In this course we study the numerical solution of partial differential equations by a *Finite Difference Method* (FDM). In a FDM the derivatives are approximated by incremental ratios called *finite differences*.

We consider three prototype linear equations belonging to the three classes of *elliptic equations*, *parabolic equations* and *hyperbolic equations*. These three prototypes are:

i) for the class of elliptic equations, the *Poisson equation*

$$\Delta u = f,$$

where $u(x_1, \ldots, x_n)$ (the unknown function) and $f(x_1, \ldots, x_n)$ are functions of the n $(n = 1, 2, 3)$ spatial scalar variables $x_1, \ldots, x_n$ and Δ is the *Laplacian operator*

$$\Delta u = \sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2};$$

ii) for the class of parabolic equations, the *heat equation*

$$\frac{\partial u}{\partial t} = c\Delta u + f,$$

where $u(x_1, \ldots, x_n, t)$ (the unknown function) and $f(x_1, \ldots, x_n, t)$ are functions of the n $(n = 1, 2, 3)$ spatial scalar variables $x_1, \ldots, x_n$ and the time variable t and $c > 0$;

iii) for the class of hyperbolic equations, the *advection equation*

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0,$$

where $u(x, t)$ (the unknown function) is a function of the spatial scalar variable x and the time variable t and $c \in \mathbb{R}$.