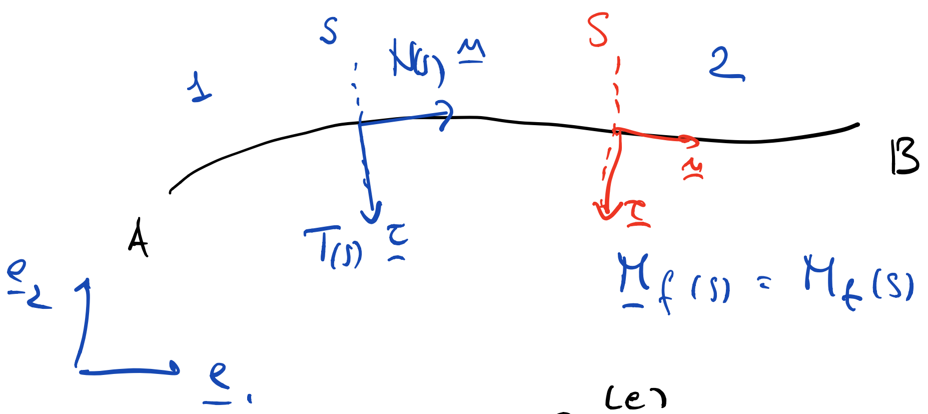
MECCANICA RAZIONALE

– LEZIONE DEL 26 MARZO 2020

– PRIMA PARTE

Sforzi interni ad un rigido

$\to$ rigidi piani $\left(N(s),\ T(s),\ M_f(s) \right)$



$$\underline{M}_f(s) = M_f(s)\,\underline{e}_3$$

$$\underline{u} = \frac{d\underline{x}}{ds}$$

$$\underline{\tau} := \underline{u} \wedge \underline{e}_3$$

attive + reattive

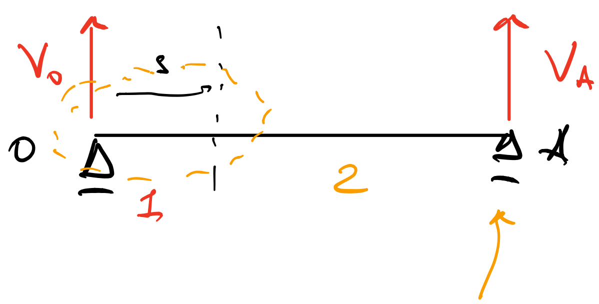
$$\underline{R}_1^{(e)} + N\underline{u} + T\underline{\tau} = \underline{0}$$

$$\underline{M}^{(e),1}_{(S)} + M_f(S)\,\underline{e}_3 = \underline{0}$$

al variare di s

$$\to \begin{cases} \dfrac{d}{ds}\left(N\underline{u}_{(s)} + T\underline{\tau}_{(s)}\right) = -\underline{f}(s) \\ \dfrac{d}{ds}\underline{M}_f(s) = \underline{T}(s) \wedge \underline{u}(s) \end{cases}$$

Asta



$\downarrow \underline{g}$

$\underline{f}(s) = \rho(s)\,\underline{g}$

$L = \overline{OA}$

$$N\underline{n} + T\underline{\tau} - V_0\underline{\tau} + m(s)\, g\, \underline{\tau} = \underline{0}$$

$$m(s) = \int_0^s \rho(x)\, dx$$

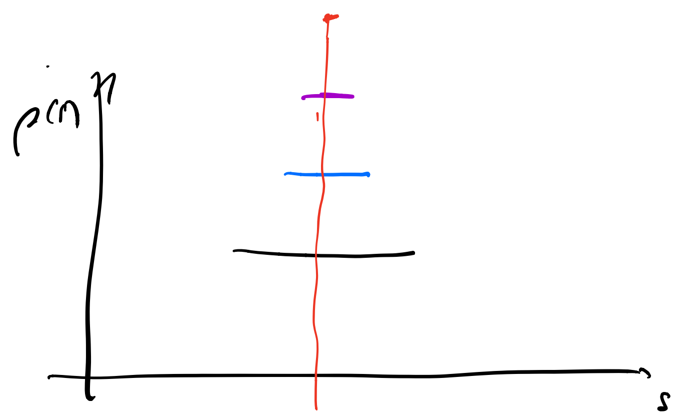
ρ = costante

ρ :

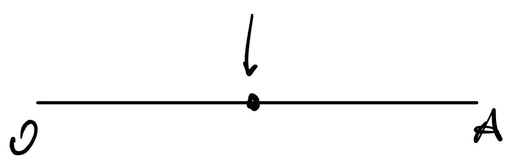
N, T, M_f

$$\rho(s) = \begin{cases} 0 & a \\ n\frac{M}{L} & b \\ 0 & c \end{cases}$$

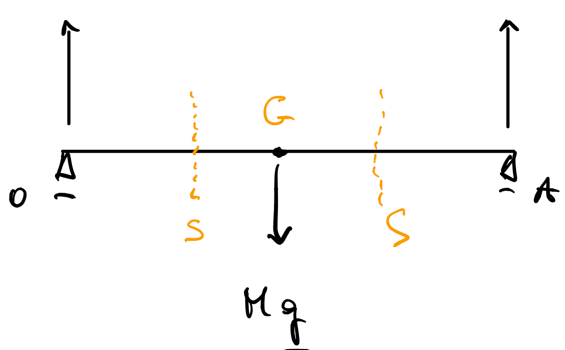
quando $n \to \infty$



$$\underline{f}(s) = \rho(s)\, \underline{g}$$



Esempio



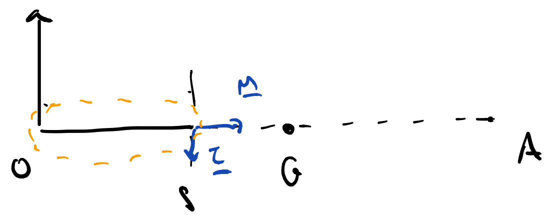
Sottoposta solo ad un carico concentrato in $\frac{L}{2}$ e pari a $M\underline{g}$

Reazioni vincolari: V_O, V_A sono uguali a quelle trovate nel caso $\rho = \frac{M}{L}$ = costante

$$\Rightarrow \quad V_O = V_A = \frac{Mg}{2}$$

Sforzi interni: eq di bilancio

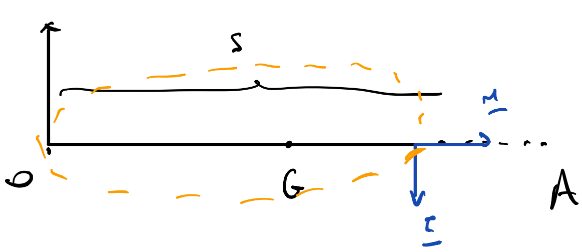
<u>$s \in OG$</u> (prima del punto di applicazione del carico $0 < s < \frac{L}{2}$)



$$N\underline{u} + T\underline{\tau} - V_O\underline{\tau} = \underline{0}$$

$$\begin{cases} N = 0 \\ T = \dfrac{Mg}{2} \end{cases} \qquad 0 < s < \frac{L}{2}$$

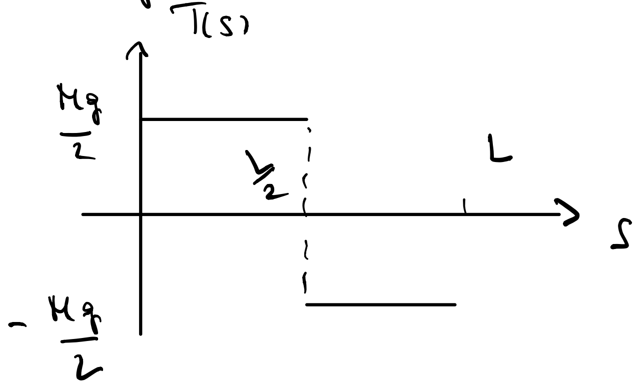
$s \in GA$

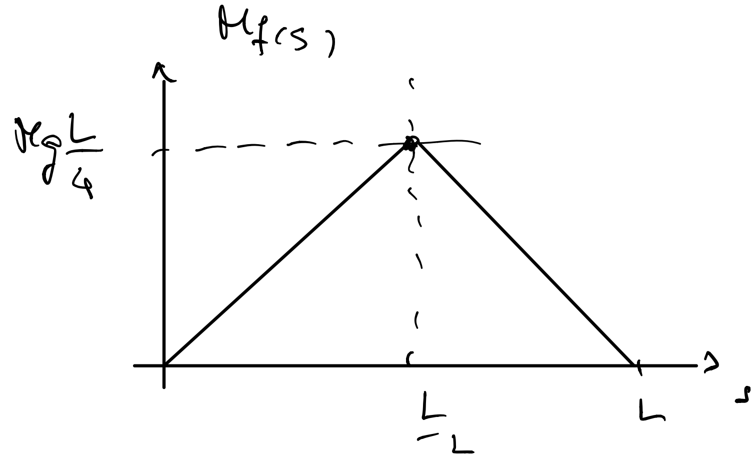


$$N\underline{u} + T\underline{\tau} - V_O\underline{\tau} + Mg\underline{\tau} = \underline{0}$$

$$\begin{cases} N = 0 \\ T = -\dfrac{Mg}{2} \end{cases}$$

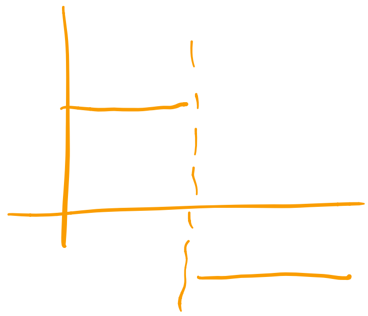
Grafico di $T(s)$



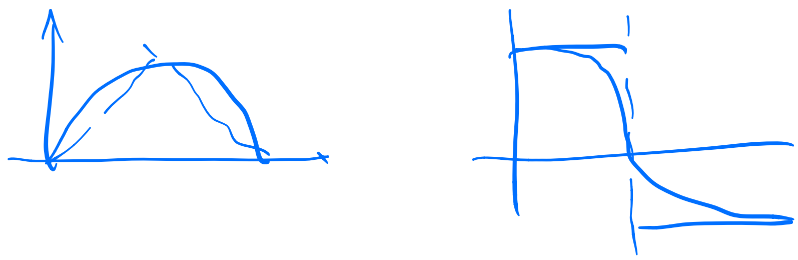


Usando

$$\frac{d}{ds} M_f = T \quad \longrightarrow \quad M_f(s) = \begin{cases} \frac{Mg}{2} s & 0 < s < \frac{L}{2} \\ -\frac{Mg}{2} s + Mg \frac{L}{2} & \frac{L}{2} < s < L \end{cases}$$



(se forze



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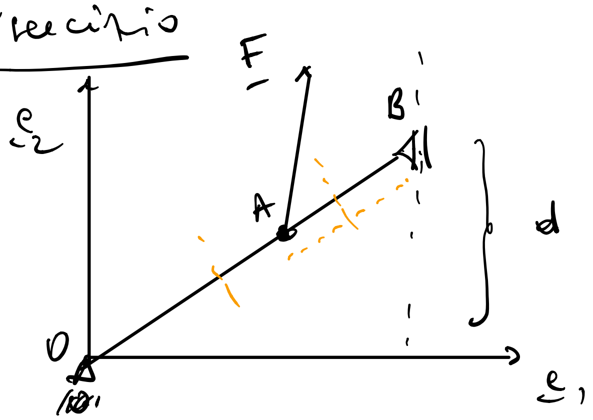
Esercizio

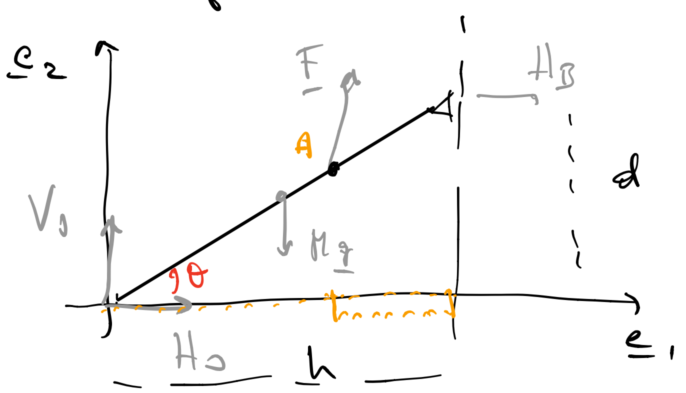


$\downarrow \underline{g}$ OB massa M e densità $\rho = \rho(s)$ nota

$OB = L$ $\quad AB = aL$ $\quad 0 < a < 1$

Piano verticale

Calcolare
- reazione vincolare in B
- sforzi interni all'asta



$\underline{F} = F_1 \underline{e}_1 + F_2 \underline{e}_2$

$L = \sqrt{d^2 + h^2}$ $\quad AB = aL$

Reazione in H_B

$h = L \cos\theta$

ECS $\to$ eq momenti rispetto ad O (non compaiono H_O, V_O)

ad esempio

$$+ \quad (\underline{x}_B - \underline{x}_O) \wedge \underline{H}_B = (h\underline{e}_1 + d\underline{e}_2) \wedge H_B \underline{e}_1$$

$$= d H_B \underline{e}_2 \wedge \underline{e}_1 = -d H_B \underline{e}_3$$

$$+ \quad (\underline{x}_A - \underline{x}_O) \wedge \underline{F} = [(h - ah)\underline{e}_1 + (d - ad)\underline{e}_2] \wedge$$

$$\wedge (F_1 \underline{e}_1 + F_2 \underline{e}_2)$$

$$= -F_1 (d - da)\underline{e}_3 + F_2 (h - ah)\underline{e}_3$$

$$+ \quad (\underline{x}_G - \underline{x}_O) \wedge (-Mg\underline{e}_2) = -\left(\overline{OG}\frac{h}{L}\underline{e}_1 + \cdots\right) \wedge$$

$$\wedge (Mg\underline{e}_2) = -Mg\,\overline{OG}\,\underbrace{\frac{h}{L}}_{\cos\theta}\underline{e}_3$$

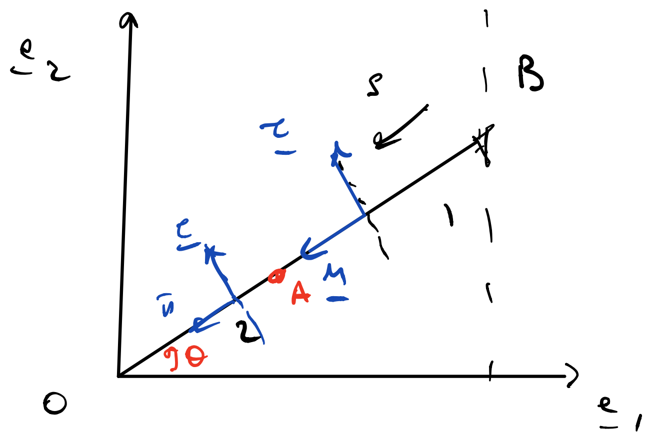
$$-dH_B - F_1(d - ad) + F_2(h - ah) - Mg\,\overline{OG}\frac{h}{L} = 0$$

$$\Rightarrow \quad H_B = -(1-a)F_1 + (1-a)\frac{h}{d} - \overline{OG}\,\frac{h}{L}\,\frac{1}{d}\,Mg$$

MECCANICA RAZIONALE

– LEZIONE DEL 26 MARZO 2020

– SECONDA PARTE



forze peso, $\underline{F}$, reazioni vincolari

→ abbiamo appena calcolato H_B

Misuriamo la lunghezza a partire dal punto B

→ $\underline{n} = -\frac{h}{L}\underline{e}_1 - \frac{d}{L}\underline{e}_2$

→ $\underline{\tau} = \underline{n} \wedge \underline{e}_3 = -\frac{d}{L}\underline{e}_1 + \frac{h}{L}\underline{e}_2$

Due casi $0 < s < aL$, $aL < s < L$

Primo caso $0 < s < aL$

$$N\underline{n} + T\underline{\tau} - m(s)\,g\,\underline{e}_2 + H_B\,\underline{e}_1 = \underline{0}$$

$$\hookrightarrow m(s) = \int_0^s \rho(x)\,dx$$

$$N = m(s)\,g\,\underline{e}_2\cdot\underline{n} - H_B\,\underline{e}_1\cdot\underline{n}$$
$$= -\frac{d}{L}m(s)\,g + \frac{h}{L}H_B =: N(s)$$

$$T = m(s)\,g\,\underline{e}_2\cdot\underline{\tau} - H_B\,\underline{e}_1\cdot\underline{\tau}$$
$$= m(s)\,g\,\frac{h}{L} - H_B\frac{d}{L} =: T(s)$$

Secondo caso: $aL < s < L$

$$N \underline{n} + T \underline{\tau} - m(s) g \underline{e}_2 + H_B \underline{e}_1 + \underline{F} = \underline{0}$$

$$N = m(s) g \underline{e}_2 \cdot \underline{n} - H_B \underline{e}_1 \cdot \underline{n} - \underline{F} \cdot \underline{n}$$

$$= N^-_{(s)} - \underline{F} \cdot \underline{n} =: N^+_{(s)}$$

$$T^+_{(s)} := T^-_{(s)} - \underline{F} \cdot \underline{\tau}$$

Mettiamo tutto insieme

$$N(s) = \begin{cases} N^-(s) = -\frac{d}{L} m(s) g + \frac{h}{L} H_B & 0 < s < aL \\ N^+_{(s)} = N^-_{(s)} - \left(-\frac{h}{L} F_1 - \frac{d}{L} F_2 \right) & aL < s < L \end{cases}$$

$$T(s) = \begin{cases} T^-_{(s)} = \frac{h}{L} m(s) g - \frac{d}{L} H_B & 0 < s < aL \\ T^+_{(s)} = T^-_{(s)} - \left(-\frac{d}{L} F_1 + \frac{h}{L} F_2 \right) & aL < s < L \end{cases}$$

Per finire $\quad M_f(s) = \int_0^s T_{(x)}\, dx$

$$M_f(s) = \begin{cases} \int_0^s T^-_{(x)}\, dx & 0 < s < aL \\ \int_0^{aL} T^-_{(x)}\, dx + \int_{aL}^s T^+_{(x)}\, dx & aL < s < L \end{cases}$$

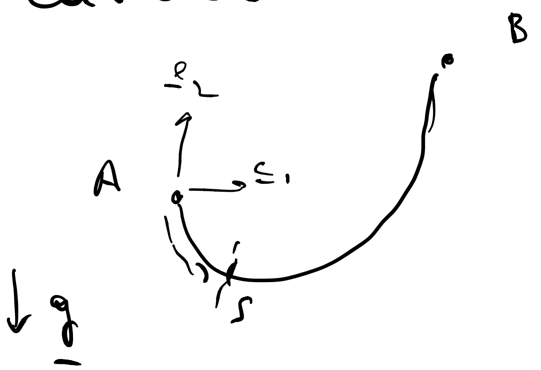
CENNI ALLA STATICA DEI CONTINUI DEFORMABILI

consideriamo un continuo 1-dim deformabile modellato come una curva

forze agenti

materiale in esame : equazione costitutiva

Catena



peso specifico

$\rho(s)\,\underline{g}$

s = lunghezza AS

$\overline{AB} = L$

All'eq. le equazioni di bilancio su un tratto → le stesse dell'arco rigido

$$\begin{cases} \dfrac{d}{ds}\left(N\underline{u} + \underline{T}\right) = -\rho\,\underline{g} \\ \dfrac{d}{ds}\,\underline{M}_f = \underline{T} \wedge \underline{u} \end{cases}$$

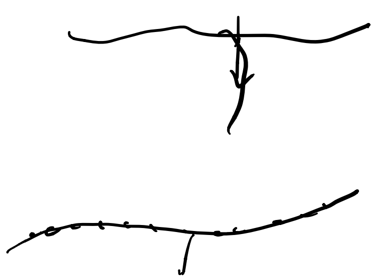
La curva è

$x = x(s)$ $\quad s \in [0, L]$ $\quad x(0) = x_A = 0$ $\quad y(0) = y_A = 0$

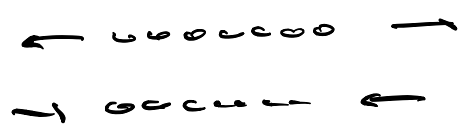
$y = y(s)$ $\quad x(L) = x_B$ $\quad y(L) = y_B$

$$\underline{u} = \frac{dx}{ds}\underline{e}_1 + \frac{dy}{ds}\underline{e}_2 = \frac{d}{ds}\underline{x}$$

<u>Equazioni costitutive</u>: la catena è filo flessibile, formato da anelli



$$\boxed{\underline{T} = \underline{0} \qquad \underline{M}_f = \underline{0} \qquad N > 0}$$



$$\rightarrow \frac{d}{ds}\left(N \frac{d\underline{x}}{ds}\right) = -\rho \underline{g}$$

Trovare $x(s)$, $y(s)$, $N(s) > 0$, tali che

$$\begin{cases} \dfrac{d}{ds}\left(N \dfrac{dx}{ds}\right) = 0 \\[2ex] \dfrac{d}{ds}\left(N \dfrac{dy}{ds}\right) = \rho g \end{cases}$$

+ condizioni al contorno

Soluzione (detta catenaria)

$$y(x) = b + a \cosh\left(\frac{x - x_0}{a}\right)$$

(a, b, x_0 sono parametri che fissano i dati di contorno)