

1) Introduction

Heat transfer deals with the rate of heat transfer between different bodies. While thermodynamics deals with the magnitude of heat exchanged in a process, heat transfer is necessary to determine the time required for a process or alternatively the size of a surface necessary to achieve a certain total rate of heat transfer.

Heat transfer analysis permits a calculation of the heat loss from a building surface to the surroundings for a given building size, window area and wall design, e.g. the level of insulation in the wall cavity. The comfort conditions for occupants in a room is determined by a balance of heat transfer from the person to the air surrounding him or her as well as the heat transfer to the walls of the interior. The size and cost of a heat exchanger is also determined by considering the heat transfer between the fluid streams in the exchanger.

In other fields, heat transfer plays a key role as well. The design of integrated microprocessors which contain very closely spaced elements, each with a finite amount of heat generation, is limited by the requirement for adequate cooling so that the operating temperature of the electronic components is not exceeded. Reentry of the space shuttle in the earth's atmosphere must be carefully programmed so that temperature extremes due to air friction are confined to the insulating tiles on the shuttle's surface.

Modes of Heat Transfer

Following thermodynamics, heat transfer is that energy transfer which takes place between two bodies by virtue of a temperature difference between the bodies. From the second law considerations it can be demonstrated that there is always a net positive energy transfer from the body at a high temperature to a second body at a lower temperature. Following the definition of heat, there are only two physical mechanisms for heat transfer: (1) electromagnetic waves produced by virtue of the temperature of a body, referred to as thermal radiation heat transfer and (2) atomic or molecular motion in a medium between the bodies exchanging energy, referred to as conduction heat transfer.

Sometimes conduction heat transfer takes place during the change of phase and is referred to as boiling or condensation heat transfer. Conduction heat transfer can also take place in the presence of fluid motion, which is called convection heat transfer.

The rate of heat transfer between two bodies is proportional to the temperature difference between the bodies and in some cases the temperature level of the bodies as well. In many instances the heat transfer process is analogous to the rate of transfer which appears in other fields. The analogy between heat transfer and DC electrical current flow will be used to illustrate some of the simpler heat transfer processes. Similarly, it can be shown that the rate of transfer of mass in an evaporation process follows a process very similar to that for heat transfer.

2) Conduction Heat Transfer

In a homogenous body which experiences a temperature gradient the rate of heat transfer due to microscopic motions is conduction heat transfer. In a gas the gas molecules in the higher temperature portion of the gas will have a higher kinetic energy. As the molecules of the gas randomly move through the gas volume there is a net energy transfer from the high temperature portion to the low temperature zones. In a solid, the energy transfer from high to low temperature may be due to the migration of electrons or the vibration of the molecular bonds.

Viewed as a macroscopic phenomena, the rate of heat transfer by conduction represented by the symbol q or Q is found to be directly proportional to the product of the local temperature gradient and the cross-sectional area available for heat transfer,

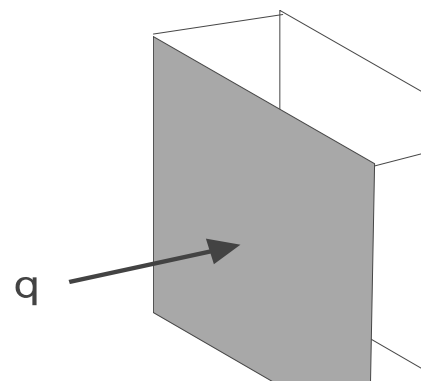


Fig 2.1 One dimensional conduction

$$q \sim -A \text{ grad}T \quad (2.1)$$

In the case of one-dimensional heat transfer normal to a

plane slab, figure 1, the conduction heat transfer can be given by Fourier's Equation,

$$q = -kA \frac{dT}{dx} \quad (2.2)$$

The constant k is known as the thermal conductivity. q has the dimensions of BTU/hr or Watts and k has the dimensions of BTU/hrft^{°F} or W/m^{°K}.

The thermal conductivity defined by equation 2.2 is a thermophysical property of the material. If the composition and thermodynamic state is known then the thermal conductivity can be found.

Table 2.1 lists the thermal conductivity of common solids, liquids and gases at normal temperatures. Note that these values span many orders of magnitude with electrically conductors having the highest thermal conductivity and high molecular weight gases generally having the lowest thermal conductivity.

Consider a slab with a steady conduction heat transfer across it in the x direction, fig. 2, with the temperature equal to T₁ and T₂ at the surfaces corresponding to x equal to 0 and L, respectively. Then q is a constant and equation 2 can be integrated to give,

$$q = kA \frac{T_1 - T_2}{L} \quad (2.3)$$

For this case the temperature varies linearly across the width of the slab. One can consider an analogy between the solution for steady conduction and for steady D.C. electric current flow, Ohm's Law,

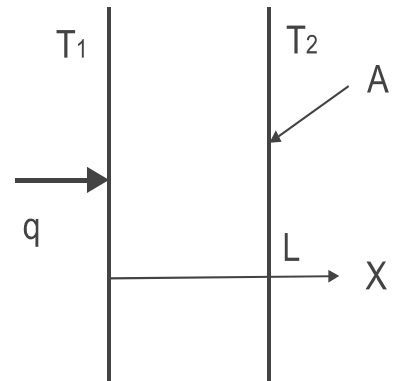


Fig. 2,2 Conduction Through a Plane Wall

$$I = \frac{V_2 - V_1}{R} \quad (2.4)$$

Table 2.1
Thermal Conductivity of
Common Materials

k (BTU/hr ft °F) (W/mK)

Solids

Copper	219	378
Aluminum	119	206
Steel	25	43
Brick, _common	0.2 - 0.1	0.17 - 0.34
Concrete	0.5 - 0.8	0.87 - 1.38
Glass	0.5	0.87
Glass fiber insulation	.03	0.05
Ice	1.3	2.2
Plastic	0.1	0.17
Wood	0.1 - 0.2	0.17 - 0.34

Liquids

Ammonia	0.3	0.5
Refrigerant-12	0.04	0.07
Light Oil	0.08	0.14
Water	0.34	0.59
Mercury	5	8.7

Gases

Air, _dry	0.015	0.026
Carbon Dioxide	0.009	0.016
Helium	0.09	0.16
Hydrogen	0.11	0.19
Water Vapor (Steam)	0.015 (at 212°F)	0.026 (at 100
C) Refrigerant-11	0.005	0.009

1.0 (BTU/hr ft °F) = 1.73 (W/m °C)

The rate of heat transfer q is analogous to the current flow I , the potential difference ΔV is analogous to ΔT and the balance of equation 2.3 is analogous to the resistance. The term thermal resistance is used; for eqn. 2.3 the thermal resistance is L/kA .

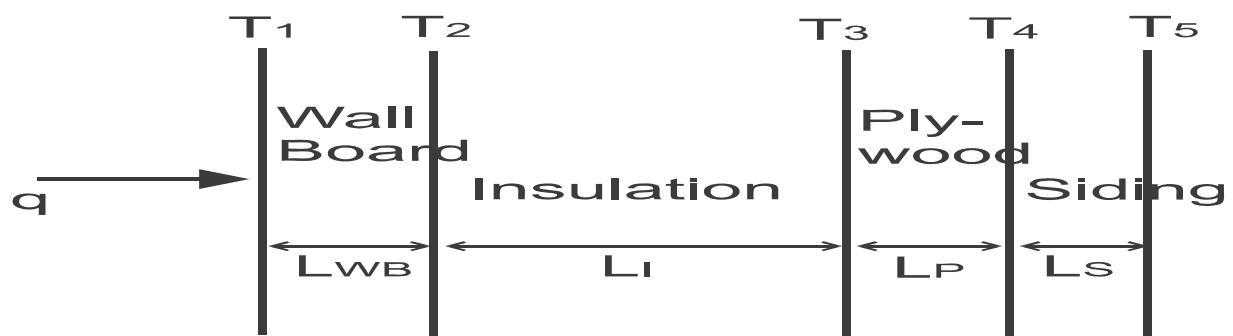


Fig. 2.3 Steady State Heat Transfer Through a Composite Wall

Consider the case of steady heat transfer through a composite wall as shown in figure 2.3. Each element of the wall has the same heat transfer rate q through it and for each an equation similar to equation 2.3 can be written. At steady state with no change in internal energy with time, no work, and no mass flows through each of the elements, the rate of heat transfer into and out of each wall element must be the same. For the wall board,

$$q = \frac{k_{WB} A}{L_{WB}} (T_1 - T_2) \quad (2.5)$$

This can be rewritten as,

$$T_1 - T_2 = q \frac{L_{WB}}{k_{WB} A} \quad (2.6)$$

For the insulation

$$T_2 - T_3 = q \frac{L_I}{k_I A} \quad (2.7)$$

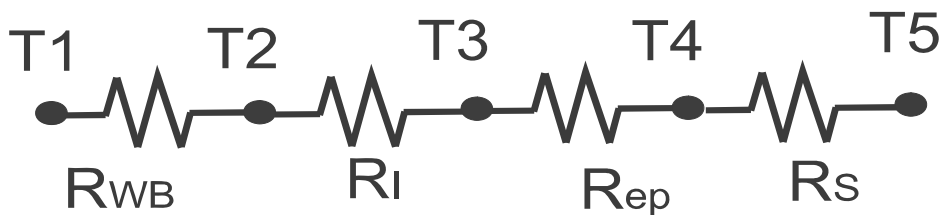
Similar equations can be written for the plywood, $T_3 - T_4$ and the siding, $T_4 - T_5$. When these equations are summed up the intermediate temperatures T_2 , T_3 and T_4 cancel and the resulting equation becomes

$$T_1 - T_5 = q \left[\frac{L_{WB}}{k_{WB} A} + \frac{L_I}{k_I A} + \frac{L_P}{k_P A} + \frac{L_S}{k_S A} \right] \quad (2.8)$$

or

$$q = \frac{T_1 - T_5}{\sum \frac{L}{kA}} \quad (2.9)$$

The electrical analogy for this case is resistors in series as shown in figure 2.4.



Then the overall solution can be easily written as,

Fig. 2.4 Electric Analogy to Steady State Heat Transfer

$$q = \frac{\Delta T}{\sum R} \quad (2.10)$$

which is identical to equation 2.9.

same so that equation 9 can be rewritten as

$$q_s = \frac{(T_1 - T_5)A}{\left[\frac{L_{WB}}{k_{WB}} + \frac{L_I}{k_I} + \frac{L_P}{k_P} + \frac{L_S}{k_S} \right]} \quad (2.11)$$

R-value of that material. Note the R-value is independent of the surface area A while the thermal resistance R_T includes the surface area. For the plywood, a typical R-value for a one inch thickness is

$$\frac{L}{k_p} = \frac{1/12}{0.1} = 0.8 \frac{hr \ ft^2 F}{BTU} \quad (2.12)$$

Note in US building practice units and dimensions are still in the imperial system.
residential wall constructed with 2 by 4 studs,

$$\frac{L}{k_I} = \frac{3.5/12}{0.028} = 11 \frac{hr \ ft^2 F}{BTU} \quad (2.13)$$

For the composite wall in figure 2.3, the R-value of the insulation dominates all of the terms in equation 2.11.

Convection Heat Transfer, Introduction

temperature on the inside wall surface, T_1 , and the outside siding surface T_5 are not generally known. Rather the interior room air temperature, T_i , and the exterior air temperature, T_e , are the known quantities. Consider a wintertime condition, when the building is at a higher temperature than the exterior air. The temperature through the built-up wall continuously decreases from the inside wall at T_1 to the outside surface at T_5 . This is shown in figure 5. The outside surface temperature T_5 is higher than the exterior air temperature T_e . In the air layer close to the building surface the air is in motion parallel to the surface. There is heat transfer by conduction from the building surface through this air layer. Because there is also energy transfer by the motion of the fluid the temperature through the air layer does not vary linearly. Rather, there is a large temperature gradient near the surface which decreases further from the surface until the temperature reaches the constant air temperature T_e . The layer over which the temperature change occurs is thin, typically one quarter of an inch or less.

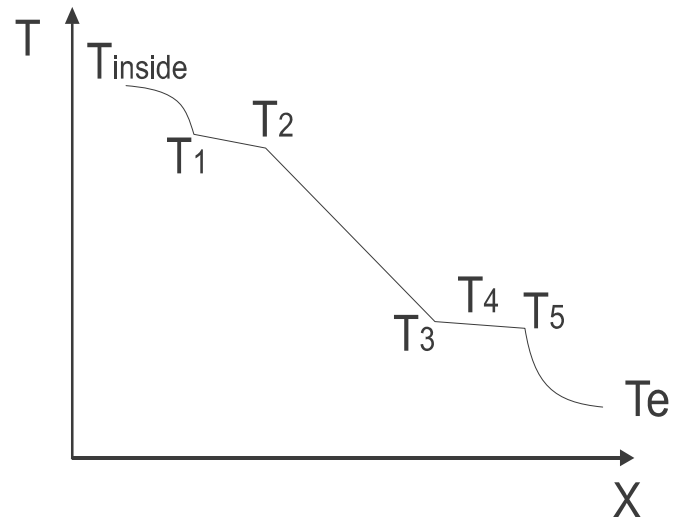


Fig. 2.5 Temperature Distribution with Convection at the Surfaces

energy transfer by fluid motion is called convection heat transfer. The rate of heat transfer is proportional to the surface area and the temperature difference between the surface and the uniform air temperature outside of the thin surface or boundary layer,

$$q \sim A(T_s - T_e) \quad (2.14)$$

The expression is changed to an equality and in the process a new quantity, h , the heat transfer coefficient is defined,

$$q = hA(T_{\text{surface}} - T_{\text{fluid far from surface}}) \quad (2.15)$$

where h has the units of BTU/hr ft² °F or W/m². Equation 2.15 is of no use until some way to calculate h is established.

Generally, the heat transfer coefficient, h , is a function of the fluid properties, the fluid velocity, the surface geometry and sometimes the temperature level. A more detailed discussion of convection will be given later. For now it is sufficient to observe that h increases as the air velocity increases and it increases with fluids of higher thermal conductivity.

There are two general forms of convection. When the air motion is set up by buoyancy effects due to the applied temperature difference between the surface and the fluid, e.g. the air flow over a hot 'radiator', the flow is natural or free convection. When the flow is due to an external source, e.g. the wind, a fan or by the motion of the surface, the flow is forced convection.

Rohsenow has presented a table which gives good estimates of the order of magnitude of h for convection heat transfer as well as boiling and condensation. It is reproduced in table 2.2.

Table 2.2
Convection Heat Transfer Coefficients
BTU/hr ft² F

Gases, Natural Convection	0.5-50
Gases, Forced Convection	2-50
Liquids, forced Convection	30-1000
Boiling Liquids	200-50,000
Phase Change	500-5,000

Now returning to the concept of thermal resistance, from equation 15 the equivalent thermal resistance, R_T for convective heat transfer is $1/hA$.

Example

For a single glazed window what is the increase in thermal efficiency if the glass is made of plastic with $k = 0.1$ BTU/hr ft °F instead of glass with a $k = 0.5$ BTU/hr ft °F? Assume that the radiation heat transfer remains the same.

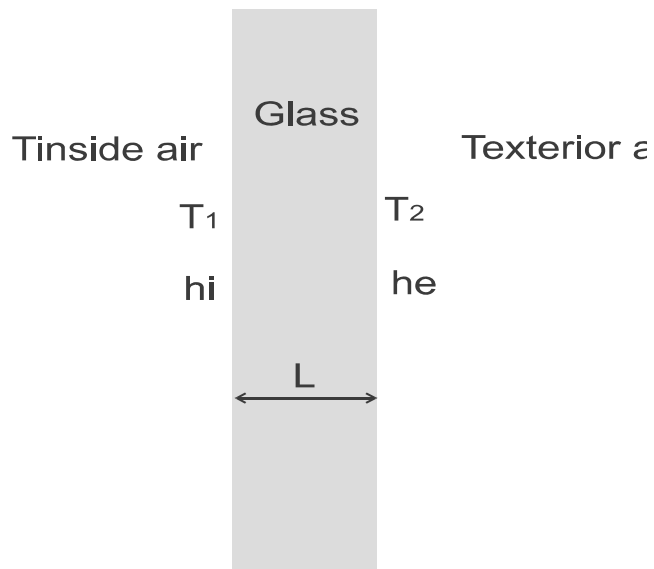


Fig. 2.6 Heat transfer through window

SOLUTION

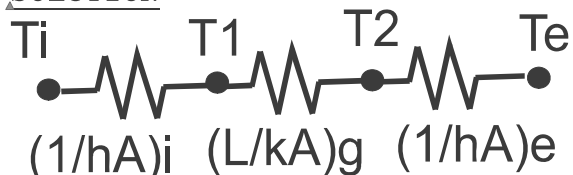


Figure 2.7 Electrical Analogy

The equivalent electrical circuit is shown on figure 7 and the steady state heat transfer, neglecting radiation, becomes

In this case convection heat transfer from the inside air at T_i to the glass surface acts in series with conduction through the glass and convection to the outside air.

$$q = \frac{(T_{inside\ air} - T_{exterior\ air})}{(1/hA)_i + (L/kA)_g + (1/hA)_e} = \frac{(T_{inside\ air} - T_{exterior\ air}) A}{(1/h)_i + (L/k)_g + (1/h)_e} \quad (2.16)$$

Using table 2 the magnitudes of h_i and h_e are

$$\frac{1}{h_i} \approx \frac{1}{1} = 1 \left(\frac{BTU}{hr ft^2 F} \right)^{-1} \quad (2.17)$$

$$\frac{1}{h_e} \approx \frac{1}{3} = 0.33 \left(\frac{BTU}{hr ft^2 F} \right)^{-1}$$

For the glass, assuming it is 1/8 inch thick,

$$\frac{L}{k_g} \approx \frac{(1/8)(1/12)}{0.5} = \frac{1}{50} \left(\frac{BTU}{hr ft^2 F} \right)^{-1} \quad (2.18)$$

Changing to plastic decreases k_g to 0.1 and increases L/k_g to 1/10 but it will only change the overall value of q , given by equation 16 by less than 10 percent.

The overall heat transfer for composite systems such as figure 2.3 or figure 2.6, represented by equations 2.9 and 2.16, respectively is sometimes rewritten in terms of an overall heat transfer coefficient U defined as

$$q = UA(T_i - T_e) \quad (2.19)$$

Although U has the same units as h , U can involve a combination of conduction and convection heat transfer and is not physically meaningful although it may be helpful for estimate purposes. A number of handbooks like the ASHRAE Handbook of Fundamentals list values of U for typical built up wall and roof construction. These values of U include convection heat transfer on the inside and outside for an assumed wind velocity and interior air circulation conditions.

Two-dimensional Heat Transfer

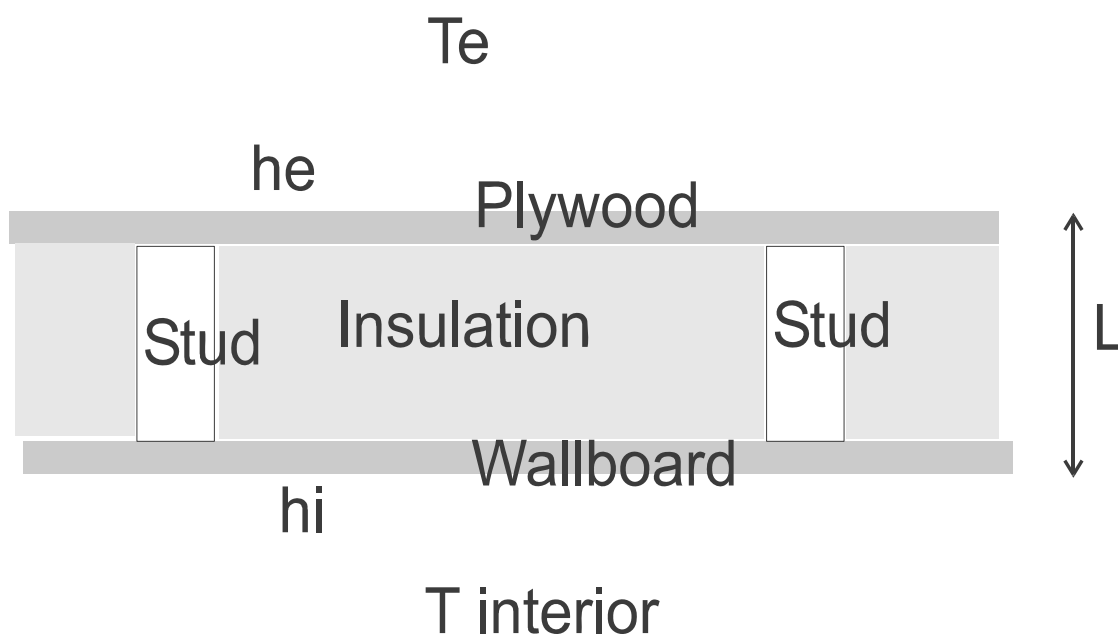


Fig. 2.8

Most walls are not uniform across their entire surface area. Wood framing using 2 by 4's has studs spaced at regular intervals in the wall cavity, fig 2.8. Clearly the heat transfer through the studs is higher than the heat transfer through an equivalent cross-sectional area containing insulation.

The heat transfer through the wall cavity is due to two parallel conduction paths, one through the studs and the other through the insulation. If the lateral resistance is very large, i.e., the lateral conductivity of the wallboard and the plywood approaches zero, then the overall heat transfer can be modeled as two separate parallel heat flow paths from T_i to T_e , shown in fig. 2.9.

The heat transfer through the studs is

$$q_{stud} = \frac{(T_i - T_e) A_{stud}}{1/h_i + (L/k)_{WB} + (L/k)_S + (L/k)_P + 1/h_e} \quad (2.20)$$

and through the insulation,

$$q_{insulation} = \frac{(T_i - T_e) A_{insulation}}{1/h_i + (L/k)_{WB} + (L/k)_{insulation} + (L/k)_P + 1/h_e} \quad (2.21)$$

The total heat transfer rate is the sum of equations 2.20 and 2.21. The true value of the two-dimensional heat transfer lies between these two limiting cases of very small lateral conductivity and very large lateral conductivity. The electrical analogy of this, in one limit, is shown in figure 2.9 with the insulation and the studs in parallel.

It is important to note that this approximation for the two-dimensional case represents one limiting case, the temperature on the inside of the wall board where it contacts the stud will be different from the temperature of the wall board in contact with the insulation. Similarly the temperature of the plywood

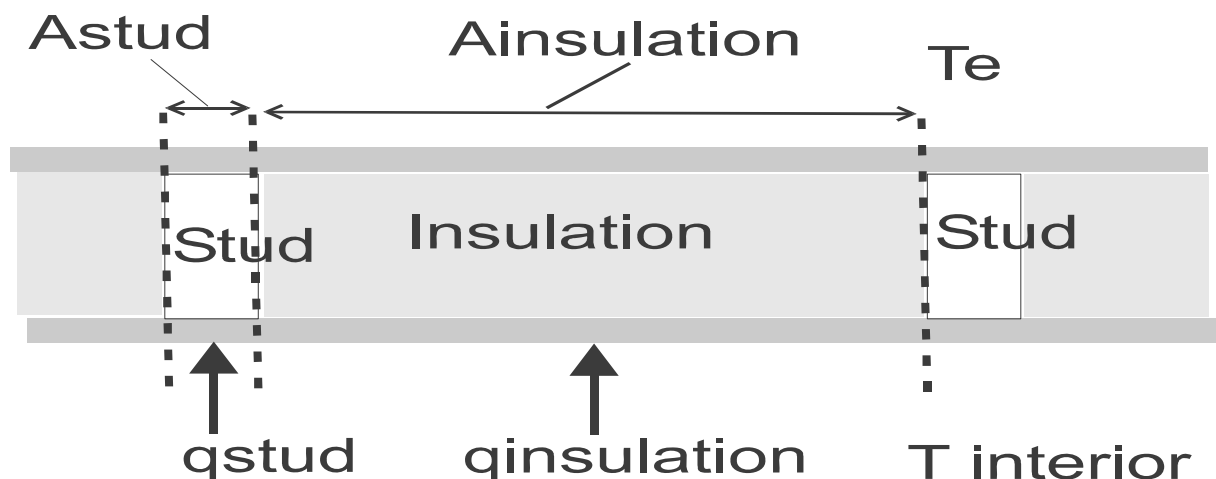


Fig.2.9 Limiting Case of Small Lateral Conductivity in Wall Board and Plywood

will differ in the lateral direction between the stud and the insulation. These temperature differences across the wall board will cause heat to flow laterally into the stud, fig. 2.10. Similarly heat will flow laterally out from the stud through the plywood. These lateral effects will enhance the total heat transfer through the stud. This will increase the overall heat transfer over the values calculated from the sum of equations 2.20 and 2.21.

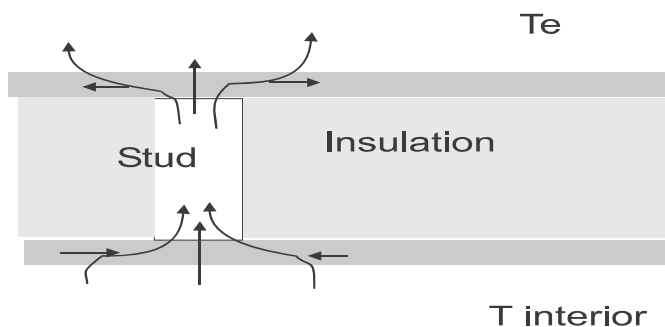


Fig. 2.10 Lateral heat flow in the vicinity of the stud

The exact analysis of these

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two-dimensional effects is difficult. We can look at the other extreme case. If the wall board and plywood were replaced by metal sheets, something sometimes seen in metal walled buildings, then the resistance to lateral conduction through them would be reduced. In the limiting case we could assume that the wall board and plywood would have a very high conductivity and the temperature would be uniform in the lateral direction across these elements. The electrical analogy for this case is shown on fig. 2.11. The solution shown in figure 10 is only valid when the lateral resistance is small, i.e., the y direction conductivity of the wall board and the plywood approaches infinity.

In this case, the heat transfer through the studs and insulation acts in parallel between T_2 and T_3 , the sheet rock and plywood temperatures, respectively. The heat transfer through the two elements is summed,

$$q = \frac{T_2 - T_3}{(L/kA)_{insulation}} + \frac{T_2 - T_3}{(L/kA)_{stud}} \quad (2.22)$$

and defining an equivalent resistance,

$$q = \frac{T_2 - T_3}{(L/kA)_{insulation} + (L/kA)_{stud}} = \frac{T_2 - T_3}{R_{equivalent}} \quad (2.23)$$

The resistance, $R_{equivalent}$, is then added in series to the

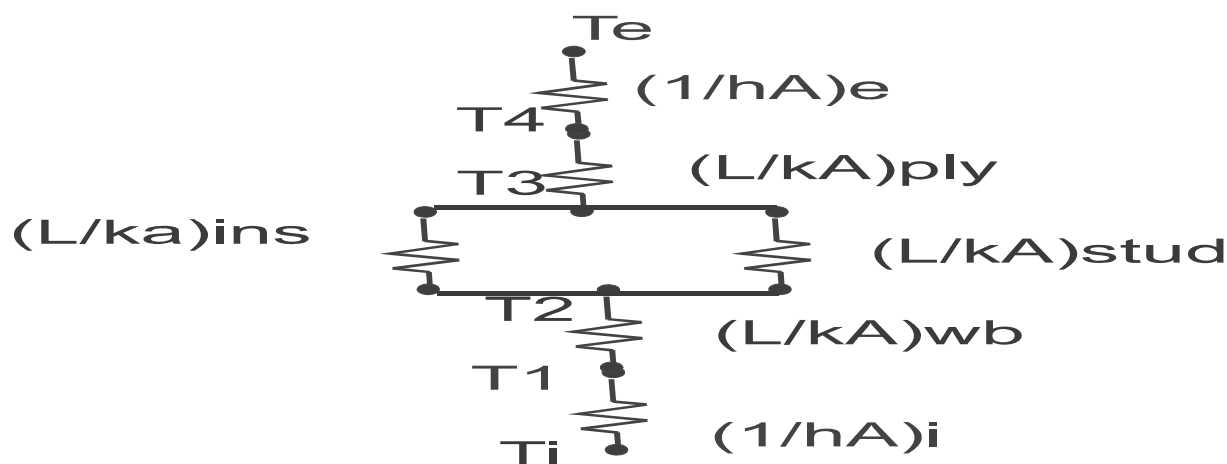


Fig 2.11 Limiting Case for
Small Lateral Resistance to Heat Transfer in Sheathing

remaining resistance shown in figure 2.11. This extreme case will result in a much larger overall calculated heat transfer than the other extreme shown in figure 2.9. The true value lies between the two and must be evaluated by analysis or practical judgment.

Transient Heat Transfer

Conditions for Uniform Temperature

When the temperature of the interior changes, the building structure may be sufficiently massive to provide significant thermal storage. This technique is used in naturally ventilated buildings, using night cooling in the summer to reduce the structural temperature. During the hot daytime the cool structure will help to maintain the interior temperature within a comfortable range. However, if the structure is very thick, it may require a substantial time interval before the temperature change is felt throughout the thickness of the structure.

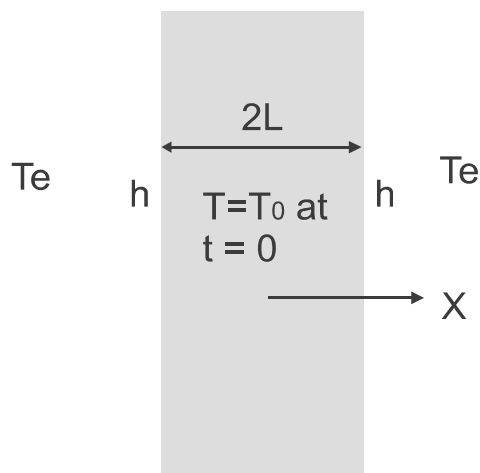


Fig. 2.12

Consider the case of a plane homogenous slab of thickness $2L$, fig. 2.12. The y and z dimensions are large compared to L so that the heat transfer can be assumed to be one dimensional, in the x direction only. Initially the slab is at a uniform temperature T_0 equal to the exterior air temperature. At time zero the air temperature suddenly increases to a new temperature level T_e . Both sides of the slab are in contact with the air and both sides have convective heat transfer coefficient h . At short times, the surface temperature of the slab will increase due to the heat transfer from the air. Some of the energy transferred to the slab will be used to raise the internal energy of the material near the surface. The balance of the heat transfer will be transferred to the next layer inside the slab where the same process occurs. The temperature distribution within the slab at some intermediate time is shown in figure 2.13. When the

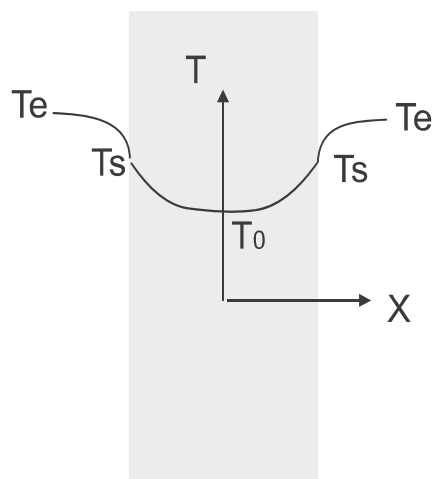


Fig. 2.13

conduction of the slab is large and the thickness is small, it is expected that the temperature differences from the surface to the center of the slab will be small at all times. A better criterion is to compare the resistance to conduction, L/k_{slab} , to the convective resistance at the surface, $1/h$. When $1/h$ is much larger than L/k_{slab} it is expected that the temperature of the slab will be uniform throughout its thickness. A handy criterion is $hL/k_{\text{slab}} < 1/6$ for uniform temperature. The term hL/k which is dimensionless is known as the Biot number.

When $hL/k < 1/6$, the slab temperature T only varies with time. Assuming the slab initially is at temperature T_0 and at time t it contacts air at a different temperature T_∞ the energy equation becomes,

$$\frac{dE}{dT} = q = -hA(T - T_\infty) \quad (2.24)$$

where A is the entire surface area of the slab which is in contact with air at a constant temperature T_∞ . When there isn't any change of phase, the energy change is given as $Mc\Delta T$, where M is the total mass of the slab. Equation 2.24 becomes,

$$Mc \frac{dT}{dt} = -hA(T - T_\infty) \quad (2.25)$$

Since T_∞ is a constant this can be rearranged to read,

$$\frac{d(T - T_\infty)}{(T - T_\infty)} = -\frac{hAdt}{Mc} \quad (2.26)$$

with the initial condition for the slab that

at $t = 0$, $T = T_0$

The solution of equation 31 is

$$T - T_\infty = (T_0 - T_\infty) \exp\left(-\frac{hAt}{Mc}\right) = (T_0 - T_\infty) \exp\left(-\frac{t}{\tau}\right) \quad (2.27)$$

where τ is the thermal time constant of the slab with convective heat transfer coefficient h and

$$\tau = \frac{Mc}{hA} \quad (2.28)$$

Example

A two inch thick steel structural beam has natural convection heat transfer over one surface; the opposite side is insulated. Find its time constant.

Solution

First it must be determined if the steel can be assumed uniform in temperature across its width. Since only one side has convective heat transfer, referring to figure 2.13, L in this case should be the full width of the steel beam. The Biot number is

$$\frac{hL}{k} \approx \frac{2(2/12)}{20} \approx \frac{1}{60} \ll 1 \quad (2.29)$$

where h is estimated for natural convection from table 2.2. The assumption of uniform temperature is clearly justified. Note that if we were considering a two inch thick concrete section with a conductivity of about 1 BTU/hr ft °F the Biot number would be 1/3 and the assumption of uniform temperature through the concrete would be questionable.

The thermal time constant for the steel beam is,

$$\tau = \frac{Mc}{hA} = \frac{\rho LAc}{hA} = \frac{\rho Lc}{h} \approx \frac{400(0.1)(2/12)}{2} \approx 3.5hrs$$

The time constant is a function of the steel properties and the heat transfer coefficient at its surface. If air was blowing over the surface at a high velocity the time constant would be substantially reduced.

3. Convection Heat Transfer

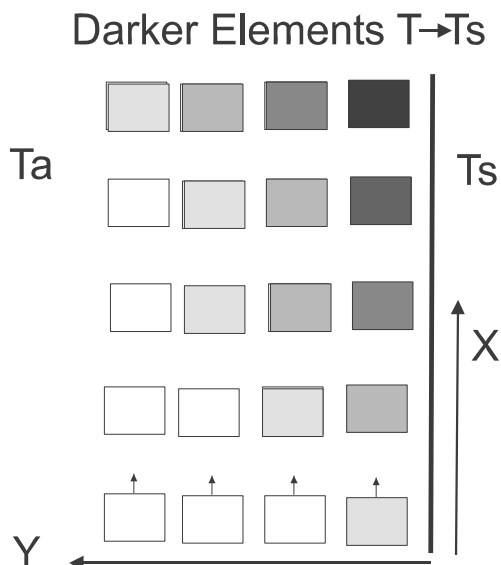


Figure 3.1 Flow over a heated plate

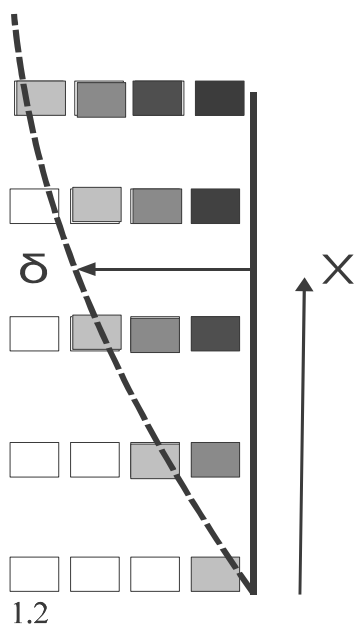
Convective heat transfer is conduction through a gas or liquid augmented by fluid motion. In these notes an introduction to the physics governing convection will be given along with some results for several different conditions.

Consider the case of air at a uniform temperature T_a blown by a fan along a flat surface which is heated to a uniform temperature T_s , Figure 3.1. Say, this is cool air in an air-conditioned room flowing over the surface of a window heated by solar radiation. The element of air closest to the heated surface has a temperature increase as it starts to move up the plate. Elements further away, at a larger y coordinate still are at T_a . As the element becomes hotter it moves up and is replaced with another element at T_a and the process is repeated. Viewed from the point of view of the room as a whole, room air at T_a approaches the

plate and a given flow of air leaves the plate at an elevated temperature somewhere between T_a and T_s . Thus there is a net energy transfer from the surface of the window to the room air.

The convective heat transfer coefficient is defined as,

$$h = \frac{q}{A(T_s - T_a)} \tag{3.1}$$



1.2

To get an estimate of how the rate of heat transfer is influenced by the parameters of the problems we have to look more closely at the layer of air moving along the plate surface. To make the explanation clear we will assume all of the elements of air are moving along the plate surface at uniform velocity and in straight lines. This is an instance of laminar flow. At any location (x,y) fixed relative to the stationary plate the temperature remains constant with time and there is a steady conduction heat transfer from the surface to the air in the y direction, normal to the plate surface. As the air continues up the plate the elements close to the plate increase their temperature. At the same time the elements further away start to rise in temperature due to conduction from the hotter elements at the plate surface..The further along the air moves in the x direction the more elements further from the plate surface feel the conduction heat transfer and rise in temperature as shown in figure 3.2. The maximum y

distance at which the thermal effects are felt, at any location x , will be specified as δ , the thermal boundary layer thickness.

At any position x the variation of air temperature normal to the plate surface, the y axis is shown on figure 3.3. The rate of heat transfer from the plate surface is determined by the conduction into the air at $y=0$. This can be calculated from,

$$q = -k_a \left. \frac{\partial T}{\partial y} \right|_{y=0} \quad (3.2)$$

As a good first estimate we can use,

$$q \approx Ck_a \frac{T_s - T_a}{\delta} \quad (3.3)$$

where the constant C should be of order of magnitude unity. Now substituting this into the definition of the convective heat transfer coefficient,

$$h = \frac{q}{(T_s - T_a)} \approx \frac{k_a(T_s - T_a)}{\delta(T_s - T_a)} \approx \frac{k_a}{\delta} \quad (3.4)$$

Thus the heat transfer coefficient is proportional to the thermal conductivity of air and inversely proportional to the size of the thermal boundary layer thickness.

We can use the estimate for h to qualitatively predict how the heat transfer coefficient will vary with the main parameters

First, if the velocity of the air is increased over the plate surface then each fluid element spends a shorter time in contact with, or close to the plate. The fluid element temperatures don't increase as much. From figure 3.1 and 3.2, in such an instance the thermal boundary layer, δ , will be smaller at a given x value. Thus an increase in air velocity should result in an increase in the heat transfer coefficient and the rate of convective heat transfer. This result is summarized in table 3.1

Consider now an increase in the air density, say, by an increase in the air pressure, assuming all other air properties and the air velocity remains the same. For the same magnitude of heat transfer the temperature increase of the element will be less since the element has a larger mass.

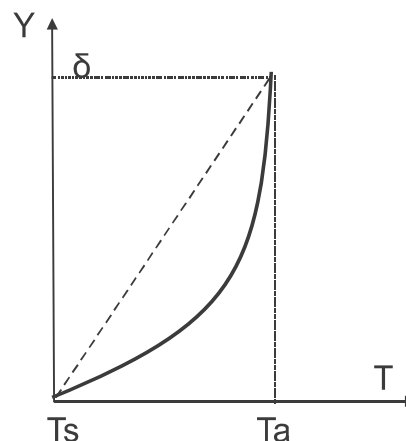


Fig. 3.3 Air temperature in thermal boundary layer

A smaller temperature increase will reduce the thermal boundary layer thickness. Therefore, an increase in the density will increase the heat transfer coefficient, one reason that liquid water has a higher convective heat transfer rate than air. An increase in the specific heat of the fluid flowing over the plate has the same behavior as an increase in density.

As the plate length is increased the air will flow over the plate for a longer time. Heat transfer will penetrate a further distance from the plate surface. The thermal boundary layer thickness will grow larger. When averaged over the entire plate length, the average thermal boundary layer thickness will become larger and the average heat transfer coefficient will be smaller. Note, the total heat transfer will be higher for the longer plate but the heat transfer per unit area will be smaller. To augment the heat transfer when possible, designers will break a long surface up into a series of smaller surfaces. This can be done by physically separating sections of the plate or by placing an array of ribs at right angles to the flow to break up the boundary layer and restart it.

Table 3.1 Key Parameters Influence on Convective Heat Transfer

Parameter	δ , thermal boundary layer thickness	h , convective heat transfer coefficient
Velocity increase	δ decreases	h increases
Density increase	δ decreases	h increases
Specific heat increase	δ decreases	h increases
Plate length L increase	δ averaged over L increases	h averaged over L decreases
Thermal conductivity k_a increase	δ increase	h increases
Transition to turbulent flow from laminar flow	δ decreases	h increases

When the thermal conductivity of the fluid passing over the plate is increased, by using a higher conductivity gas or liquid there are two elements in play. The thermal boundary layer thickness will increase because of augmented means of heat transfer through the fluid. Remember that h is proportional to the ratio of conductivity to boundary layer thickness. In this case k increases faster than δ and the heat transfer coefficient increases. It should be expected that when we increase the mechanism for heat transfer, in this case the molecular conductivity, that the rate of convection will increase.

Turbulent Flow

At low velocity, the fluid flows in very smooth paths about parallel to the plate surface. As the velocity is increased a point is reached where the fluid motion is much more chaotic characterized by eddies in the flow near the plate surface. This is termed turbulent flow. Turbulent flow over a flat plate is found to occur when the Reynolds number $\rho Vx/\mu$ exceeds 300,000. x is used to indicate the distance from the leading edge; the front of the flat plate can have laminar flow while the rear experiences turbulent flow. The distinction between laminar and

turbulent flow is important because the eddies in the turbulent flow tend to bring fluid at the ambient temperature T_a much closer to the heated plate surface. In effect the eddies reduce the distance δ for conduction heat transfer and can markedly increase the heat transfer coefficient, sometimes by an order of magnitude or more.

As can be seen that the convective heat transfer is a function of the fluid properties such as density and conductivity, the flow conditions and the surface geometry. Given below are a few dimensional expressions that can be used for specific cases. The constants in the equations already include the fluid properties.

Laminar flow expressions for h

Using air properties at room temperature h for laminar flow over flat plates can be found as

$$h = 0.71 \left[\frac{V}{L} \right]^{0.5} \quad (3.5)$$

while for water,

$$h = 12.7 \left[\frac{V}{L} \right]^{0.5} \quad (3.6)$$

In this form h is in BTU/hrft²F, V is in ft/sec and L is in ft. Note, water gives a much higher heat transfer coefficient than air because it has a much higher thermal conductivity as well as a higher density and specific heat.

Turbulent Flow

At low velocity, the fluid flows in very smooth paths about parallel to the plate surface. As the velocity is increased a point is reached where the fluid motion is much more chaotic characterized by eddies in the flow near the plate surface. This is termed turbulent flow. Turbulent flow over a flat plate is found to occur at higher velocities and longer plate lengths. Also flowing liquids will reach turbulent flow at lower velocities than gases.

For air near room temperature turbulent flow is reached when the product of plate length and air velocity exceeds,

$$VL \geq 50 \frac{ft}{s} ft \quad (3.7)$$

$$h = 0.55 \frac{V^{0.8}}{L^{0.2}} \quad (3.8)$$

For water

$$VL \geq 3.9 \frac{ft}{s} \quad (3.9)$$

$$h = 21.2 \frac{V^{0.8}}{L^{0.2}}$$

Again, all of the parameters are in Imperial units.

Flow Inside Tubes

The other important flow geometry is gas or liquid flow inside tubes. A similar development exists for the convective heat transfer with the exception that h is defined based on the mean temperature T_M of the fluid within the tube at the location in question. For a section within the tube of axial length between x and $x+\Delta x$,

$$h = \frac{q}{\pi D \Delta x (T_S - T_M)} \quad (3.10)$$

where T_M is the mean fluid temperature at x .

Almost all practical cases of tube flow, the flow is turbulent. Exceptions are flows through very small tube diameters or the flow of viscous fluids such as oil. For turbulent tube at room temperature, the relationship becomes,

$$h = 0.34 \frac{V^{0.8}}{D^{0.2}} \quad (3.11)$$

while for water we get,

$$h = 13 \frac{V^{0.8}}{D^{0.2}} \quad (3.12)$$

where V is given in ft/sec and D is in feet.

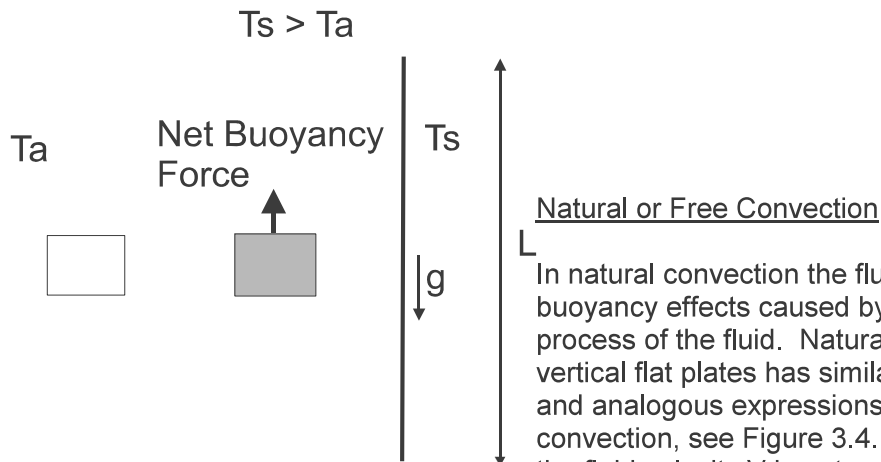


Figure 3.4 natural convection on a vertical heated plate

In natural convection the fluid motion is solely due to buoyancy effects caused by the heating or cooling process of the fluid. Natural convection flow over vertical flat plates has similar physical considerations and analogous expressions to those used for forced convection, see Figure 3.4. However, in this case the fluid velocity V is not set by external fans or by the motion of the heated body. The air density varies with $1/T$. The air close to the plate is at a higher

temperature and has a lower mass. This results in a net upward buoyancy force on these elements that accelerates them. We would expect the velocity and the heat transfer coefficient is a function on the temperature difference $T_s - T_a$.

Just as forced flow, laminar flow exists at low velocities. The exact expression for natural convection of air over a vertical plate at room temperature is

$$\Delta TL^3 < 1000 \text{ } ^\circ F \text{ ft}^3$$

$$h = 0.29 \left(\frac{\Delta T}{L} \right)^{1/4} \quad (3.13)$$

For turbulent flow,

$$\Delta TL^3 \geq 1000$$

$$h = 0.21 (\Delta T)^{1/3} \quad (3.14)$$

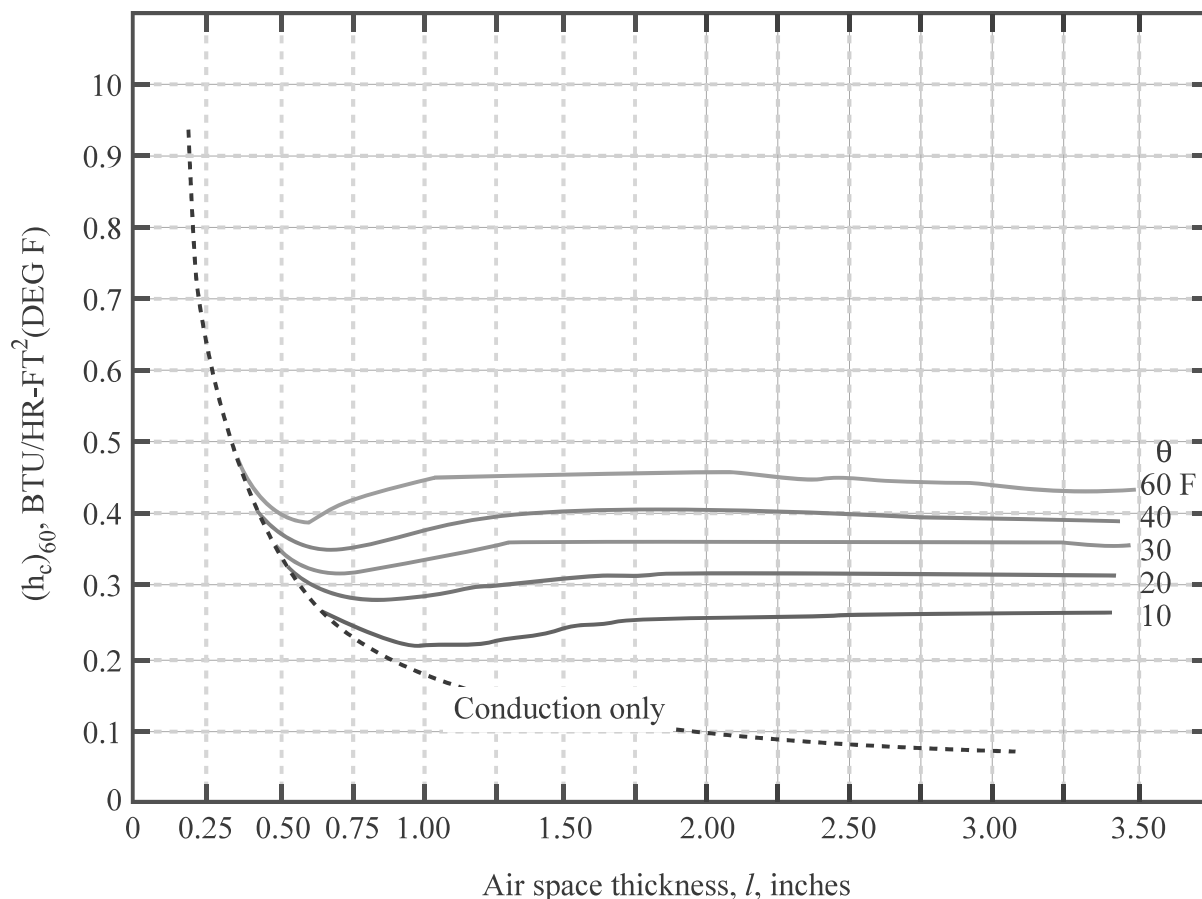
Natural Convection in Enclosed Spaces

In an open wall cavity between interior and exterior walls air circulation will take place from the hot to the cold wall. The air will rise along the hot wall move horizontally to the cold wall at the upper end of the wall cavity (and at other vertical locations as well). The warm air will then flow down the cold wall. This process will result in energy transfer from the hot to the cold

wall. The overall heat transfer can be represented by a heat transfer coefficient defined in terms of the two wall temperatures,

$$q = h_c A (T_{hot\ wall} - T_{cold\ wall}) \quad (3.15)$$

Figure 3.5 shows measured results for eight foot high walls. h_c is a function of the spacing between the walls and θ , the temperature difference from the hot to the cold wall. At a small spacing and/or a small temperature difference the buoyancy effects are minimal and h_c is simply the ratio of air conductivity to wall spacing. As the spacing is increased, h_c reaches a constant, typically a spacing between 1/2 and 3/4 inch is optimum. Radiation heat transfer across the cavity must be added to the convection. If there is infiltration of outside air into the cavity the energy transfer may be increased considerably.



Data taken from Housing Research Paper 32, "The Thermal Insulating Value of Airspaces", Housing and Home Finance Agency Director of Housing Research, Washington D.C., April 1954.

Figure by MIT OpenCourseWare.

4) Radiation Heat Transfer

Thermal radiation and conduction are the two fundamental mechanisms of heat transfer. Bodies, such as the sun, emit electromagnetic energy by virtue of their temperature level. The electromagnetic energy is exchanged between bodies at different temperatures giving rise to a net heat transfer.

For heat transfer by radiation the electromagnetic energy generally falls in the visible range, with wavelengths from 0.4 to 0.7 μm , the near infrared, from 0.7 to 25 μm in wavelength, or the far infrared from 25 to 1000 μm . Electromagnetic radiation in other wavelengths such as X-Rays, and Radio waves is not thermally induced and will be excluded from consideration.

A black body is a body which absorbs all of the radiation incident on its surface over all wavelengths of importance for heat transfer. The absorbed energy represents an energy transfer to the black body which can contribute to the internal energy increase or it may be transferred through the body by one or more forms of heat transfer.

For a black body at a uniform temperature T , the rate of radiation heat transfer emitted by the body and leaving the surface, over all wavelengths, is given as

$$q_{emitted} = A\sigma T^4 \quad (4.1)$$

where T is the absolute temperature and σ is the Stefan-Boltzmann constant, which has the value 0.17×10^{-8} BTU/hr $\text{ft}^2 \text{ } ^\circ\text{R}^4$ or 5.7×10^{-8} W/ $\text{m}^2 \text{ } ^\circ\text{K}^4$. Because of the non-linear nature of the expression, radiation becomes progressively more important at higher temperatures although there is considerable radiative heat transfer between bodies at room temperature.

The black body radiation is emitted over a range of wavelengths. The emitted energy at a single wavelength λ , $q_{re\lambda}$, can be defined so that

$$q_{emitted} = \int_0^{\infty} q_{re\lambda} d\lambda \quad (4.2)$$

Figure 20 shows the distribution of $q_{re\lambda}$ as a function of wavelength for black bodies at three temperature levels. For solar radiation, the sun has an effective black body temperature near 10,400 $^\circ\text{R}$, a majority of the emitted energy is in the visible wavelengths and the balance of the energy is in the near infrared below 3 μm . In contrast, the energy emitted by a black body at room temperature falls in the infrared wavelength range from 8 μm to 40 μm . For any black body at temperature T eighty percent of the total black body emissions occurs when the product of the wavelength and the absolute temperature, λT , is between 4000 (μm)($^\circ\text{R}$) and 17,000 (μm)($^\circ\text{R}$). One half of the total radiation is emitted at wavelengths equal to or below the value of λT equal to 7400 (μm)($^\circ\text{R}$).

Material Properties

In the visible wavelengths many common building materials are not good absorbers. The fraction of the black body radiation absorbed by a body at a given wavelength λ is given by the monochromatic absorptivity α_λ , the values of α_λ lie between zero and one. In the visible wavelengths, white paint, wood and plaster have α_λ values between about 0.1 and 0.5 see Table 3. In the infrared above 4 or 5 μm wavelengths almost all common building materials except shiny metals have α_λ of 0.8 or higher. Plate glass which is transparent in the visible

wavelength becomes a very good black body in the infrared wavelengths with α_λ greater than 0.9.

Heat Transfer Between Black Bodies

Consider two flat plates, A_1 and A_2 , parallel to each other with the spacing between the surfaces small compared to the width of the plates, see Figure 4.1. The air between the plates can be considered transparent.

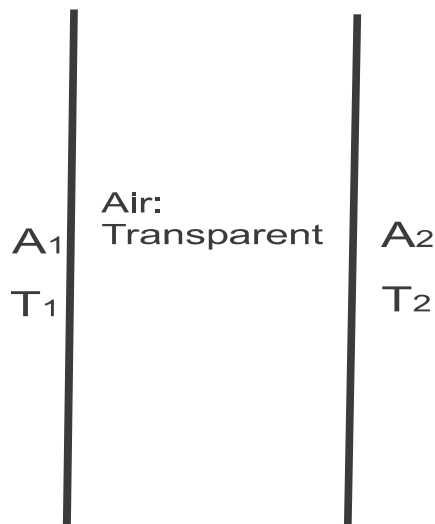


Fig. 4.1 Radiation between parallel black plates

Each plate is a black body at a uniform temperature T_1 and T_2 , respectively. The radiant energy emitted by the first plate is

$$q_{1 \rightarrow 2} = A_1 \sigma T_1^4 \quad (4.3)$$

All of the energy emitted by plate 1 is absorbed by plate 2. Similarly all of the energy emitted by plate 2, $A_2 \sigma T_2^4$ is absorbed by plate 1. The net radiative heat transfer is

$$q_1 = A_1 \sigma (T_1^4 - T_2^4) \quad (4.4)$$

This heat transfer takes place independent of any conduction or convection heat transfer assuming the temperature T_1 and T_2 can be maintained. Thus the total heat transfer is the sum of the radiative heat transfer given by Eqn. 70 plus the convective heat transfer between the two plates.

In many cases it is useful to linearize the relationship given by Eqn. 70. Taking the mean temperature as $(T_1 + T_2)/2$ and ΔT as the temperature difference $T_1 - T_2$ then $T_1 = T_M + \Delta T/2$ and $T_2 = T_M - \Delta T/2$ and

$$T_1^4 - T_2^4 = (T_1 - T_2)(T_1 + T_2) \left(2T_M^2 + 2\frac{\Delta T^2}{4} \right) \quad (4.5)$$

When $\Delta T \ll T_M$, where T_M is measured in absolute temperatures, K or R,

$$\sigma(T_1^4 - T_2^4) \approx 4\sigma T_M^3 (T_1 - T_2) \quad (4.6)$$

An effective or radiative heat transfer coefficient can be defined so that

$$q_r = h_r A (T_1 - T_2) \text{ where } h_r = 4\sigma T_M^3 \quad (4.7)$$

Table 4.1 gives values of h_r for different values of mean temperature. These values should be compared to the values given for convection in Table 2. At room temperatures heat transfer

Table 4.1 h_r for radiation at mean temperature T

Mean Temperature °F	h_r (BTU/hr ft ² F)	h_r (W/m ² K)
0	0.7	3.8
60	1.0	5.7
200	2.0	11
500	6.0	34
1500	51	290

by radiation is comparable in magnitude to natural convection. For heat transfer between two glass sheets that make up a double glazed window, infrared radiation is important. To reduce the radiation heat transfer one of the glass layers is coated with a very thin metallic layer. For residential buildings this layer allows visible solar radiation to pass but reflects most of the infrared radiation. The layer is referred to as a low e coating. The net infrared radiation can be expressed as,

$$q_1 = \epsilon A_1 \sigma (T_1^4 - T_2^4) \quad (4.8)$$

where ϵ is the emissivity of the glass covered by the metallic layer. Typically ϵ can be less than 0.1, substantially reducing the net radiation transfer.

For radiation between walls and ceiling or between a person and surrounding surfaces which are each black bodies at different temperatures the geometry of the configuration must be included. For example the radiation heat transfer between a person and a large glazed wall can be expressed as

$$q_1 = A_1 \sigma F_{12} (T_1^4 - T_2^4) \quad (4.9)$$

where F_{12} is the configuration factor between surface 1, the person, and the glazed surface. F represents the fraction of radiation from the entire surface of the person that is directed toward the glazed wall. When the person is very close to a large glazed surface extending from floor to ceiling, the value of F will be close to $\frac{1}{2}$. On the other hand when the person moves further from the window F will decrease. If the person sits in the center of a cubic room, F from the person to the glazed wall drops to $\frac{1}{6}$.

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