Exercise 1

Let θ_1 and θ_2 be real valued parameters of the model

$$y = \frac{\theta_1 x}{\theta_2 + x}.$$

a. Choose two suitable prior distributions for θ_1 and θ_2 and use HMC algorithm to find their posterior distributions, conditioning on the observations

$$x = (28, 55, 110, 138, 225, 375)$$
$$y = (0.053, 0.060, 0.112, 0.105, 0.099, 0.122).$$

b. Discuss how different parameters for both priors and the HMC algorithm lead to different estimates.

c. Plot the most reliable posterior distributions, according to convergence checks on the traces.

Exercise 2

A bivariate Gibbs sampler for a vector $x = (x_1, x_2)$ draws iteratively from the posterior conditional distributions in the following way:

- choose a starting value $p(x_1 | y, x_2^{(0)})$
- for each iteration i:
  - draw $x_2(i)$ from $p(x_2 | y, x_1^{(i-1)})$
  - draw $x_1(i)$ from $p(x_1 | y, x_2^{(i)})$

a. Supposing that samples are drawn from a bivariate normal distribution

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right],$$

implement a Gibbs sampler for x which takes as inputs the number of iterations `iters` and the number of warmup draws `warmup` .

b. Use your implementation of Gibbs sampler to infer the parameters $\theta = (\theta_1, \theta_2)$ from **Exercise 1**.