Lecture 3 : Basic elements of game theory

- Game Theory is a tool to understand situations in which two or more decision-makers interact.
- Outcomes of economic decisions frequently depend on others' actions
  - effect of price policy depends on competitors
  - outcome of wage negotiations depends on choices of both sides
  - outcome of elections depends on others' votes
- Decision makers should thus take expectations of others' decisions into account
- Such situations are plausibly modeled as a "*game*", a model of interactions where the outcome depends on others' as well as one's own actions

Motivating Example:

Sherlock vs Moriarty

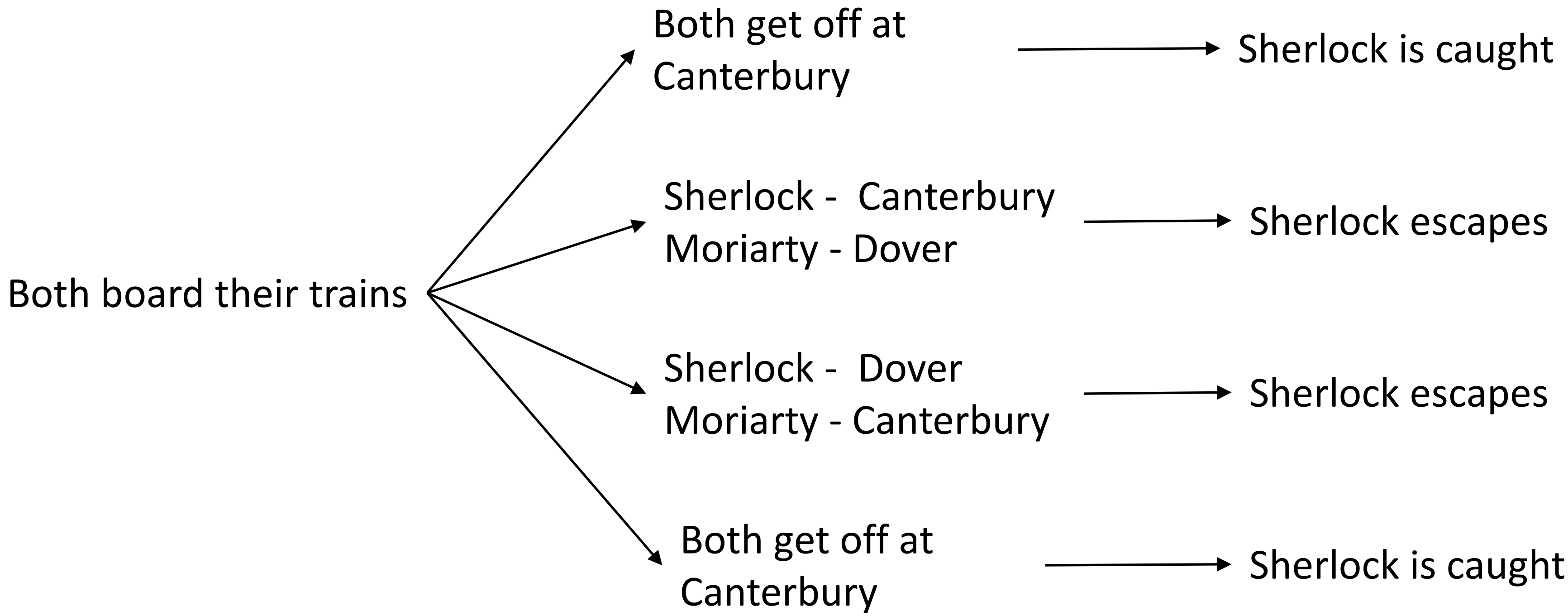
Sherlock Holmes boards a train from London to Dover to escape his arch enemy, Professor Moriarty. When the train leaves, Holmes catches sight of Moriarty on the platform, and Moriarty has spotted him.

Holmes knows that right after his train a faster train will leave which will reach Dover sooner. There is only a single stop before Dover - in Canterbury - for both trains.

If Holmes can make it to Dover without meeting Moriarty, he will be able to escape to the Continent.

But if he runs into Moriarty, either in Canterbury or in Dover, he will most likely be killed.

What should Holmes do?



The point of this example is not what they should do, but how important it is for each of them to hide the decision from the other.

If Moriarty can find out where Holmes will get off the train, he will be able to kill him.

If Holmes can figure out where Moriarty will wait, he will escape.

Definition of a game

A game is a formal representation in which a number of agents interact in a setting of strategic interdependence. By that, we mean that each individual's welfare depends not only on her own actions but also on the actions of the other individuals.

Moreover the actions that are best for her to take may depend on what she expect the other players to do

To describe such situations we need four things:

1) **The players**: who is involved?
2) **The rules**: Who moves when? What do they know when they move? What they can do?
3) **The outcomes**: for each possible set of actions by the players, what is the outcome of the game?
4) **The payoffs**: what are the players' preferences over the possible outcomes?

Example 1: Matching Pennies v. 1

1) **The players**: John and Paul
2) **The rules**: Each player simultaneously puts a penny down either Head Up or Tail Up
3) **The outcomes**: if the two pennies match John pays 1 euro to Paul, otherwise Paul pays 1 euro to john
4) **The payoffs**: Paul and John prefer receive 1 euro instead to pay

Note that the previous example of Sherlock vs Moriarty belong to this class of games where one player (John) likes to act differently from the other, while the other player (Paul) likes to choose the same action.

Then Sherlock is in the same position of John and Moriarty is in the same position of Paul

Example 2: Prisoner dilemma

Consider first two students who have handed in two good, but suspiciously similar, exams and are therefore suspected of cheating.

They are being separately interrogated by the Professor, without a chance to coordinate their statements. The professor offers each of them the following deal:

If one confesses and the other does not, the confessor will get a B, but the other will fail the exam.

If both confess, they will both pass, but with the worst possible grade, say, D. If both hold out, they will both pass, but with a C.

1) **The players**: Student A, Student B
2) **The rules**: Each player either confesses or holds out
3) **The outcomes**: if both confess, they get grade D, if both hold out, they get grade C. If one confesses and the other not, the first gets grade B, the other fails the exam
4) **The payoffs**: Students prefer to pass the exam with the highest possible grade

Example 3: meeting for dinner

Suppose you are on the station platform, ready to board the train, and you meet an old friend who has reserved a seat in a different car from yours. You agree to meet in the dinner. After you board the train a steward comes through making reservations, and you discover that there is a first-class dinner and a second-class buffet car. You would rather eat in first class, but you suspect that your friend prefers the buffet car. You would prefer to make a reservation that coincides with hers. Do you choose the buffet car or the first-class dinner?

1) **The players**: You, your friend
2) **The rules**: Each player either books the first-class dinner or the buffet car.
3) **The outcomes**: dinner together either in the first-class or in the buffet car. Separate dinner.
4) **The payoffs**: both prefer to dine together in the preferred place. They also prefer to dine in the less preferred place compared to have a separate dinner. In the case of separate dinner they prefer to dine in the preferred place.

Extensive Form Representation

An **extensive form** representation of a game specifies:

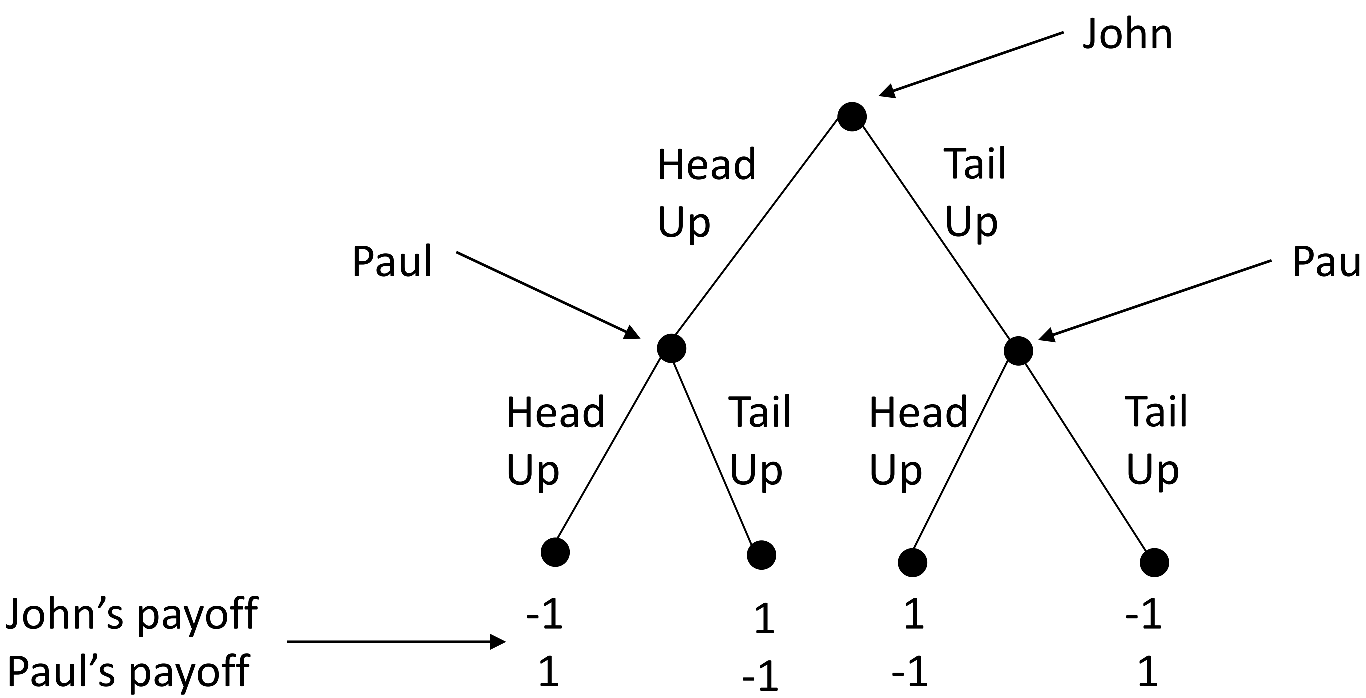
- Players
- When each player has to move
- The actions a player can use at each of his opportunities to move
- What a player knows at each of his opportunities to move
- Payoffs received by each player for each possible outcome

The extensive form relies on the conceptual apparatus known as a *game tree*

To illustrate the extensive form we start with the following example based on Matching Pennies game

Matching Pennies v. 2

1) **The players**: John and Paul
2) **The rules**: John puts a penny down either Head Up or Tail Up. Paul observes the decision of John and puts a penny down either Head Up or Tail Up.
3) **The outcomes**: if the two pennies match John pays 1 euro to Paul, otherwise Paul pays 1 euro to john
4) **The payoffs**: Paul and John prefer receive 1 euro instead to pay



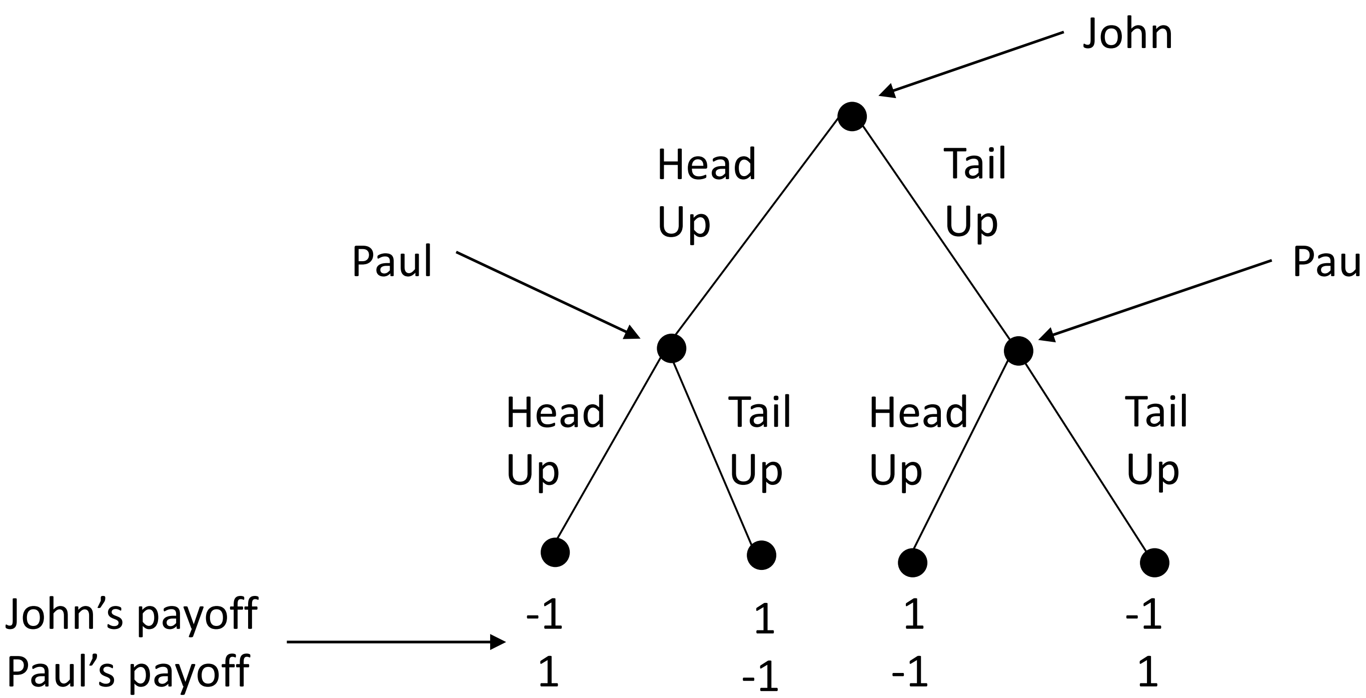
The game start at an ***initial decision node*** (*John's decision node*) where John decides his action (either Head Up or Tail Up)

Each of the two possible action is represented by a ***branch*** from the initial decision node

At the end of each branch there is another ***decision node***, at which Paul can choose between Head Up and Tail Up, after seeing John 1's choice.

These actions are represented by branches from *Paul's decision nodes*.

At the end there are the ***terminal decision nodes***, where we list the payoffs arising from the sequences of moves leading to that terminal node



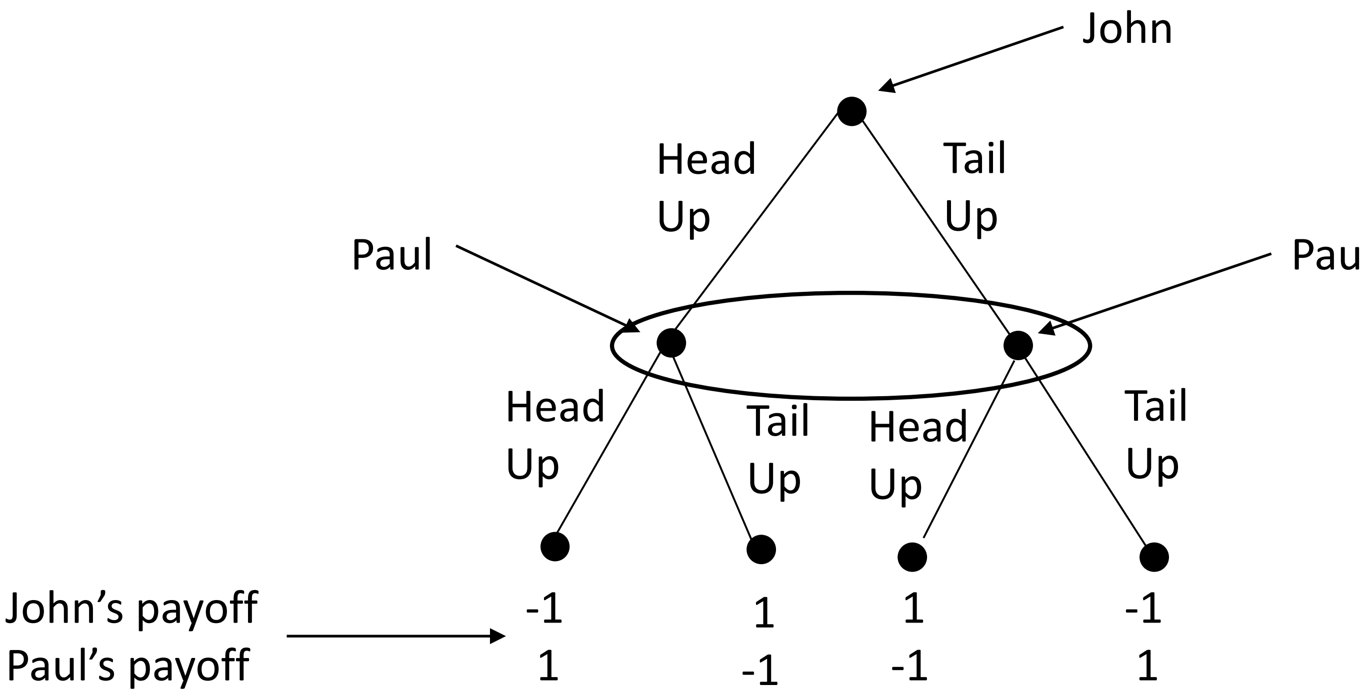
In the previous example when Paul has to move, she is able to observe John previous move. This is a game of ***perfect information***.

The concept of ***information set*** allows us to accommodate the possibility that Paul is not able to observe John's decision.

To introduce this concept we use the following example:

Example: Matching Pennies v. 3

1) **The players**: John and Paul
2) **The rules**: John puts a penny down either head up or tail up. After 1 minute the decision of John and without the possibility to know the decision of John, Paul puts a penny down either head up or tail up.
3) **The outcomes**: if the two pennies match John pays 1 euro to Paul, otherwise Paul pays 1 euro to john
4) **The payoffs**: Paul and John prefer receive 1 euro instead to pay



The circle around Paul's nodes indicates that these two nodes are in a single information set.

The meaning of this information set is that Paul does not know in which node he is because he has not observed the action played by John

Note that Paul has the same two possible actions at each of the two nodes in the information set

This is a game of ***imperfect information***

The use of information sets allows us to represent simultaneous games using the extensive form.

Consider Matching pennies v.1 where John and Paul simultaneously put a penny down either head up or tail up.

For them this game is strategically equivalent to Matching pennies v.3.

In Version 3, John is unable to observe the action of Paul because he moved first. Paul cannot observe the move of John (by the rule of the game).

In Version 1 they are not able to observe the other's decision because they move simultaneously.

As long as they cannot observe each other's choice, the timing of the moves is irrelevant.

Then we can use the extensive form of Version 3 to represent Version 1

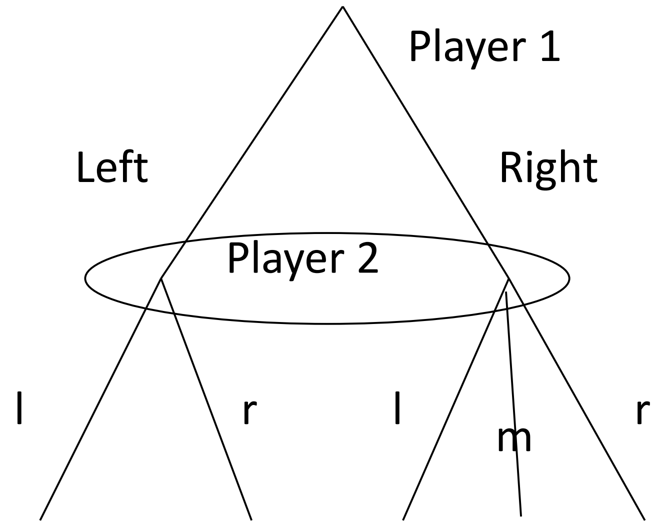
Information set

- It is a collection of decision nodes where:
  - The player has to move at every node in the information set
  - When a player has to move, he cannot distinguish the nodes belonging to the same information set
  - Matching Pennies v.2: John has one info set, Paul has two info sets
  - Matching Pennies v.1 and v.3 : John has one info set, Paul 2 has one info set

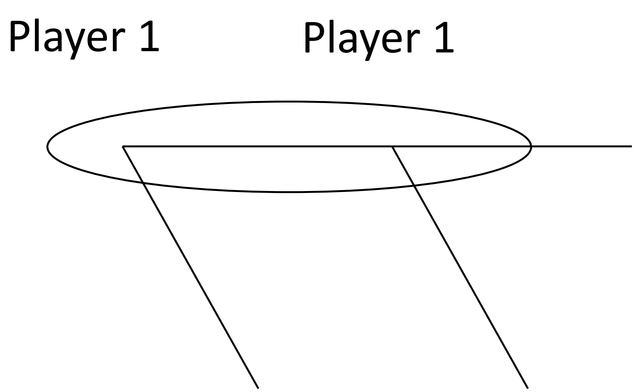
Two restrictions:

1) At every node within a given information set, a player must have the same set of possible actions
2) Players have *perfect recall*: a player does not forget what she once knew

Note: What can an info set NOT look like



The two nodes in the information set have different number of available actions, then player 2 can distinguish the node



This could be true only assuming that player 1 does not remember his move in the first node: *perfect recall* is violated

Dynamic games of perfect and imperfect information

- **perfect information,** i.e. when choosing an action a player knows the actions chosen by players moving before her

  i.e. all previous moves are observed before the next move is chosen

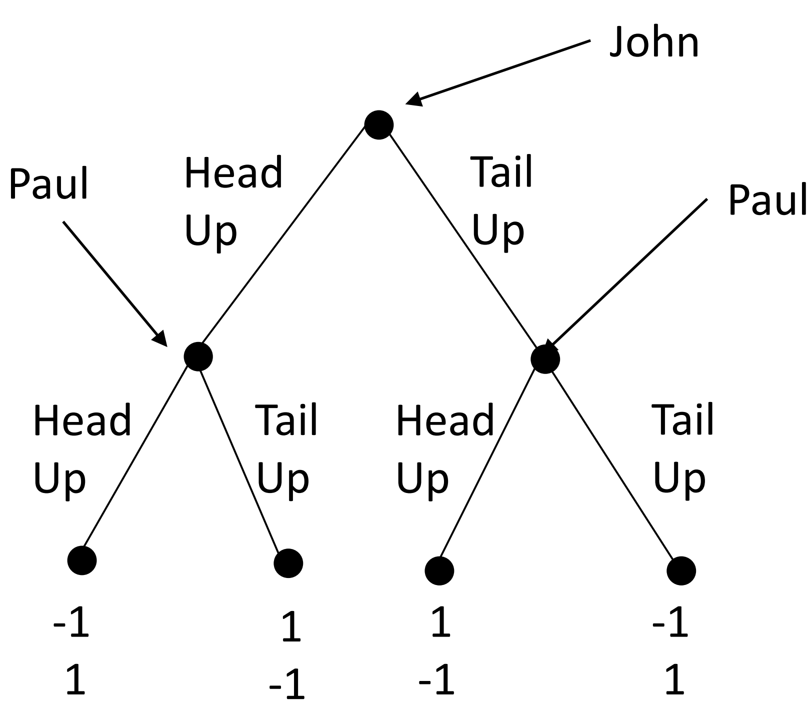
- **Imperfect information** when at least one player does not know the history by the time he chooses.

  At least one player does not know all the actions chosen by players moving before him

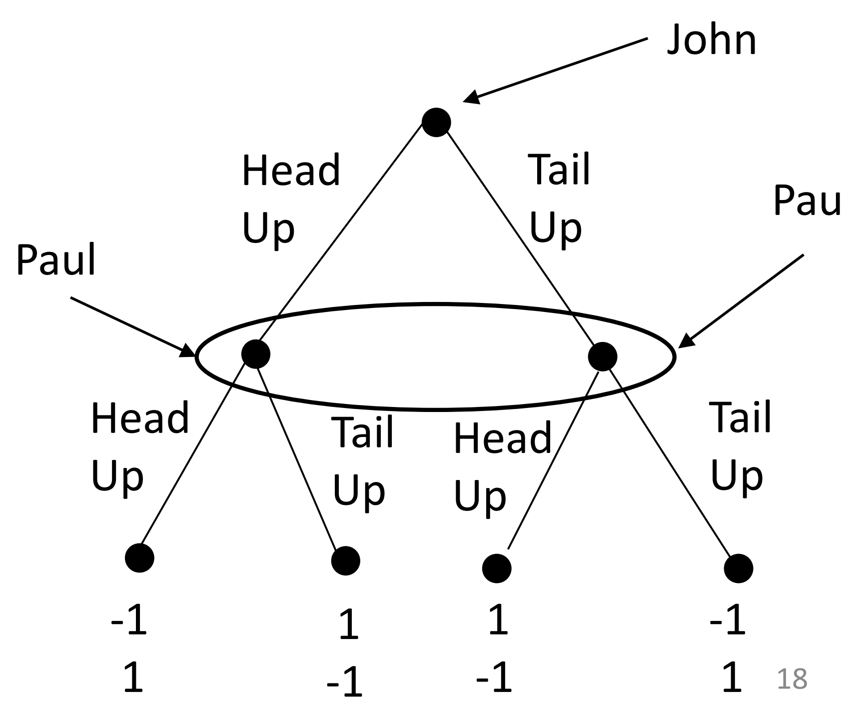
Definition:

a game is one of *perfect information* if each information set contains a single decision node. Otherwise, it is a game of *imperfect information*.

perfect information



imperfect information.



Summarizing:

- An extensive form game can be represented in a **game tree**
- This shows
  - who moves when (at the ***nodes***)
  - available actions (the ***branches***)
  - available information (***information sets***)
  - the payoffs over all possible outcomes (at the ***terminal nodes***)

Strategies

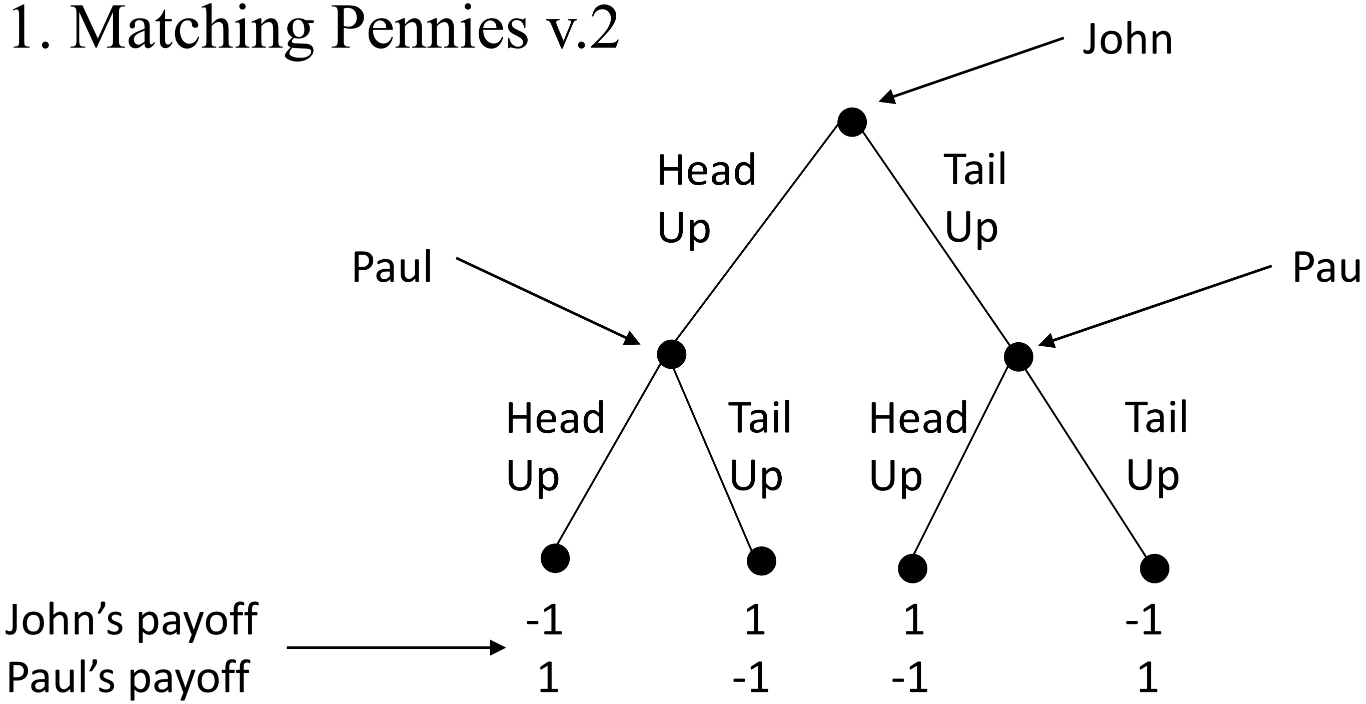
A central concept of game theory is the notion of *strategy*.

A *strategy* is a *complete contingent plan*, or *decision rule* that specifies how a player will act in every possible distinguishable circumstances in which she might be called upon to move.

In other words, a *strategy* is a complete description of a player's actions in all the information sets where he has to move.

Consider the following examples:

1. Matching Pennies v.2



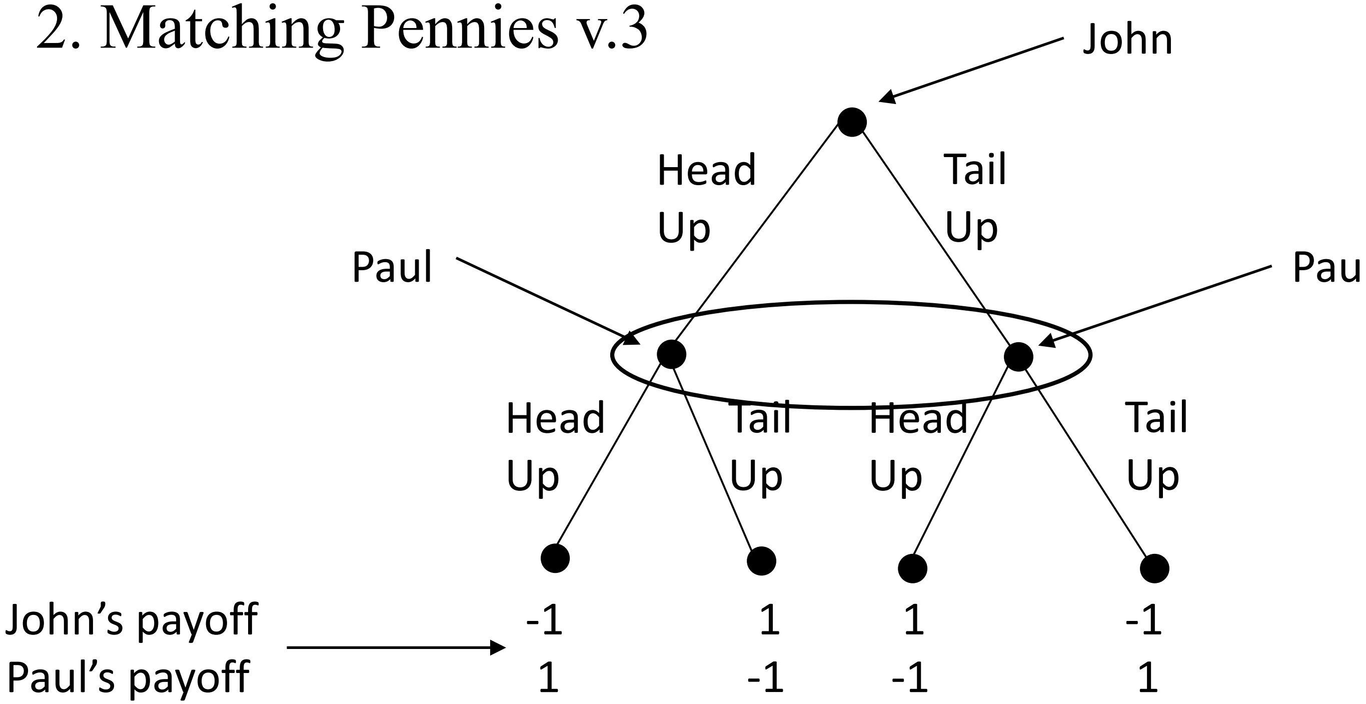
John has only one information set, then his strategy simply says the action he like to play.

Paul has two information sets, then his strategy has to list two actions: one in the case John displays Head Up and another in the case John displays Tail Up, for example Head Up after Tail Up and Tail Up after Head Up

How many possible strategies has John? And Paul?

Answer: John has 2 strategies, Paul has 4 strategies

2. Matching Pennies v.3

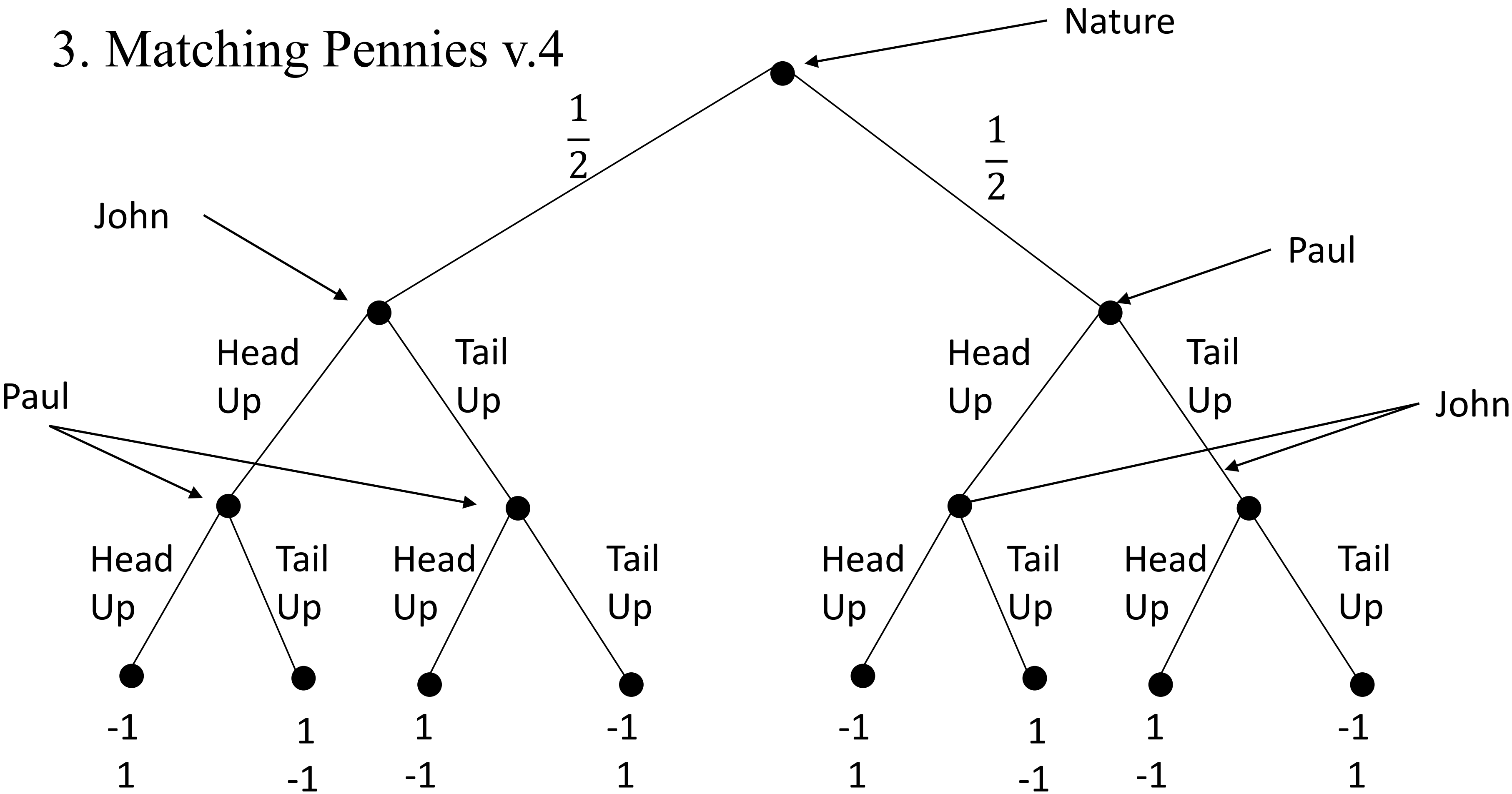


John and Paul have only one information set, then their strategy simply says the action they like to play in the unique information set.

How many possible strategies has John? and Paul?

Answer: 2 each

3. Matching Pennies v.4



John and Paul have 3 information sets, then their strategy has to list 3 actions, one for each of their three information sets. Paul's strategy(John's strategy) has to list:

1. The action in the case he moves first
2. The action in the case he moves second and John (Paul) displays Tail Up
3. The action in the case he moves second and John (Paul) displays Head Up

Definition of subgame:

A subgame starts at an information set with a single node ***n***

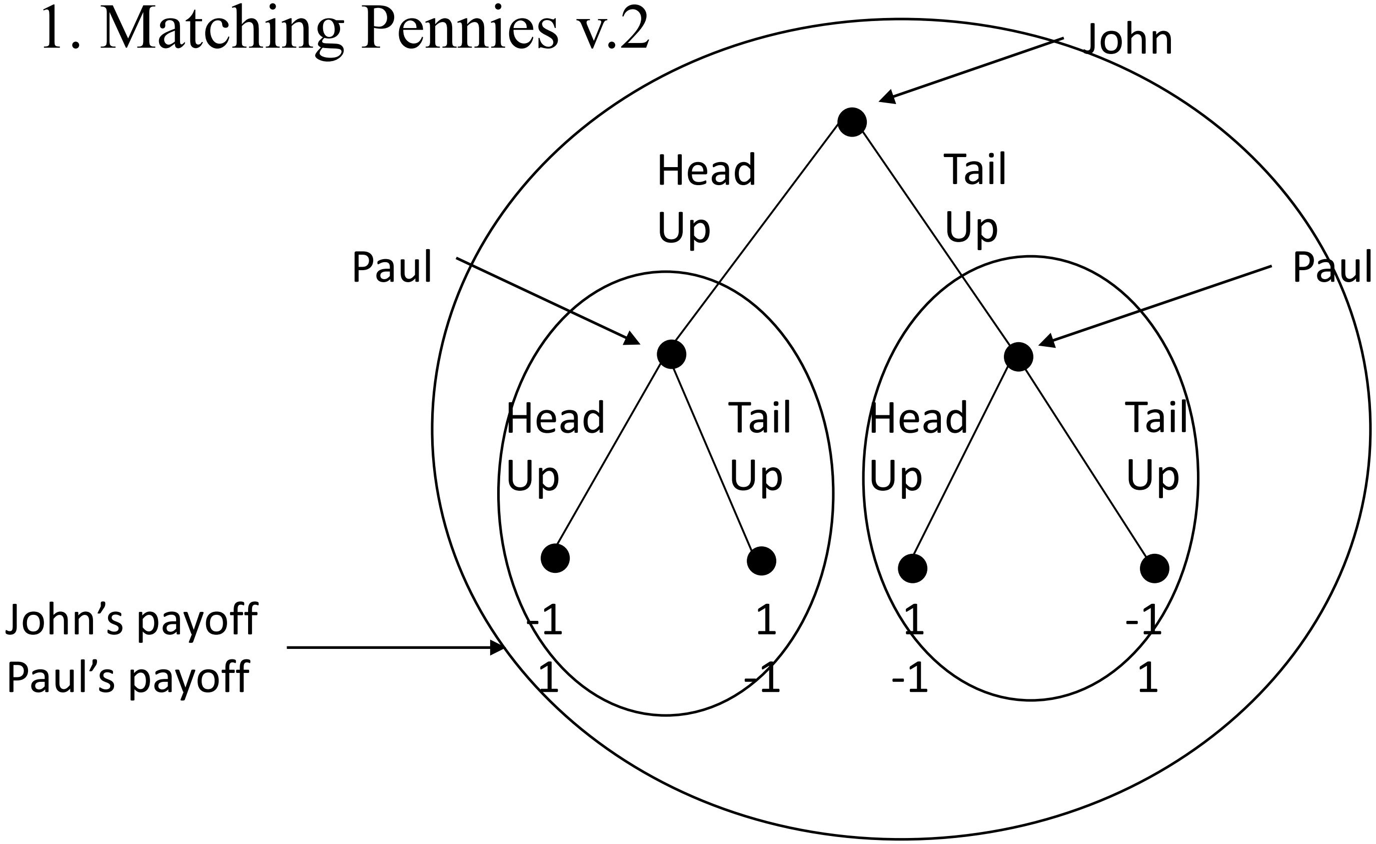
- it contains all decision and terminal nodes following ***n***
- an information set cannot belong to two different subgames

Note: someone considers the whole game a subgame, others do not consider the whole game a subgame.

In the following we use the first approach

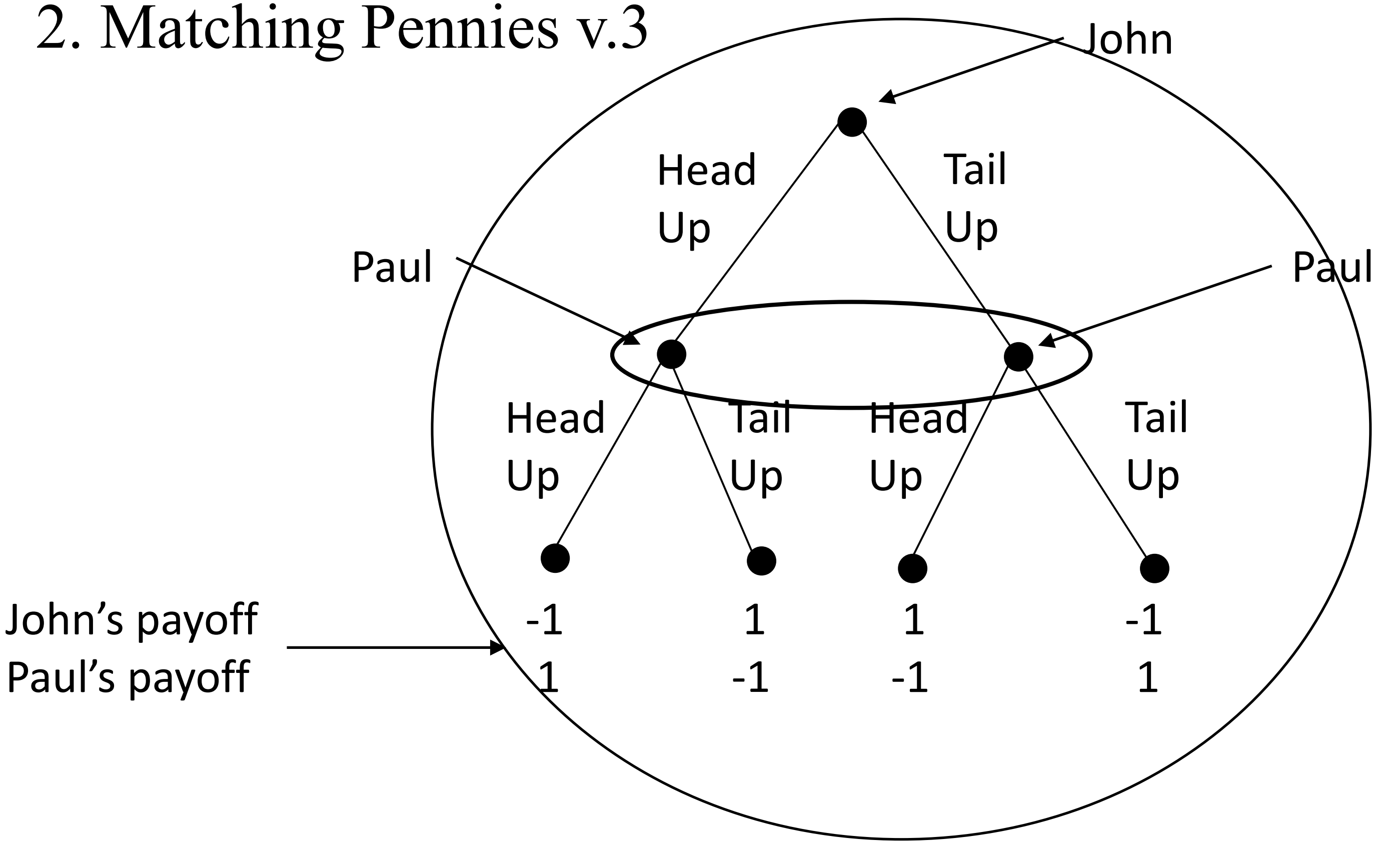
Look at the following examples:

1. Matching Pennies v.2



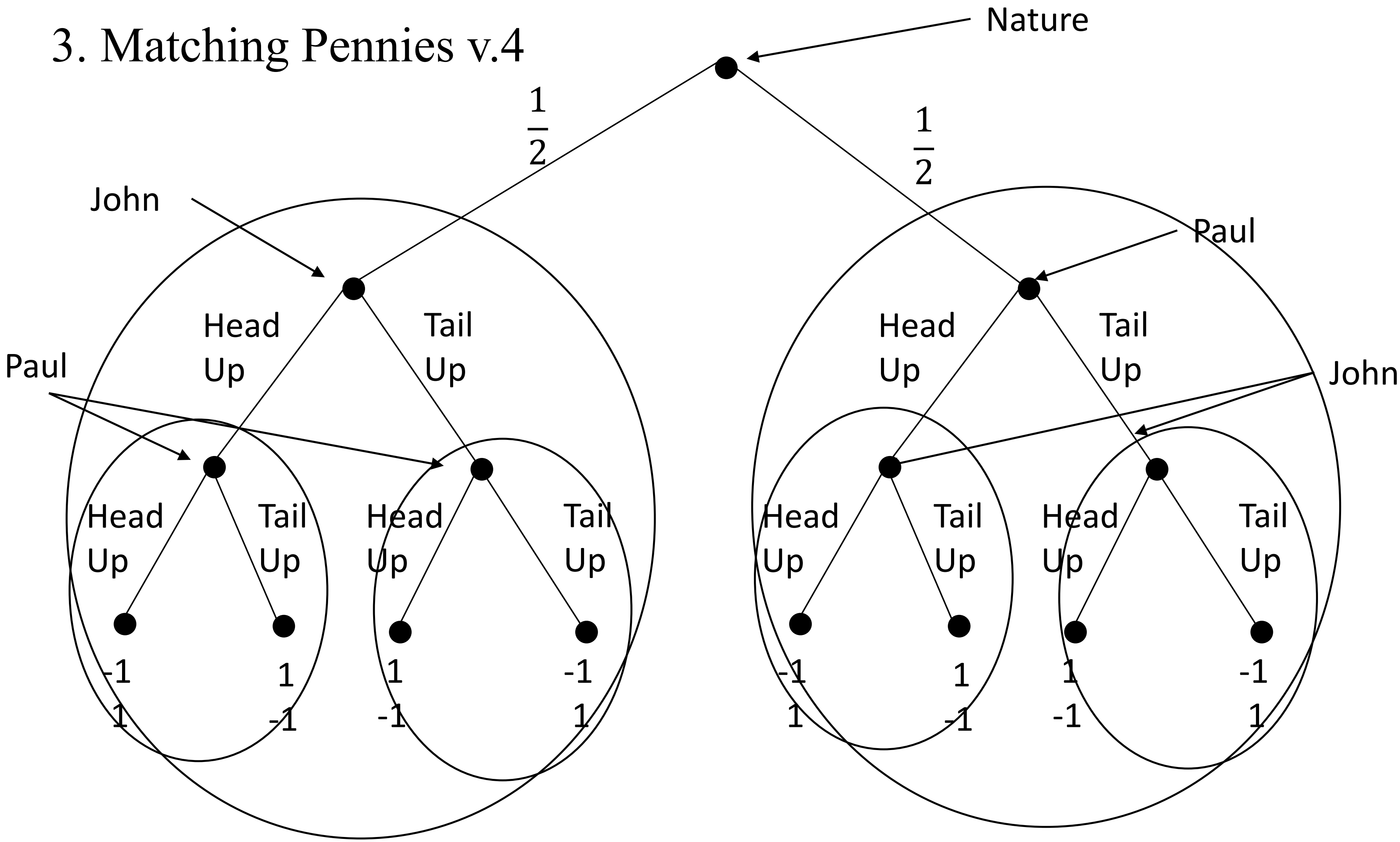
It has three subgames: the whole game and a subgame starting from each decision node of Paul

2. Matching Pennies v.3



It has an unique subgame: the whole game

3. Matching Pennies v.4

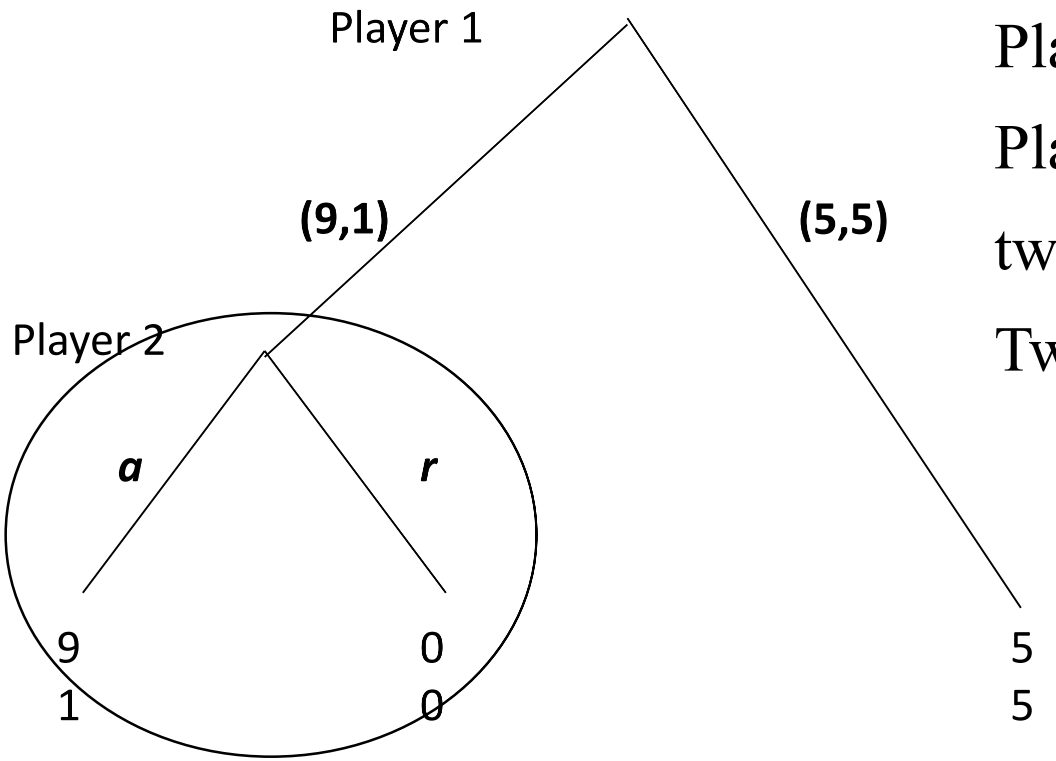


It has seven subgames: the whole game and a subgame starting from each decision node

Summarizing examples on information, information sets, subgames and strategies

Example 1: Mini Ultimatum Game

- Proposer (Player 1) can suggest one of two splits of £10: (5,5) and (9,1).
- Responder (Player 2) can decide whether to accept (*a*) or reject (*r*) (9,1), but has to accept (5,5). Reject leads to 0 for both



Perfect information

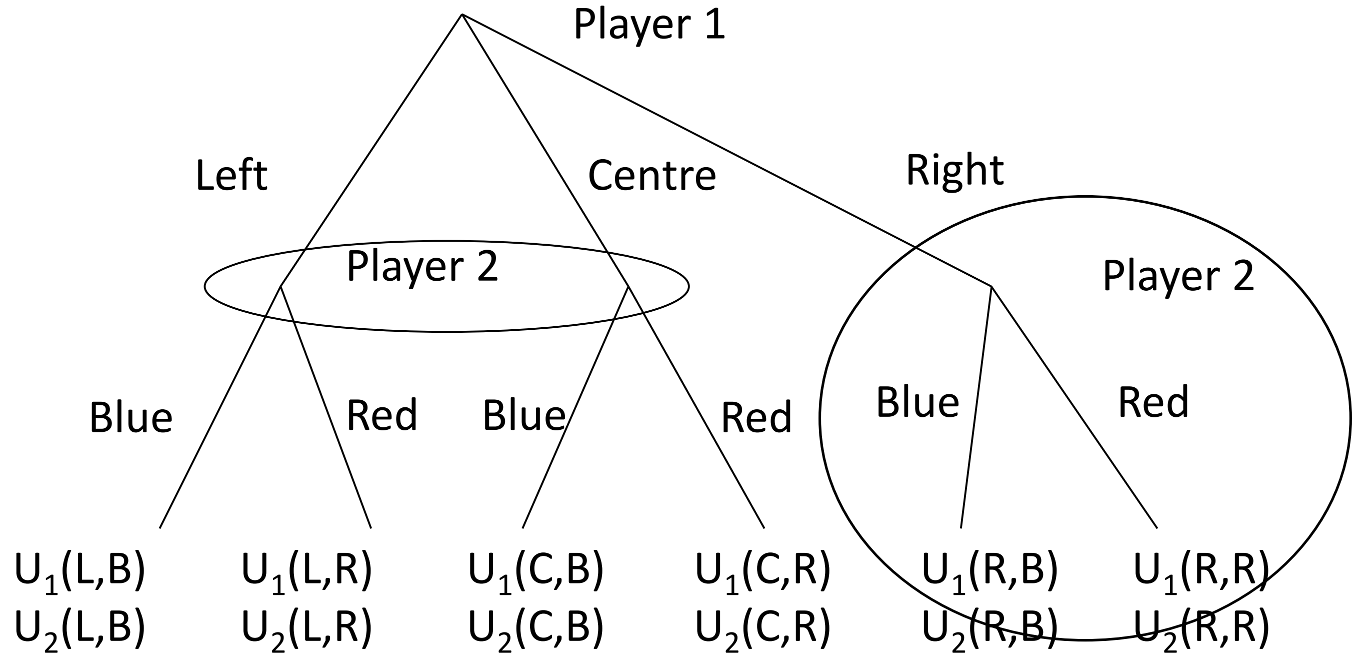
Player 1 has one information set

Player 2 has one information set

two subgames

Two strategies for each player

Example 2



Imperfect information

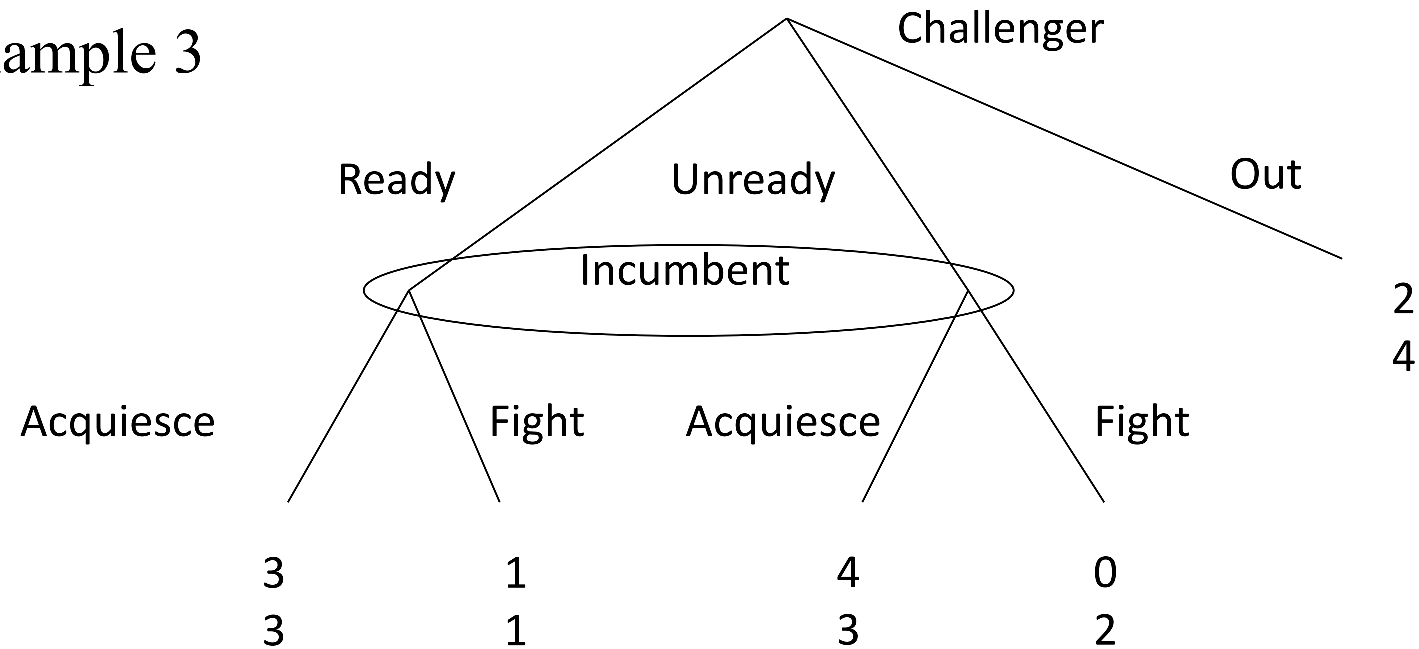
Player 1 has one information set, Player 2 has two information sets

Two subgames

Player 1 has three possible strategies: Left, Centre, Right

Player 2 has 4 possible strategies: (Blue, Blue), (Red, Red) (Blue, Red), (Red, Blue) [the left action refers to information set on the left]

Example 3



Imperfect information

Challenger: one information set

Incumbent: one information set

One subgame

Strategies: 3 for the Challenger and 2 for the incumbent

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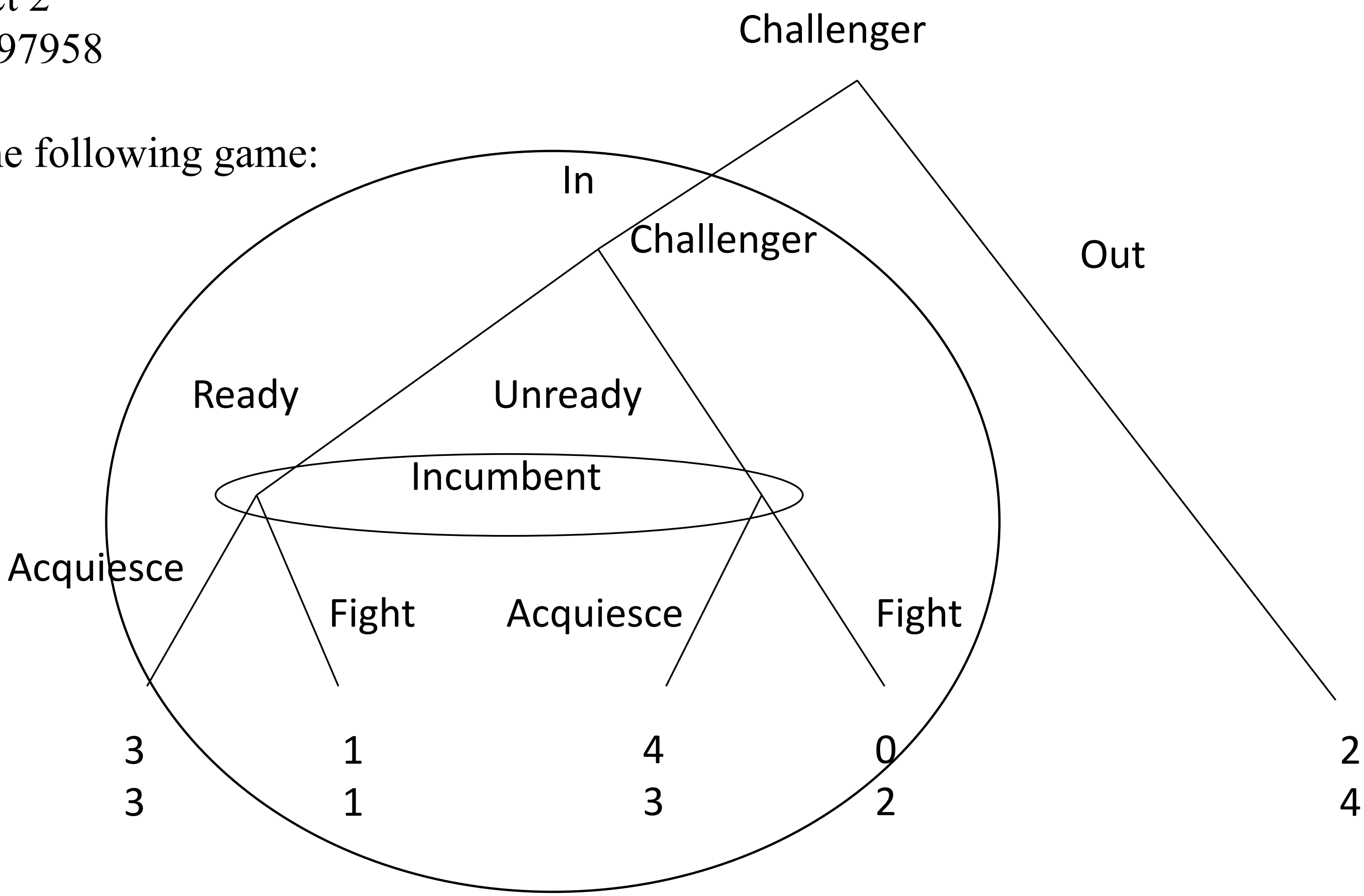
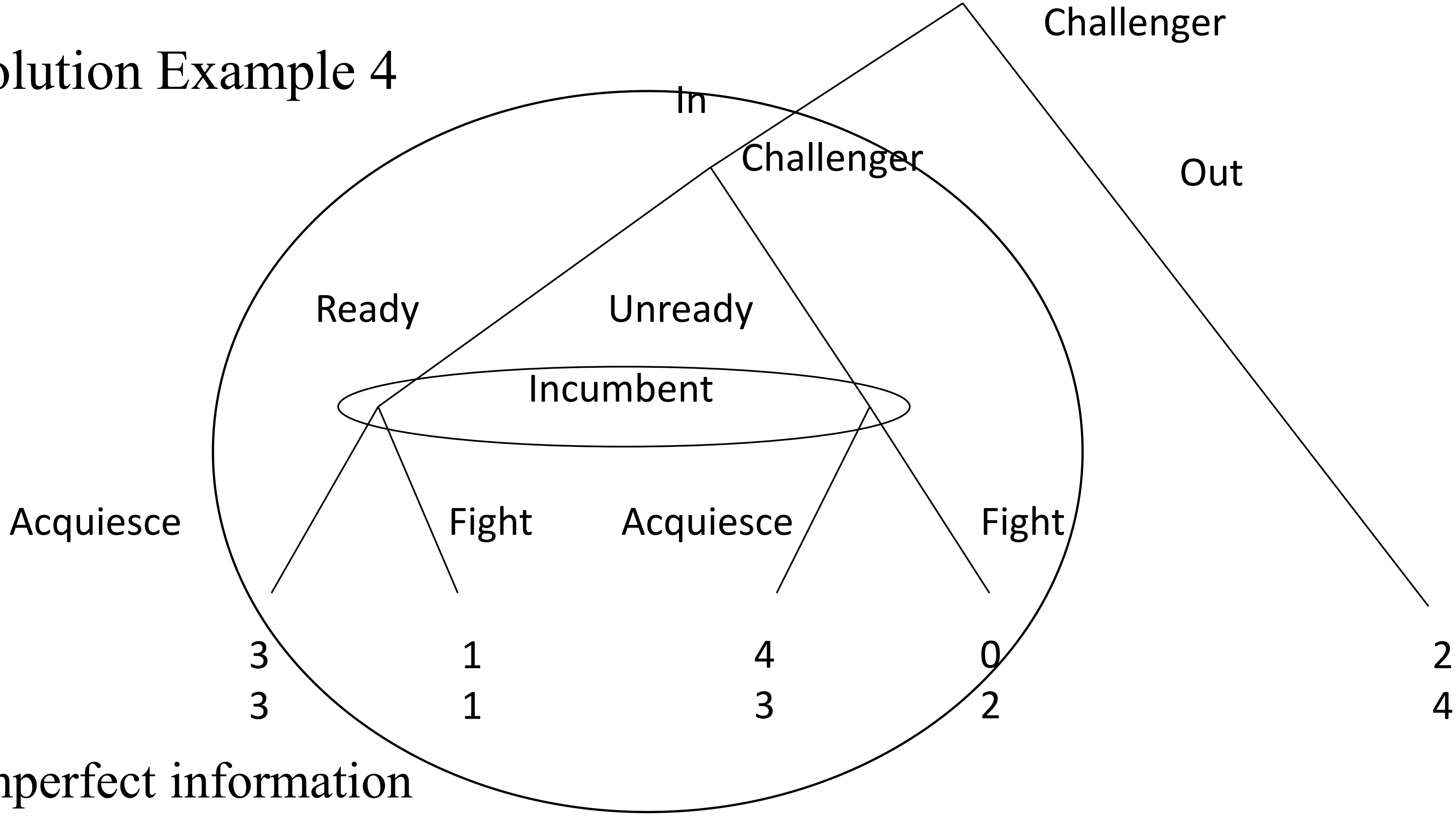
Question set 1
Use code 335528

Question set 2
Use code 497958

Consider the following game:



Solution Example 4



Imperfect information

Challenger: two information sets

Incumbent: one information set

Two subgames

Four strategies for the Challenger (In, Ready) (In, Unready) (Out, Ready) (Out, Unready)

Two strategies for the Incumbent (Acquiesce or Fight)

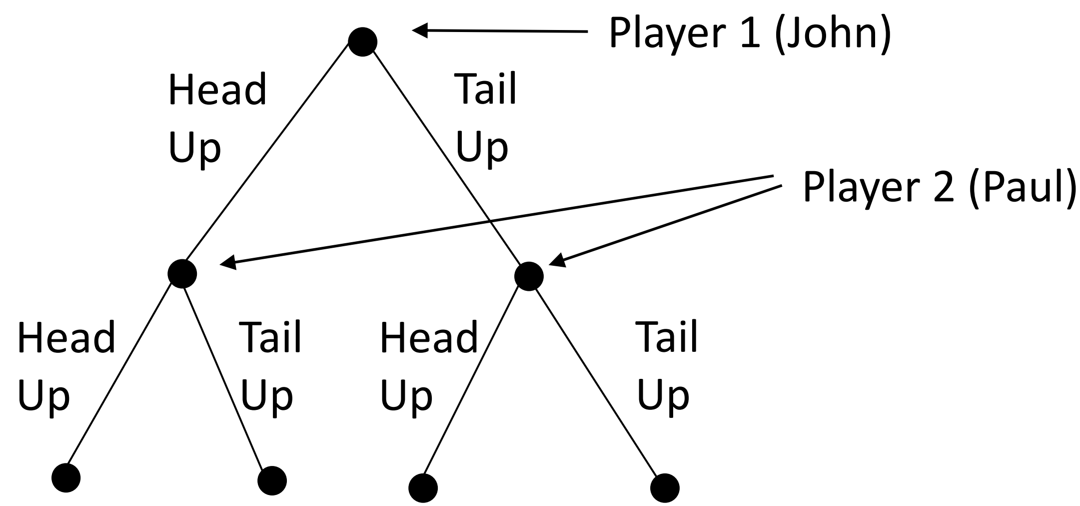
Normal form games

Recall that a pure strategy assigns a choice to each and every information set of a player:

- Given a pure strategy for each opponent, therefore, a player can figure out what she should do in all contingencies that may arise during the course of the game.
- If all players have done this, each of them has chosen a pure strategy and these pin down which outcomes may materialize (up to chance moves).
- Hence, after all players have decided on a pure strategy, they may well submit those to an umpire, who can then uniquely reconstruct what will happen in the game if it were actually played.

each agent i has a **set of available strategies** denoted by $\boldsymbol{S_i}$ where $\boldsymbol{s_i}$ is an element of $\boldsymbol{S_i}$, i.e. $s_i \in S_i$

For example consider matching pennies v.2



$S_1 = \{Head\ Up, Tail\ Up\}$

s_1 could be player 1's strategy "$Head\ Up$"

$S_2 = \{(Tail\ Up, Tail\ Up), (Head\ Up, Tail\ Up), (Tail\ Up, Head\ Up), (Head\ Up, Head\ Up)\}$

s_2 could be player 2's strategy "$(Head\ Up, Tail\ Up)$"

Then in a game,

1. $s = (s_1 \ldots s_i \ldots s_n)$ denotes a ***strategy profile***, i.e. one strategy for each player.
2. The ***set of all strategy profiles*** is denoted by S, that is: $s \in S =$

$$S_1 \times S_1 \times \ldots\ldots S_n$$

   This is the cartesian product of the n sets S_i.

In the previous example:

A strategy profile s could be:

Player 1 plays "$Head\ Up$", Player 2 plays $(Head\ Up, Tail\ Up)$. i.e. $s = (Head\ Up, (Head\ Up, Tail\ Up))$

The set of all strategy profile S is (one strategy profile per cell):

Head Up, (Head Up, Head Up)	Head Up, (Head Up, Tail Up)	Head Up, (Tail Up, Head Up)	Head Up, (Tail Up, Tail Up)
Tail Up, (Head Up, Head Up)	Tail Up, (Head Up, Tail Up)	Tail Up, (Tail Up, Head Up)	Tail Up, (Tail Up, Tail Up)

Definition

A normal form specifies:

1. the agents in the game,
2. for each agent, the set of available strategies S_i and the set of all strategy profiles S
3. The payoff received by each player for each strategy profile $s \in S$. For player i it is denoted by:

$$u_i(s)$$

   where s = $(s_1 \ldots s_i \ldots s_n)$ is a strategy profile resulting from the decisions of the players.

Representation of a game using the normal form

Case of two players: 1 and 2

Use a table and:

- Label the rows of the normal form with the player 1's strategies
- Label the columns of the normal form with the player 2's strategies
- In each cell report the payoffs of both players given by the combination of the row strategy with the column strategy

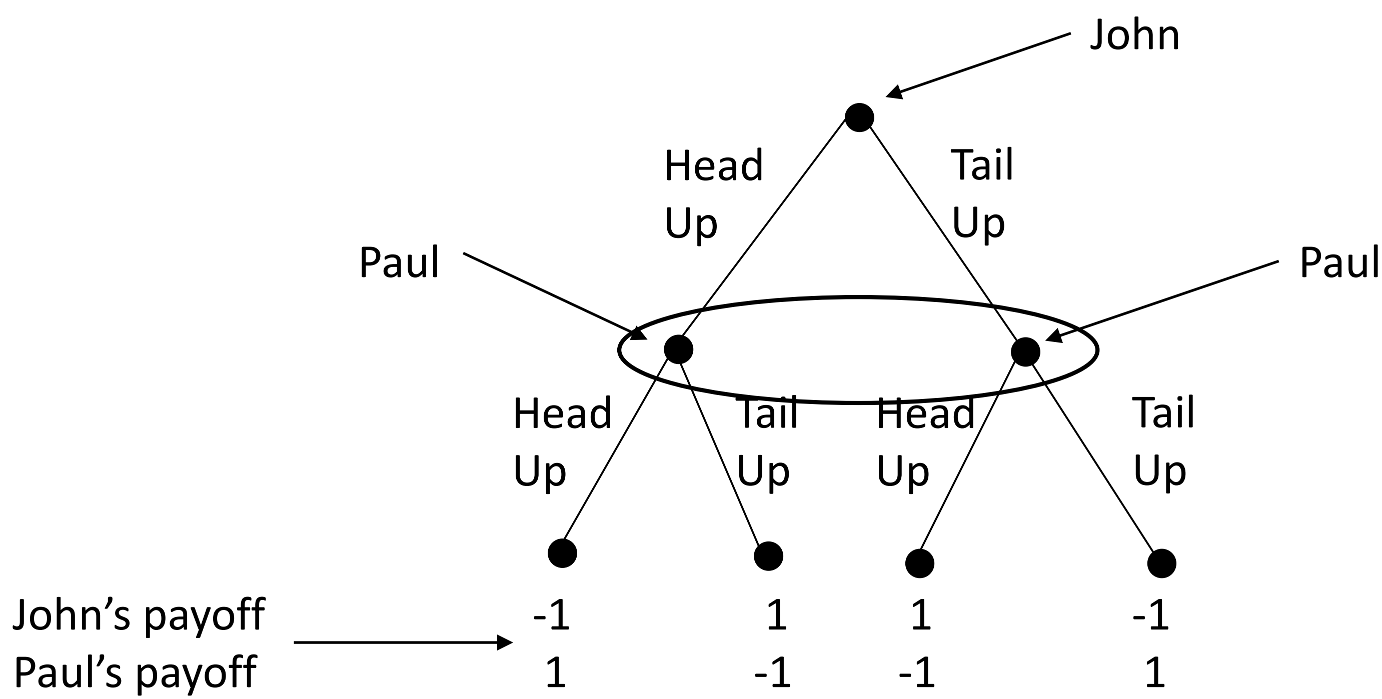
Look at the following examples:

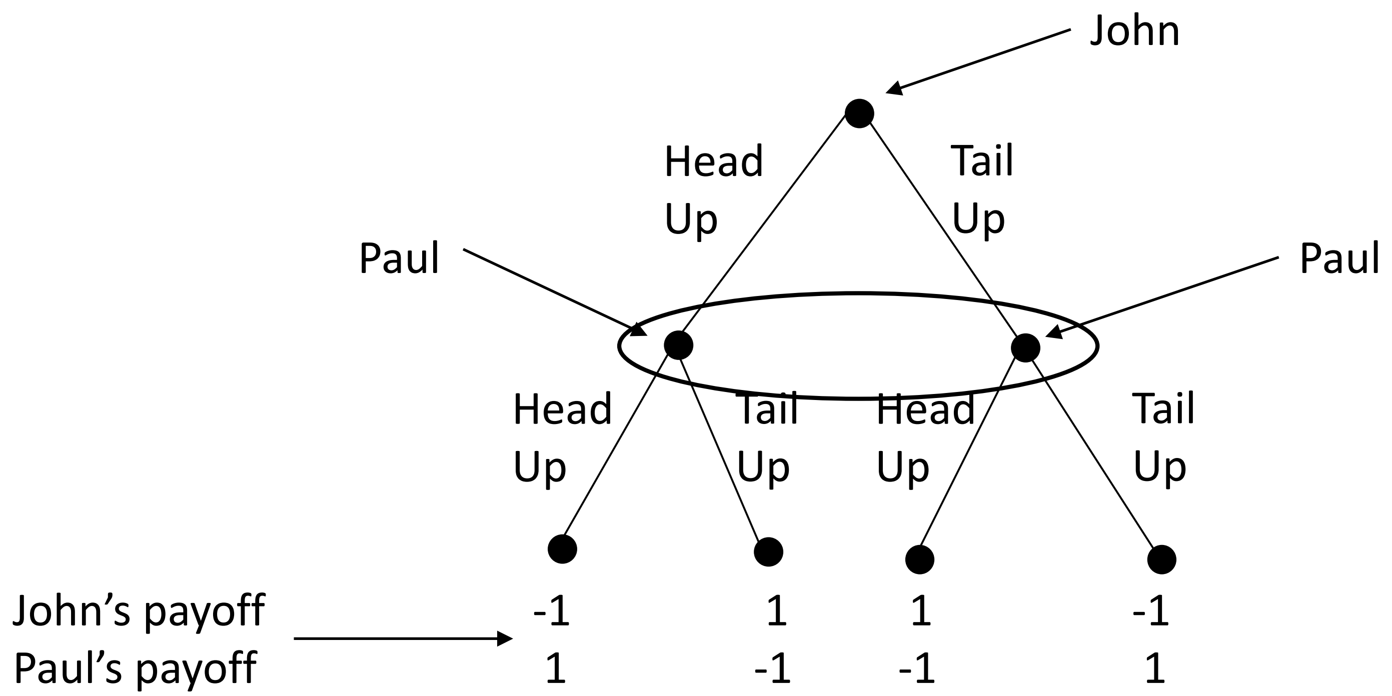
Example 1.

Consider again Matching Pennies v. 1 where John and Paul simultaneously puts a penny down either Head Up or Tail Up.

If the two pennies match John pays 1 euro to Paul, otherwise Paul pays 1 euro to John

The extensive form of this simultaneous game is:





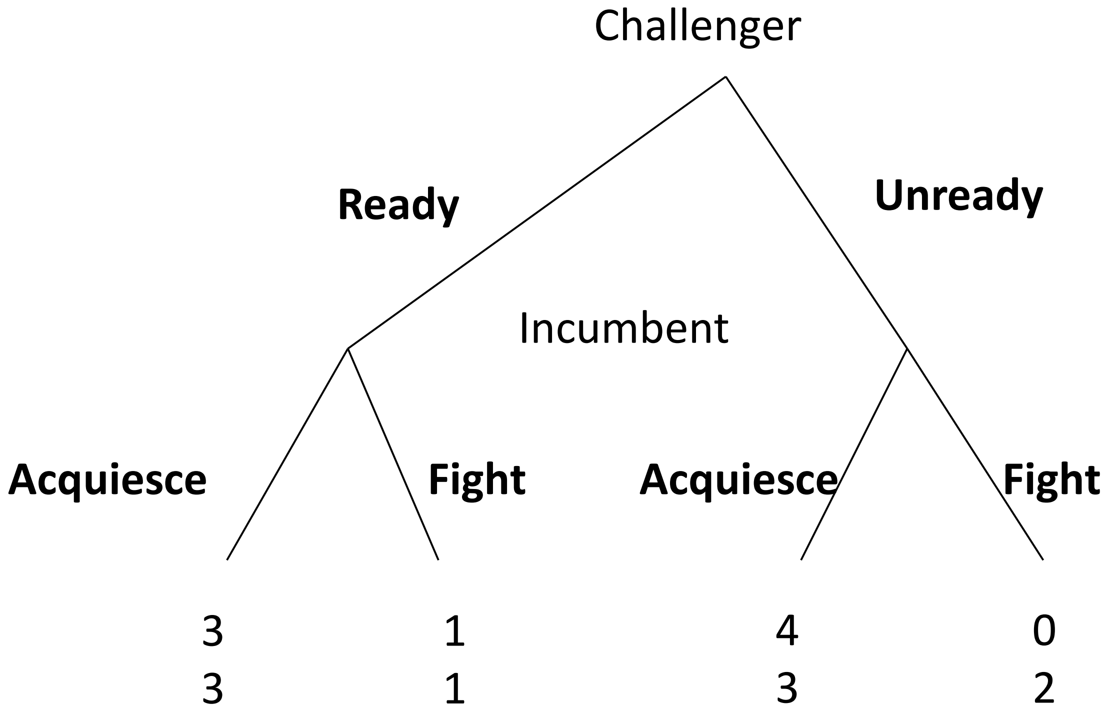
Set of Paul' strategies is $S_P = \{\text{Head Up, Tail Up}\}$

Set of John' strategies is $S_J = \{\text{Head Up, Tail Up}\}$

The normal form is:

		Paul	
		Head Up	Tail Up
John	Head Up	-1, 1	1, -1
	Tail Up	1, -1	-1, 1

Example 2



Acquiesce is denoted by A and Fight by F

Set of Challenger's strategies: $S_I = \{Ready, Unready\}$

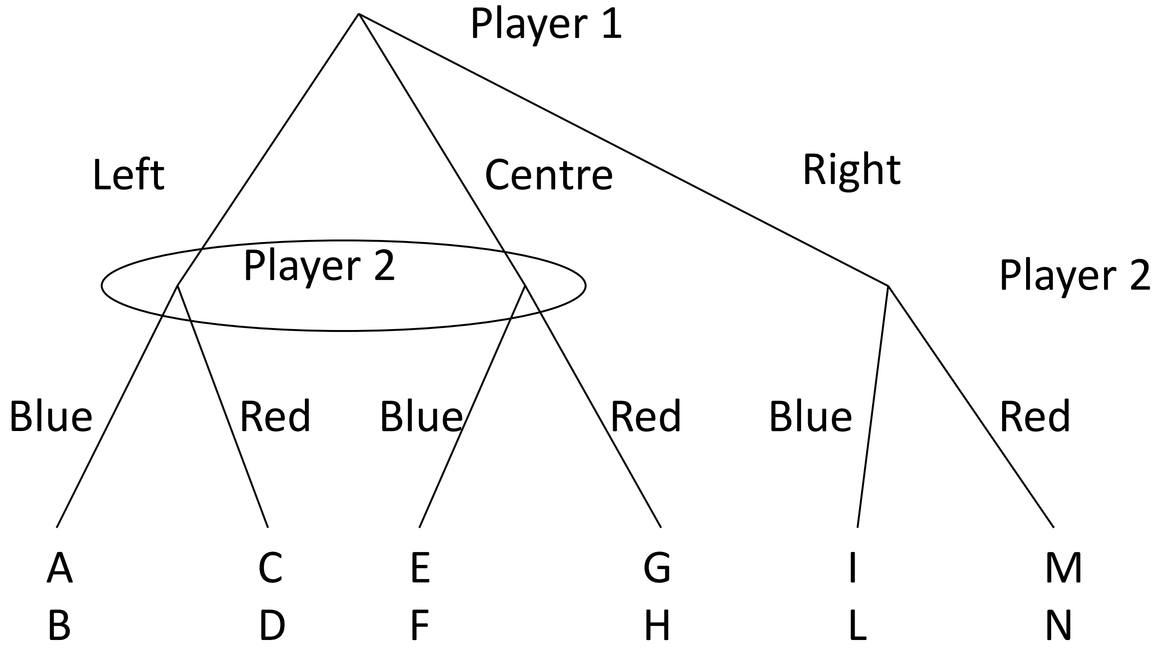
Set of Incumbent's strategies: $S_C = \{(A, A), (A, F), (F, A), (F, F)\}$

Set of Challenger’s strategies: $S_I = \{Ready, Unready\}$
Set of Incumbent’s strategies: $S_C = \{(A, A), (A, F), (F, A), (F, F)\}$

		Incumbent			
		(A, A)	(A, F)	(F, A)	(F, F)
Challenger	Ready	3, $\underline{3}$	$\underline{3}$, $\underline{3}$	1, 1	$\underline{1}$, 1
	Unready	$\underline{4}$, $\underline{3}$	0, 2	$\underline{4}$.$\underline{3}$	0, 2

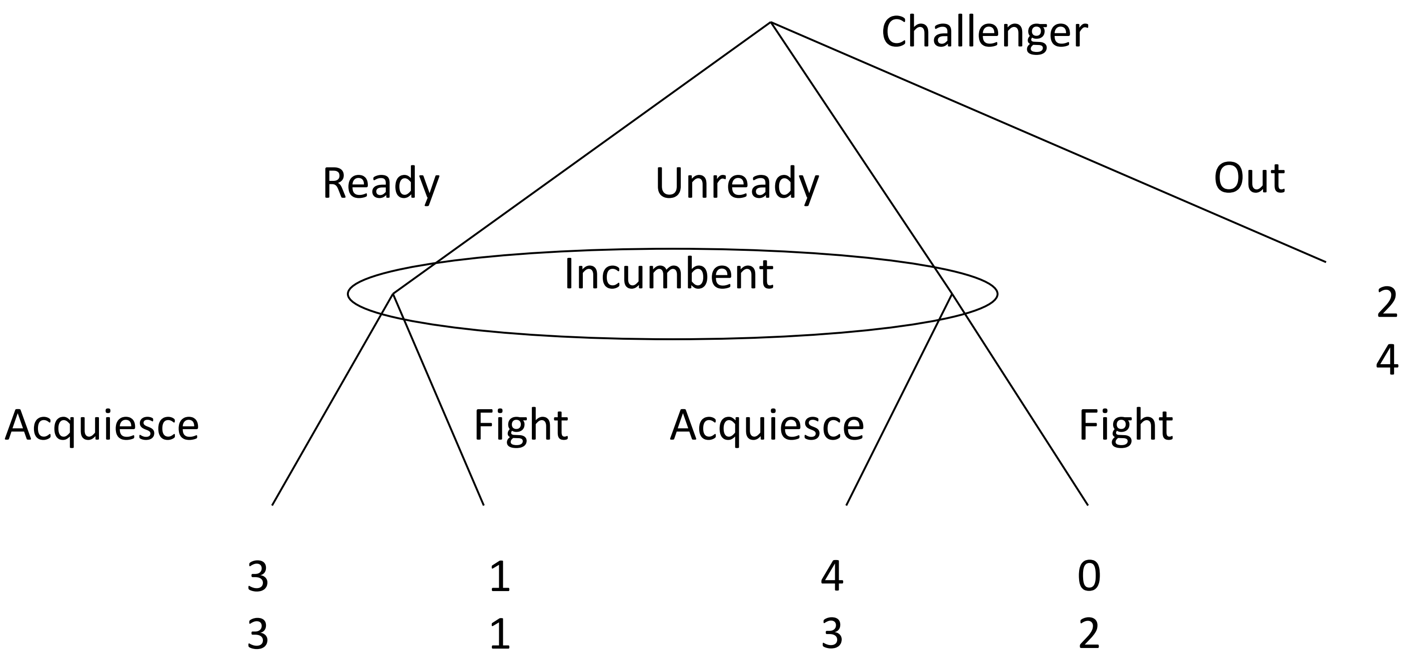
Example 3

		Player 2			
		Blue, Blue	*Blue, Red*	*Red, Blue*	*Red, Red*
Player 1	Left	A, B	A, B	C, D	C, D
	Centre	E, F	E, F	G, H	G, H
	Right	I, L	M, N	I, L	M, N



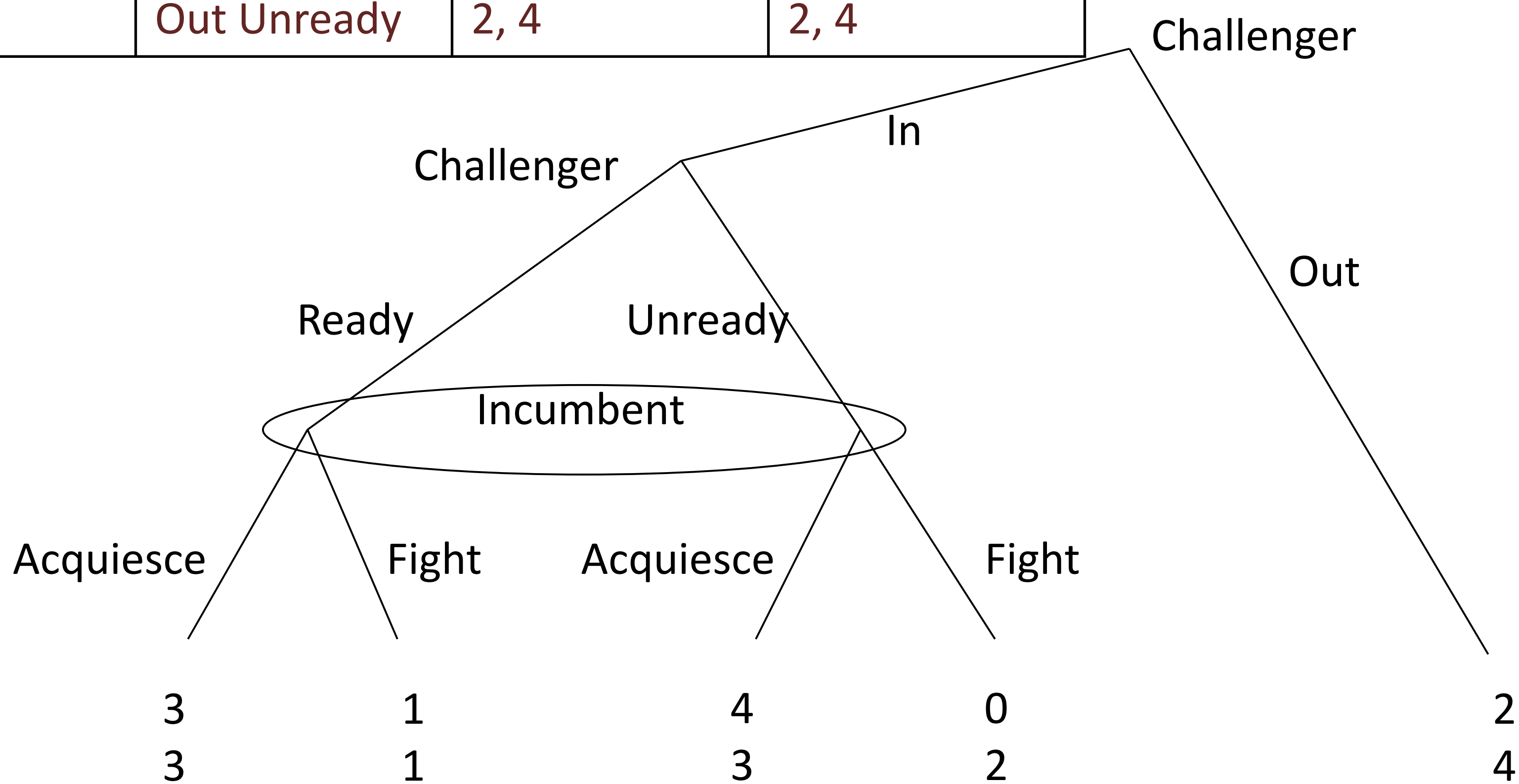
Example 4

		Incumbent	
		Acquiesce	Fight
Challenger	Ready	3, $\underline{3}$	1,1
	Unready	$\underline{4}$, $\underline{3}$	0, 2
	Out	2, $\underline{4}$	$\underline{2}$, $\underline{4}$



Example 5

		Incumbent	
		Acquiesce	Fight
Challenger	In Ready	3, 3	1,1
	In Unready	4, 3	0, 2
	Out Ready	2, 4	2, 4
	Out Unready	2, 4	2, 4



Solving normal form games

Iterated elimination of strictly dominated strategies

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Question set 3
Use code 124030

Consider the Prisoner Dilemma game

		Player 2	
		Not confess	Confess
Player 1	Not confess	-1, -1	-9, 0
	Confess	0, -9	-6, -6

		Player 2	
		Not confess	Confess
Player 1	Not confess	-1, -1	-9, 0
	Confess	0, -9	-6, -6

- If player 2 is going to play "*Not confess*", then player 1 prefers "*Confess*"
- If player 2 is going to play "*Confess*", then player 1 prefers "*Confess*"
- Player 1 prefers "*Confess*" in both cases
- We say that for player 1 playing "*Not confess*" is dominated by playing "*Confess*"

This example is easy to solve because the optimal choice for each player does not depend on what the other does:

"confess" leads to a higher payoff independently from the choice of the other.

In this example a rational player will never play "not confess"

In general a rational player will never use a particular strategy when she has another strategy that does better than the first.

Furthermore, if every players behave rationally, no player can be expected to use a strategy that is "dominated" by another

Definition of dominated strategy

In a normal form game a strategy s_i is **strictly dominated** by s_i' if:

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i}) \ \forall s_{-i} \in S_{-i}$$

(for each combination of other players' strategies s_{-i}) where:

- s_{-i} is a strategy profile of all other players except player i
- S_{-i} is the set of all s_{-i}

Note:

1) rational players do not play strictly dominated strategies (because a strictly dominated strategy is not optimal for all possible beliefs)

2) s_i is ***weakly*** **dominated** by s_i' if:

$$u_i(s_i', s_{-i}) \geq u_i(s_i, s_{-i}) \ \forall s_{-i} \in S_{-i}$$

with strict inequality for some $s_{-i} \in S_{-i}$

Example 1

		Player 2	
		L	R
Player 1	T	2,3	5,0
	M	3,2	1,1
	~~B~~	~~1,0~~	~~4,1~~

Note that in the original game only strategy B is dominated.
In the original game strategy R is undominated because it is good against strategy B.
But if player 2 knows that his opponent will never use strategy B (dominated), then she can judge her strategies only against T and M. Then results that strategy R is dominated

		Player 2	
		L	~~R~~
	T	2,3	~~5,0~~
Player 1	M	3,2	~~1,1~~

Then player 1 knows that player 2 will never use R, so can evaluate her strategy only against strategy L.

		Player 2
		L
	T	~~2,3~~
Player 1	M	3,2

		Player 2
		L
Player 1	M	3,2

Then the only strategic combination is given by Player 1 playing M and Player 2 playing L

- This process is called ***"iterated elimination of strictly dominated strategies" (*****or** ***iterated dominance)***
- It requires that rationality is common knowledge and it is based on the idea that rational players do not play strictly dominated strategies.
- Important property: ***independence from the order of elimination:***

  If there are two strictly dominated strategies the order of elimination does not affect the final result
- It could produce no accurate predictions (see example 2)
- We do not use this process with weakly dominated strategies because
  - the independence from the order of elimination does not hold
  - A rational player could play a weakly dominated (we see this later)

Example 2

		Player 2			
		L	C	~~R~~	
Player 1	~~T~~	~~2,3~~	~~2,2~~	~~5,0~~	3
	Y	3,2	5,3	~~3,1~~	
	Z	4,3	1,1	~~2,2~~	
	~~B~~	~~1,2~~	~~0,1~~	~~4,4~~	1
				2	

Example 3

Two firms produce an homogeneous good.
They face the following market

If the total production is 1 the price will be 26
If the total production is 2 the price will be 21
If the total production is 3 the price will be 16
If the total production is 4 the price will be 11
If the total production is 5 the price will be 6
If the total production is 6 the price will be 0

Where the total production is the sum of the quantities produced by the two firms

To produce one unit of this good has a cost of 2

		Firm 2						
		0	1	2	3	4	5	6
Firm 1	0	0, 0	0, 24	0, 38	0, 42	0, 36	0, 20	0, -12
	1	24, 0	19, 19	14, 28	9, 27	4, 16	-2, -10	-2, -12
	2	38, 0	28, 14	18, 18	8, 12	-4, -8	-4, -10	-4, -12
	3	42, 0	27, 9	12, 8	-6, -6	-6, -8	-6, -10	-6, -12
	4	36, 0	16, 4	-8, -4	-8, -6	-8, -8	-8, -10	-8, -12
	5	20, 0	-10, -2	-10, -4	-10, -6	-10, -8	-10, -10	-10, -12
	6	-12, 0	-12, -2	-12, -4	-12, -6	-12, -8	-12, -10	-12, -12

Go on www.menti.com

Using iterated deletion, how many units will produce firm 1?

Question set 3
Use code 124030

		Firm 2						
		0	1	2	3	4	5	6
Firm 1	0	0, 0	0, 24	0, 38	0, 42	0, 36	0, 20	0, -12
	1	24, 0	19, 19	14, 28	9, 27	4, 16	-2, -10	-2, -12
	2	38, 0	28, 14	18, 18	8, 12	-4, -8	-4, -10	-4, -12
	3	42, 0	27, 9	12, 8	-6, -6	-6, -8	-6, -10	-6, -12
	4	36, 0	16, 4	-8, -4	-8, -6	-8, -8	-8, -10	-8, -12
	5	20, 0	-10, -2	-10, -4	-10, -6	-10, -8	-10, -10	-10, -12
	6	-12, 0	-12, -2	-12, -4	-12, -6	-12, -8	-12, -10	-12, -12

		Firm 2						
		0	1	2	3	4	5	6
Firm 1	0	0, 0	0, 24	0, 38	0, 42	0, 36	0, 20	0, -12
	1	24, 0	19, 19	14, 28	9, 27	4, 16	-2, -10	-2, -12
	2	38, 0	28, 14	18, 18	8, 12	-4, -8	-4, -10	-4, -12
	3	42, 0	27, 9	12, 8	-6, -6	-6, -8	-6, -10	-6, -12
	4	36, 0	16, 4	-8, -4	-8, -6	-8, -8	-8, -10	-8, -12
	5	20, 0	-10, -2	-10, -4	-10, -6	-10, -8	-10, -10	-10, -12
	6	-12, 0	-12, -2	-12, -4	-12, -6	-12, -8	-12, -10	-12, -12

		Firm 2						
		0	1	2	3	~~4~~	~~5~~	~~6~~
Firm 1	0	0, 0	0, 24	0, 38	0, 42	~~0, 36~~	~~0, 20~~	~~0, -12~~
	1	24, 0	19, 19	14, 28	9, 27	~~4, 16~~	~~-2, -10~~	~~-2, -12~~
	2	38, 0	28, 14	18, 18	8, 12	~~-4, -8~~	~~-4, -10~~	~~-4, -12~~
	3	42, 0	27, 9	12, 8	-6, -6	~~-6, -8~~	~~-6, -10~~	~~-6, -12~~
	~~4~~	~~36, 0~~	~~16, 4~~	~~-8, -4~~	~~-8, -6~~	~~-8, -8~~	~~-8, -10~~	~~-8, -12~~
	~~5~~	~~20, 0~~	~~-10, -2~~	~~-10, -4~~	~~-10, -6~~	~~-10, -8~~	~~-10, -10~~	~~-10, -12~~
	~~6~~	~~-12, 0~~	~~-12, -2~~	~~-12, -4~~	~~-12, -6~~	~~-12, -8~~	~~-12, -10~~	~~-12, -12~~

		Firm 2						
		0	1	2	3	4	5	6
Firm 1	0	0, 0	0, 24	0, 38	0, 42	0, 36	0, 20	0, -12
	1	24, 0	19, 19	14, 28	9, 27	4, 16	-2, -10	-2, -12
	2	38, 0	28, 14	18, 18	8, 12	-4, -8	-4, -10	-4, -12
	3	42, 0	27, 9	12, 8	-6, -6	-6, -8	-6, -10	-6, -12
	4	36, 0	16, 4	-8, -4	-8, -6	-8, -8	-8, -10	-8, -12
	5	20, 0	-10, -2	-10, -4	-10, -6	-10, -8	-10, -10	-10, -12
	6	12, 0	12, -2	12, -4	12, -6	12, -8	12, -10	12, -12

		Firm 2						
		0	1	2	3	4	5	6
Firm 1	0	0, 0	0, 24	0, 38	0, 42	0, 36	0, 20	0, -12
	1	24, 0	19, 19	14, 28	9, 27	4, 16	-2, -10	-2, -12
	2	38, 0	28, 14	18, 18	8, 12	-4, -8	-4, -10	-4, -12
	3	42, 0	27, 9	12, 8	-6, -6	-6, -8	-6, -10	-6, -12
	4	36, 0	16, 4	-8, -4	-8, -6	-8, -8	-8, -10	-8, -12
	5	20, 0	-10, -2	-10, -4	-10, -6	-10, -8	-10, -10	-10, -12
	6	12, 0	12, 2	12, 4	12, 6	12, 8	12, 10	12, 12

		Firm 2						
		0	1	**2**	3	4	5	**6**
Firm 1	0	0, 0	0, 24	0, 38	0, 42	0, 36	0, 20	0, -12
	1	24, 0	19, 19	14, 28	9, 27	4, 16	-2, -10	-2, -12
	2	38, 0	28, 14	**18, 18**	8, 12	-4, -8	-4, -10	-4, -12
	3	42, 0	27, 9	12, 8	-6, -6	-6, -8	-6, -10	-6, -12
	4	36, 0	16, 4	-8, -4	-8, -6	-8, -8	-8, -10	-8, -12
	5	20, 0	-10, -2	-10, -4	-10, -6	-10, -8	-10, -10	-10, -12
	6	12, 0	12, 2	12, 4	12, 6	12, 8	12, 10	12, 12

Each firm will produce two units and has a profit of 18