Carl-Gustaf Arvid Rossby

Born: December 28, 1898
Stockholm, Sweden

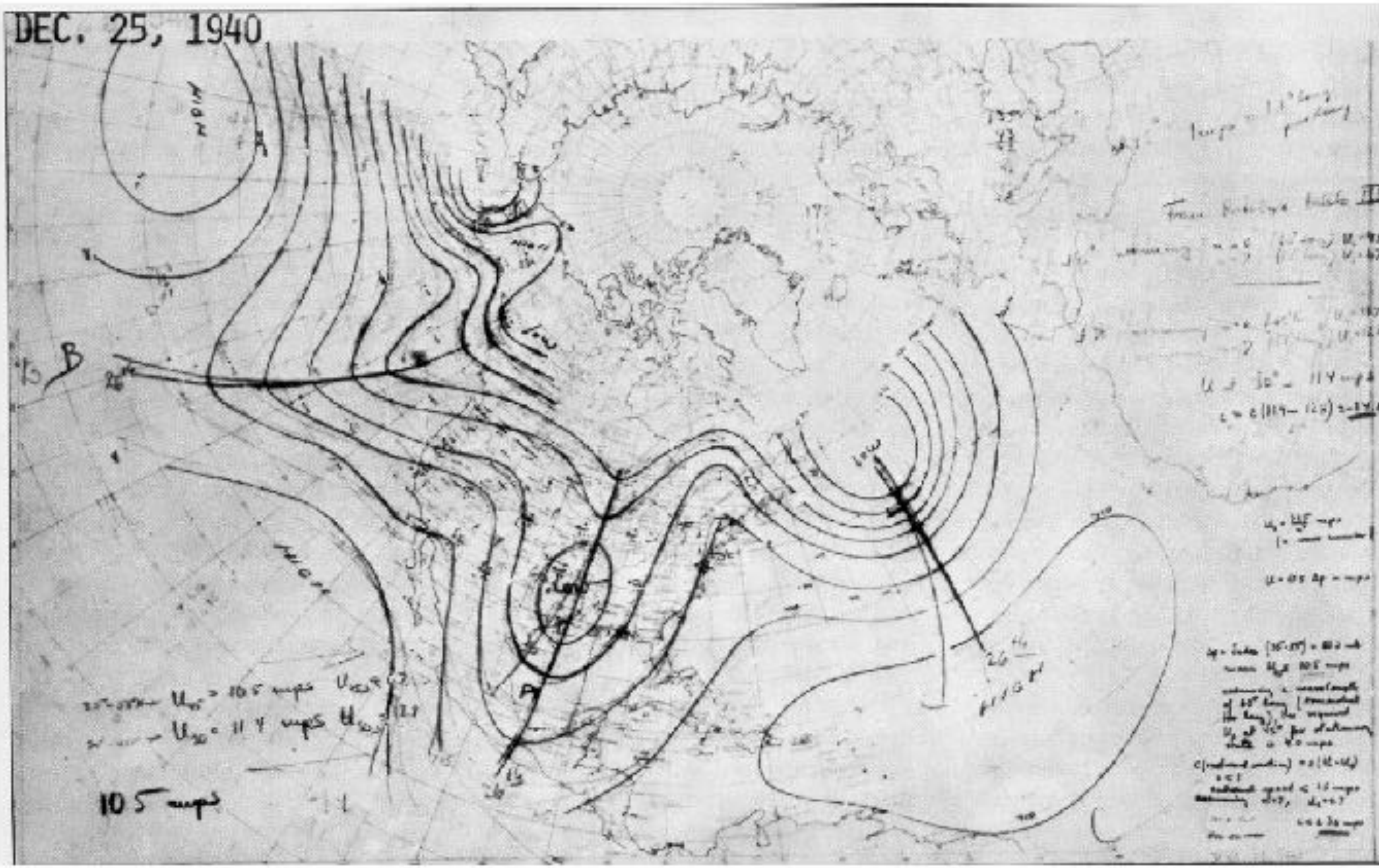
Massachusetts Institute of Technology

University of Chicago

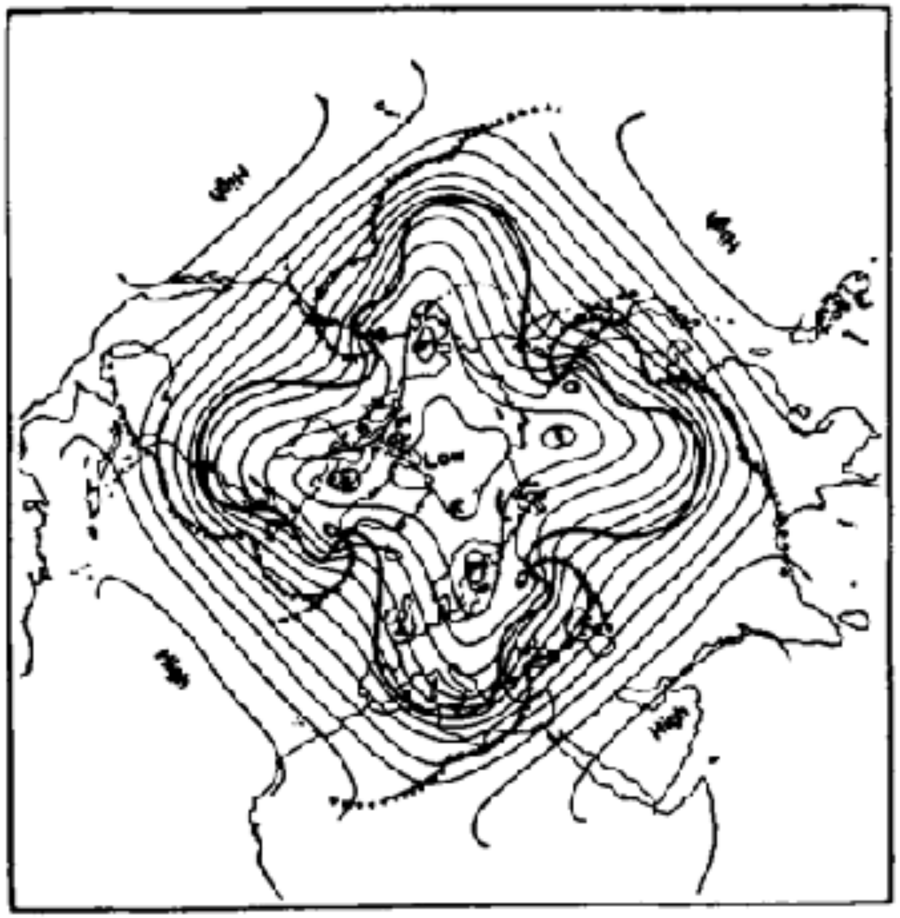
Woods Hole Oceanographic Institution

Swedish Meteorological and

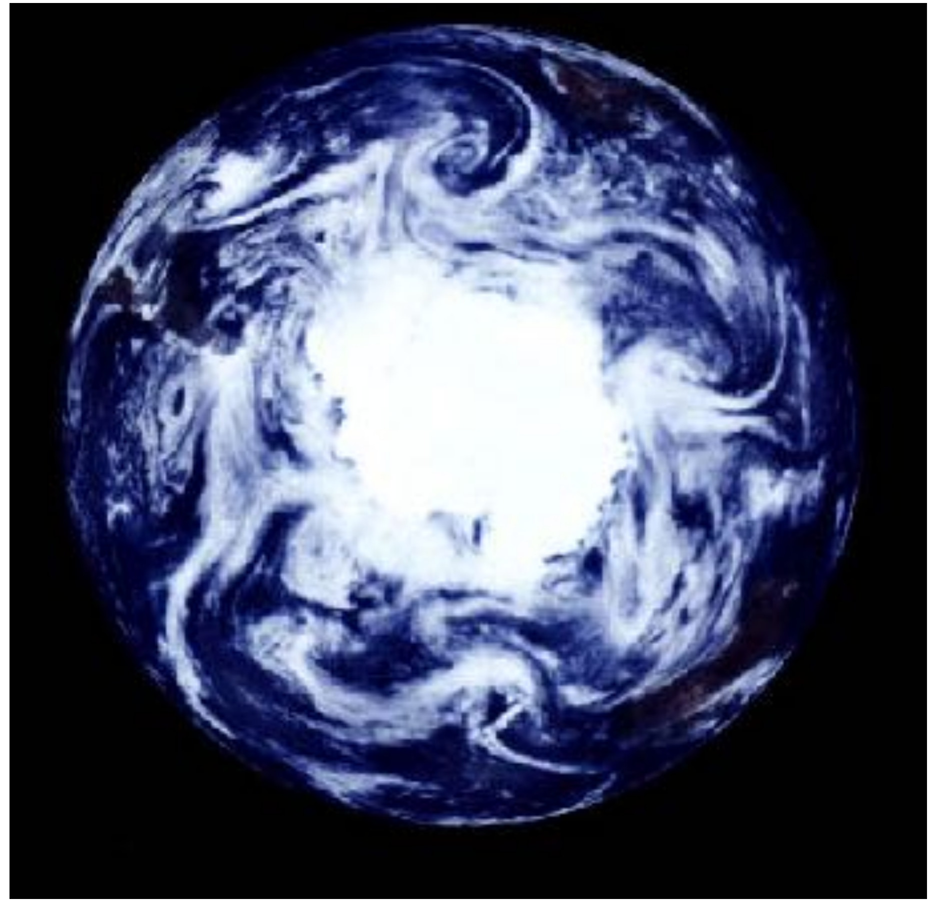
Rossby Waves – as seen by Rossby



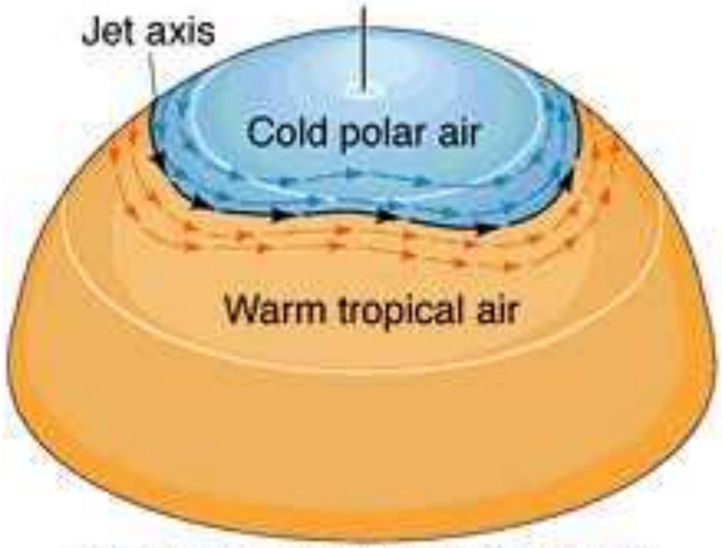
Platzman 1968



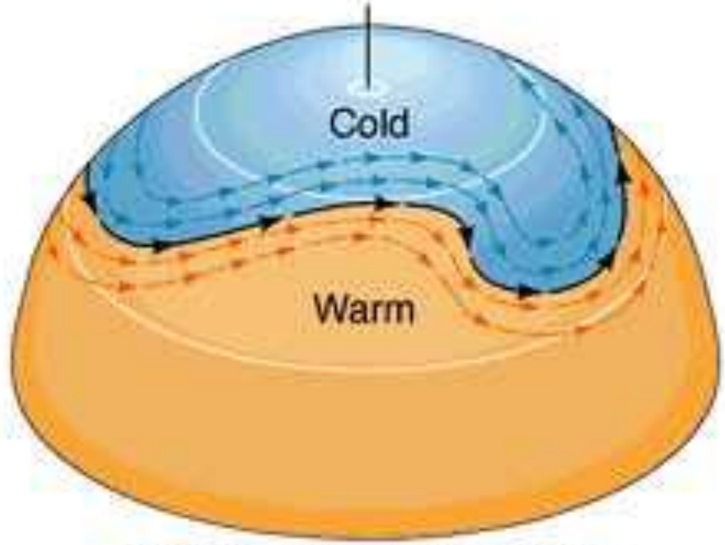
Palmen 1949



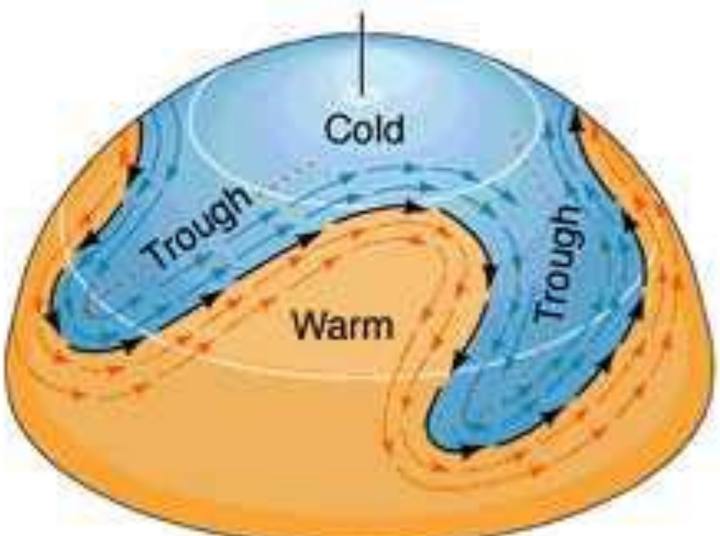
The picture is taken from above the South Pole, shows a number of mid latitude cyclones circling Antarctica.



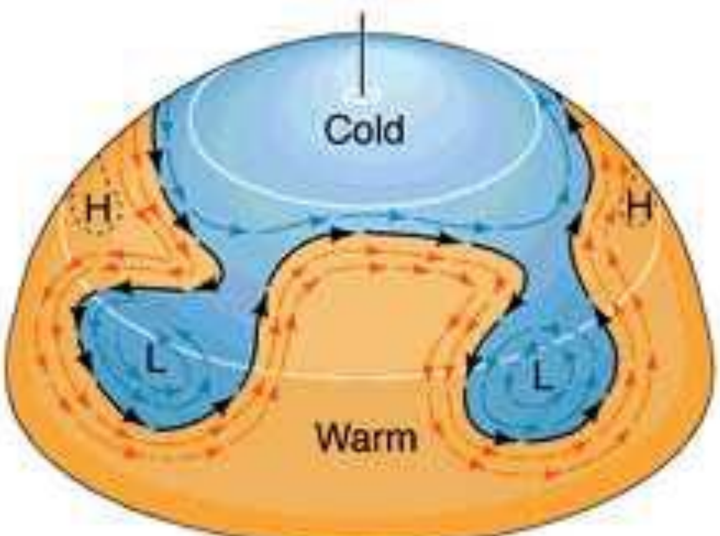
The jet stream begins to undulate.



Rossby waves begin to form.



Waves are strongly developed. The cold air occupies troughs of low pressure.



When the waves are pinched off, they form cyclones of cold air.

Rossby waves (hydrodynamics)

Momentum equation in rotating frame (with angular velocity $\mathbf{\Omega}$)

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p - 2\mathbf{\Omega} \times \mathbf{u} + \mathbf{g},$$

where $-2\rho\mathbf{\Omega} \times \mathbf{u}$ is the Coriolis force.

Ratio of convective and Coriolis terms is called a Rossby number

$$\mathrm{Ro} = \frac{U}{L\Omega}$$

When $\mathrm{Ro} << 1$ then the rotation effects are significant.

Rectangular coordinates:

$$\frac{\partial u_x}{\partial t} - f u_y = -g \frac{\partial h}{\partial x},$$

$$\frac{\partial u_y}{\partial t} + f u_x = -g \frac{\partial h}{\partial y},$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\left(H u_x\right) + \frac{\partial}{\partial y}\left(H u_y\right) = 0.$$

$f = 2\Omega \sin\vartheta$ is the Coriolis parameter.

$\vartheta = 90^0 - \theta$ is the latitude.

These equations can be cast into one equation

$$\frac{\partial}{\partial t}\left[\frac{1}{c^2}\left(\frac{\partial^2}{\partial t^2}+f^2\right)-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]u_y-\frac{\partial f}{\partial y}\frac{\partial u_y}{\partial x}=0.$$

$c=\sqrt{gH}$ is the surface gravity speed.

If one neglects the surface elevation $h \approx 0$ or $\frac{h}{H} << 1$

$$\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u_y+\frac{\partial f}{\partial y}\frac{\partial u_y}{\partial x}=0.$$

This approximation eliminates surface gravity (or Poincare) waves and induces small change in Rossby wave dispersion relation.

At this point came up Rossby with his $\beta -$ plane approximation

When spatial scales of considered process is less than sphere radius then one can expand the Coriolis parameter at a given latitude as

$$f = f_0 + \beta y,$$

$$\beta = \frac{\partial f}{\partial y} = \frac{2\Omega}{R}\cos\vartheta = const.$$

$$\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)u_y + \beta\frac{\partial u_y}{\partial x} = 0.$$

Fourier analysis of the form $\exp(-i\tilde{\omega}t + ik_x x + ik_y y)$ leads to the dispersion relation of Rossby (or planetary) waves

$$\tilde{\omega} = -\frac{\beta k_x}{k_x^2 + k_y^2}.$$

Rossby waves always propagate in the opposite direction of rotation!

For purely toroidal propagation $\tilde{\omega} = -\frac{\beta}{k_x}.$

Phase speed $v_{ph} = \frac{\tilde{\omega}}{k_x} = -\frac{\beta}{k_x^2 + k_y^2}.$

Long wavelength waves propagate faster!

Group speed $\mathbf{v_g} = \left(\frac{\partial\tilde{\omega}}{\partial k_x}, \frac{\partial\tilde{\omega}}{\partial k_y}\right) = \left(-\beta\,\frac{k_y^2 - k_x^2}{\left(k_x^2 + k_y^2\right)^2}, \beta\,\frac{2k_x k_y}{\left(k_x^2 + k_y^2\right)^2}\right)$

If there is a constant zonal flow then the phase speed can be written as (Rossby 1939)

$$c = U - \frac{\beta L^2}{4\pi^2},$$

It appears that the waves become stationary when

$$c = U - \frac{\beta L_s^2}{4\pi^2} = 0, \quad L_s = 2\pi\sqrt{\frac{U}{\beta}}.$$

$$c = U\left(1 - \frac{L^2}{L_s^2}\right),$$

Long wavelength waves propagate westward and short wavelength waves propagate eastward!

$\beta = 1.6\ 10^{-11}\ m^{-1}\ s^{-1}$ at mid-latitudes.

For the wavelength of 10000 km, one gets the period of 5.6 days.

Phase speed - 20 m/s.

The observed Rossby wave period on the Earth is 4-6 days (Yanai and Maruyama 1966, Wallace 1973, Madden 1979).

The ratio of Rossby wave and Earth rotation periods is around 6!