

# Course: Principles of radiation detection and measurement

## Problems sets.

The following is a set of simple problems aimed at familiarizing the student with the application of the concepts and formulas studied in the course.

### 1. Cross section

•A 100 GeV proton of cosmic origin enters the earth atmosphere. Calculate the probability it reaches ground without interacting, if the total interaction cross section with nitrogen nuclei is 100 mb. Notes:

- consider the atmosphere as equivalent to a mass thickness of  $1000\text{g/cm}^2$  of nitrogen.
- do not consider Coulomb energy loss phenomena

•Let's consider a pion impinging upon a  $^{208}\text{Pb}$  nucleus, which we consider as a homogeneous sphere of radius=7fm. If the pion moves along a diameter which is the probability it exits the nucleus without interacting? Evaluate first its mean free path, for the 2 cases of a 100 MeV/c and a 300MeV/c positive pion, where  $\sigma_{\text{tot}}(100\text{MeV})=5\text{mb}$  and  $\sigma_{\text{tot}}(200\text{MeV})=200\text{mb}$ .

•A measurement of the proton flux outside the atmosphere and at ground level gives a survival value of 0.5% for 100 GeV protons. Evaluate the total inelastic cross section for 100GeV/c protons and their mean free path. Consider the atmosphere as equivalent to a mass thickness of  $1000\text{g/cm}^2$  of nitrogen.

•In an experiment with a 500MeV pion beam  $\Lambda$ -hyperons are produced via the  $\pi^+ n \rightarrow K^+ \Lambda$  which has a cross section of  $\sim 1\text{mb}$ .

If the pion flux is of  $10^{**6}/\text{s}$  and the target is liquid deuterium target ( $\rho=160\text{Kg/m}^{**3}$ ) 1 cm thick, evaluate the running time to be allocated in order to accumulate  $10^5 \Lambda$

## 2. $dE/dx$ and range.

1. The Tevatron (Fermilab) circulates 1000 GeV protons.

Show that in the 6 km ring there would be an energy loss of 1.3 GeV/turn if it was filled with air.

Data:  $\rho_{\text{air}} \sim 1.3 \text{ mg/cm}^3$ ;  $(dE/dX)_{\text{min-ion.air}} \sim 1.7 \text{ MeV/g}\cdot\text{cm}^2$

2. Estimate the thickness of iron ( $7.87 \text{ g/cm}^3$ ) necessary to stop a 200 GeV muon in the hypothesis that the energy loss is by em. collision.

And for 200 GeV protons?

If the total nuclear cross section on iron is 400 mb, evaluate its mean free path in iron.

How much iron is needed to stop the protons?

3. The integral cosmic rays flux is of  $100 \text{ min}^{-1}/100 \text{ cm}^2$ . Calculate the total number of cosmic rays passing through a person per year and the total dose released. Model the person as a cube of 40 cm on a side. Consider the cosmic rays as impinging on the upper face at 90 degs and made of minimum ionizing muons only.

Compare the absorbed dose to the mean yearly background dose of 0.2 rad (100 rad = 1 Gy = 1 joule/kg).

4. Evaluate the mean energy loss of 20 MeV protons in 0.05 mm of lead ( $\rho = 10.35 \text{ g/cm}^3$ ). Use the tables of the protons stopping power.

5. Evaluate the mean energy loss of 100 MeV and 20 MeV deuterons in 1 cm water and in 0.05 mm of Pb. Use the tables of the protons stopping power.

6. Evaluate the energy loss of 5.5 MeV alpha particles in  $3 \text{ mg/cm}^2$  of Al and in  $3.5 \text{ mg/cm}^2$  of Au.

$dE/dx$  and range tables for protons in:

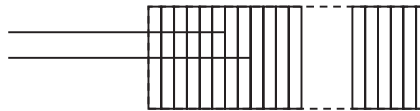
<http://physics.nist.gov/PhysRefData/Star/Text/PSTAR.html>

## 2. Problems about: $dE/dx$ and range (continued)

7. In order to discriminate between low energy ( $E \sim < 300$  MeV) pions and protons it is planned to use a range measurement (the energy loss is not recorded).

If the detector is made of 20, 1 cm thick slabs of plastic scintillator ( $\rho = 1 \text{ g/cm}^3$ ) each individually read out with a 'true'/'false' response, evaluate if and in which range of energies the discrimination is possible, once the energy is known. Do the same for Kaons and protons.

Assume that a difference of at least 2 slabs is needed to state a range difference between 2 particles.



Use the range curves for scintillators to make an approximate evaluation.

8. A vertex detector tracking system, made of 4 layers of  $300 \mu\text{m}$  thick silicon detectors, is used to trace the particles emitted after a heavy ion collision. Approximately  $10^4$  hits are recorded on the first layer due to the passage of charged particles. Assuming that the particles are 500 MeV pions evaluate how many hits will be recorded by the second layer.

Hint: assume that an energy of at least 60 KeV is needed on average to a  $\delta$ -ray to reach the second layer and that all the  $\delta$ -rays produced are emitted in the forward direction. Evaluate how many  $\delta$ -rays are produced in  $300 \mu\text{m}$  of silicon with such a minimum energy.

9. A tumor of 8mm diameter is centered at a depth of 2 cm inside the eye. What energy has to be used to stop protons at the centre of the tumor and how much energy is released in the healthy tissue? Assume the eye is made of water.

10. A pion emerges normally from a 4-mm-thick Lucite slab ( $1.19 \text{ g cm}^{-3}$ ) with an energy of 15MeV. What was its energy when it entered the slab?

### Energy straggling:

10. Determine the shape of the energy distribution you expect to get from a) a  $300 \mu\text{m}$  thick silicon detector and 2) a 3 mm thick plastic scintillator at the passage of 200MeV/c and 1000 MeV/c protons (calculate Landau's parameter  $\kappa = \Delta E / E_{\text{max}}$ )

11. A 1 mm thick scintillator is used to detect pions in an energy range between 20 MeV and 300 MeV. Determine the shape of the energy loss distribution (in terms of  $\kappa = \Delta E / E_{\text{max}}$ ) and its spread (FWHM) at 20,50 and 300 MeV)

### 3. Problems about: multiple coulomb scattering

#### Coulomb Multiple scattering

Consider the multiple coul. scattering of a 1 GeV proton in a 300 $\mu$ m silicon detector. Evaluate the RMS angle due to coulomb multiple scattering. What will be the RMS distance between the real and ideal (i.e. with no coul. M. scatt.) points at a distance of 1 cm from the detector? If you position a detector in that position, what kind of position resolution would you ask for it?

Use the practical formula  $\Theta_{\text{RMS}} = (21.2 \text{ MeV}/(pv)t)^{1/2} z$

where  $t$  is the path (absorber thickness) in units of radiation lengths,  $p$  is the momentum,  $v$  the velocity of the particle,  $z$  the particle charge.

In a heavy-ions collision several  $\Lambda$ -hyperons are produced, which are identified from their decay into  $p\pi^-$  pairs. In the hypothesis of  $p_p, p_{\pi^-} \sim 200 \text{ MeV}/c$  (very low energy  $\Lambda$ ) evaluate the resolution (=pitch/sqrt(12)) you require to 4 silicon microstrip detectors, 300 $\mu$ m thick and positioned at 1 cm from each other, which are used to trace the charged particles.

As a criterion assume that the spatial resolution of the silicon detectors must be of the same order of the minimal ( $p$  or  $\pi^-$ ) spread due to multiple scattering.

#### $\gamma$ -rays interactions.

-Calculate the minimum energy of a photon of 0.1, 1, 10 and 100 MeV after Compton scattering.

-A beam of 2 MeV  $\gamma$ 's of fluence  $\Phi = 10^6 \text{ cm}^{-2}\text{s}^{-1}$  impinges upon a 5 cm thick lead foil used as a shield. Evaluate the number of  $\gamma$ 's exiting the shield per second. (for the absorption coefficients see the link below)

The photoelectric cross section for  $E_\gamma = 0.6 \text{ MeV}$   $\gamma$ -rays is  $\sim 18 \text{ barn}$ .

Evaluate the approximate value of the same cross section in Uranium at the same energy ( $Z(\text{Pb})=82, Z(\text{U})=92$ ).

#### Cherenkov emission.

-Evaluate the Cher. Emission angle for 2 GeV pions, kaons and protons in water.

-Evaluate the Cher. Emission threshold for pions, kaons and protons in 4He gas.

-Evaluate the minimum length of a Cherenkov detector used to detect the passage of pions in the range 150-200 MeV if the minimum detectable threshold is 200 photons.

Absorption coefficients tables can be found at

<http://physics.nist.gov/PhysRefData/XrayMassCoef/tab3.html>