ESTIMATION OF STEADY STATE MEANS

$(X_t)_{t \geq 0}$ $\quad [X_{0:T}]$ $\cdot$ X_∞ : steady state distribution , $\underbrace{X_\infty = \lim_{t\to\infty} X_t}_{X_T \,\tilde{\approx}\, X_\infty \text{ for } T \text{ very large}}$

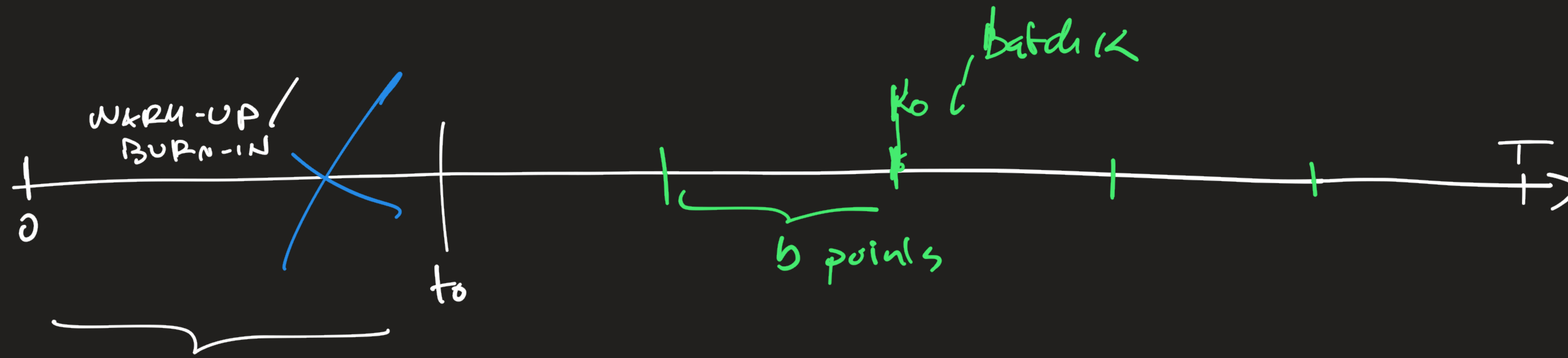
$\mathbb{E}[f(X_\infty)]$?

ERGODIC THEOREM

$$\underbrace{\mathbb{E}[f(X_\infty)]}_{\text{STATE AVERAGE}} = \underbrace{\lim_{T\to\infty} \frac{1}{T}\int_0^T f(X_t)\,dt}_{\text{TIME AVERAGE}} \quad (\text{ERGODIC } X_t)$$

We simulate a SINGLE TRAJECTORY for a very long time and use ergodic theorem

$$\boxed{Y_j = f(X_j) \qquad y_j = f(x_j)}$$



"time to reach stationarity"

$$\hat{Y} = \left(\sum_{j=t_0}^{T} y_j\right) \cdot \frac{1}{T-t_0} \qquad T - t_0 = n \Rightarrow \hat{Y} = \frac{1}{n}\sum_{j=0}^{n} y_{t_0+j}$$

$\boxed{\text{BATCH MEANS}}$: FIX $b>0$ BATCH SIZE,

SPLIT the n points in K batches of size b $\left(K = \frac{n}{b}\right)$

$\forall$ batch k, we compute $\hat{Y}_k = \frac{1}{b}\sum_{i=0}^{b-1} y_{k_0+i}$

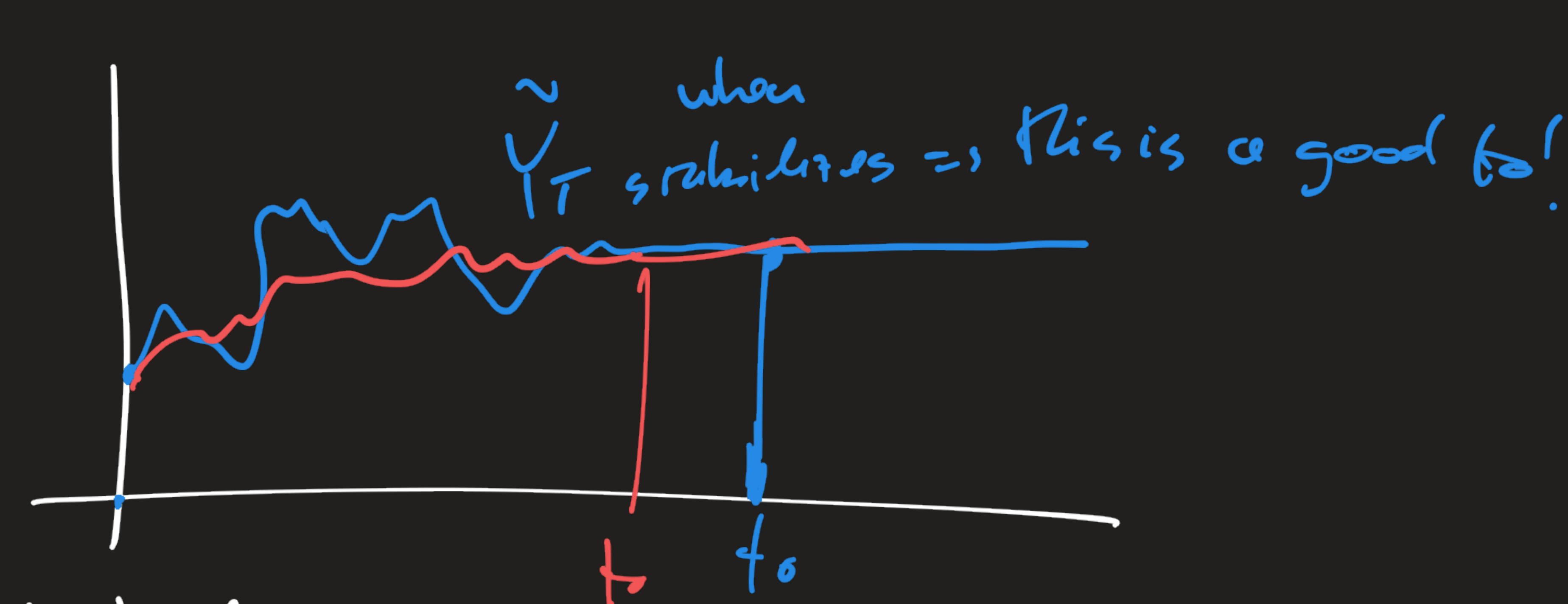
FIRST INDEX IN BATCH K.

$$\hat{Y} = \frac{1}{K}\sum_{j=1}^{K}\hat{Y}_j \qquad VAR[\hat{Y}_j] = \frac{1}{K-1}\sum_{j=1}^{K}\left(\hat{Y}_j - \hat{Y}\right)^2$$

$$VAR[\hat{Y}] \approx \frac{VAR[\hat{Y}_j]}{K}$$ (true in the limit of $n \to \infty, b \to \infty$

- HOW TO CHOOSE t_0?

$y_{0:T}$ $\qquad \tilde{Y}_T = \frac{1}{(T+1)} \sum_{j=0}^{T} y_j$



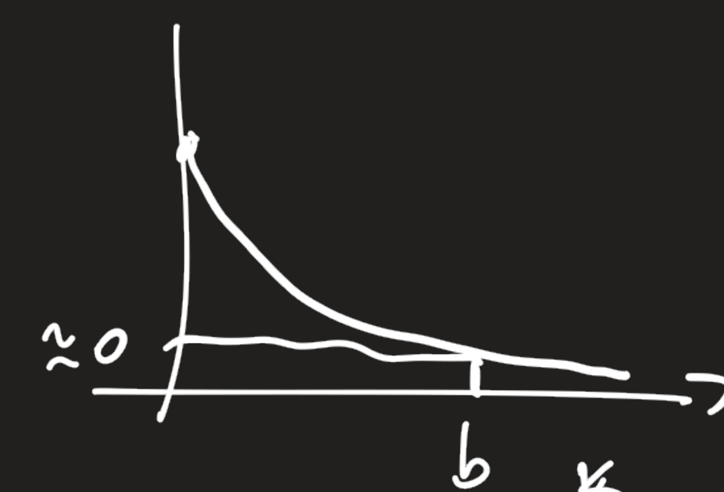
better strategy: simulate m (few) chains (m=5 or m=10)

take at each time $\bar{y}_j = \frac{1}{m} \sum_{i=1}^{m} y_{ji}$

- HOW TO CHOOSE b?

<u>AUTOCOVARIANCE</u> $\quad X_t$

$$\rho_k = \mathrm{cov}\Big[\overset{y_i}{f(X_i)}, \overset{y_{i+k}}{f(X_{i+k})}\Big]$$



$$\hat{\rho}_k = \frac{1}{T-k} \sum_{i=1}^{T-k} \big[\overset{y_i}{f(X_i)} - \hat{Y}\big] \big[\overset{y_{i+k}}{f(X_{i+k})} - \hat{Y}\big]$$

($\hat{\rho}_k$ becomes bad for large k)