} = P^n$, where $P^n$ is the $n$-th power of the matrix $P$.

*Hints:* Recall that $P^0 = I$ and $P^1 = P$. Use the memory-less property and the time homogeneity of the DTMC.

### Exercise 2

The limiting distribution is defined as a vector $\pi = [\pi_1, \dots, \pi_N]$, such that $\pi_i = \lim_{n\to\infty} Pr(X_n = i)$.

Show that if the limiting distribution exists, it satysfies $\pi_j = \sum_{i=1}^{N} \pi_i \cdot p_{i,j}$, for $j \in S$ and $\sum_{j=1}^{N} \pi_j = 1$.