

# Problem set 3

- 1) Represent using the extensive form the two examples in slides 9 and 10 (Prisoner dilemma and meeting for dinner)
- 2) Two individuals have to agree on how to divide 4 dollars. Two divisions are being considered: an even split that would give 2 dollars to each of them, and an asymmetric division that would leave 3 dollars with one of the players (labelled player 1) and one dollar with the other (player 2). The following allocation procedure is considered. First, player 1 has to make a proposal (i.e. one of the previous two possibilities), to which player 2 then has to respond with acceptance or rejection. If the proposal is accepted the four dollars are divided accordingly, whereas in the alternative case neither of them receives any money at all.
  - a. Represent this situation using the extensive form
  - b. Write all possible strategies of player 1
  - c. Write all possible strategies of player 2
- 3) A three-man board, composed of **A**, **B**, and **C**, has held hearings on a personnel case involving an officer of the company. This officer was scheduled for promotion but, prior to final action on his promotion, he made a decision that cost the company a good deal of money. The question is whether he should be (1) promoted anyway, (2) denied the promotion, or (3) fired. The board has discussed the matter at length and is unable to reach unanimous agreement. In the course of the discussion it has become clear to all three of them that their separate opinions are as follows:

**A** considers the officer to have been a victim of bad luck, not bad judgment, and wants to go ahead and promote him but, failing that, would keep him rather than fire him.

**B** considers the mistake serious enough to bar promotion altogether; he'd prefer to keep the officer, denying promotion, but would rather fire than promote him.

**C** thinks the man ought to be fired but, in terms of personnel policy and morale, believes the man ought not to be kept unless he is promoted, i.e., that keeping an officer who has been declared unfit for promotion is even worse than promoting him.

These preferences are summarized in the following table

	Promote	Keep	Fire
A:	Best	Middle	Worst
B:	Worst	Best	Middle
C:	Middle	Worst	Best

Assume that everyone's preferences among the three outcomes are fully evident as a result of the discussion. The three must proceed to a vote.

Consider the following voting procedure. First A proposes an action (either promote or keep or fire). Then it is B's turn. If B accepts A's proposal, then this becomes the final

decision. If B disagrees with A's proposal, then C makes the final decision (which may be any of the three: promote, keep or fire).

Assume that the best outcome gives an utility equal to 3, the middle outcome gives an utility equal 2 and the worst outcome gives an utility equal 1.

- a. Represent this situation using the extensive form
- b. Write all possible strategies of player A
- c. Write all possible strategies of player B
- d. Write all possible strategies of player C

- 4) A pack of matchsticks are arranged on the table in a triangular shape, one in the first row, three in the second row.



The player who makes the first move may pick as many matchsticks as he likes from one, but only one, row. Then the other player picks as many matchsticks as he likes from one row. This goes on until one of the players has to pick the last matchstick, and loses; Answer the following questions:

- a. Is it a game of perfect or imperfect information
- b. Represent it using the extensive form
- c. How many subgames?
- d. How many information sets has each player?
- e. How many strategies has each player?

- 5) Two players use a "pack" of three distinct cards,  $C \equiv \{h(\text{high}), m(\text{medium}), l(\text{low})\}$ , to participate in the following game. First, player 1 picks a card, sees it, and then decides to either "Bet" (B) or "Pass" (P). If player 1 bets, then player 2 picks a card out of the two remaining ones, sees it, and chooses as well to either "Bet" (B'), or "Pass" (P'). If both players bet, the player who has the highest card (no ties are possible) receives a hundred dollars from the opponent. On the other hand, if at least one of the players does not bet, no payments at all are made.

- a. Is it a game of perfect or imperfect information
- b. Represent it using the extensive form
- c. How many subgames?
- d. How many information sets has each player?
- e. How many strategies has each player?

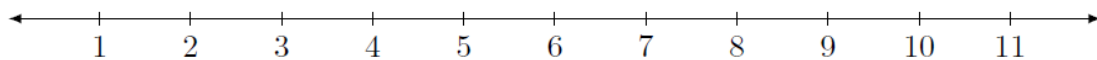
6) Consider the following normal form game :

		Player 2		
		L	M	R
Player 1	T	3, 0	2, 1	1, 0
	C	2, 1	1, 1	1, 0
	B	0, 1	0, 1	0, 0

- What is the set of pure strategies  $S_i$  for each player  $i = 1, 2$ ? What is the set of all pure strategy profiles  $S$ ?
- Which player has a strategy that is strictly dominated? State the strategy that is dominated and the strategy that dominates it, explain why this is an example of a strictly dominated strategy
- Find the solution(s) of this game by applying the iterated elimination of strictly dominated strategies.

#### 7) Location Game

Assume that the players are two vendors who simultaneously choose a location. Following the vendors choice of location, the customers then choose the vendor that they will purchase from - customers will always buy from the vendor that they are closest to. The profit for each vendor equals the number of customers it has attracted (if customers are indifferent between vendors assume that each vendor receives 0.5 of a customer each). To be more specific, assume that vendors choose a location from the following subset of the natural numbers:  $A = \{1, 2, \dots, 11\}$  Further, let these locations be viewed as points on the real line, and that at each location there is exactly one customer. Since we have 11 locations this can be visualised as below:



- What is the set of pure strategies,  $S_i$ , for each vendor  $i = 1, 2$  What is the set of all pure strategy profiles  $S$ ?
- If vendor 1 chooses to locate at 4,  $s_1 = 4$ , and vendor 2 chooses to locate at 8,  $s_2 = 8$ , what are their respective payoffs? What is the general formula for vendors' payoffs in this game?
- Find the solution(s) of the location game by using iterated elimination of strictly dominated

### 8) Announcing Integers

The following game is played between two players: each player simultaneously announces an integer between 0 and 100. Let  $s_1$  be the announcement of player 1 and  $s_2$  be the announcement of player 2. Their payoffs are determined as follows:

- If  $s_1 + s_2 \leq 100$ : Player 1 receives  $s_1$  and Player 2 receives  $s_1$ ;
- If  $s_1 + s_2 > 100$  and  $s_1 > s_2$ : Player 1 receives  $100 - s_1$  and Player 2 receives  $s_2$ ;
- If  $s_1 + s_2 > 100$  and  $s_1 < s_2$ : Player 1 receives  $s_1$  and Player 2 receives  $100 - s_1$ ;
- If  $s_1 + s_2 > 100$  and  $s_1 = s_2$ : both players receive 50;

- a) What is the set of pure strategies,  $S_i$ , for each player  $i = 1, 2$ ? What is the set of all pure strategy profiles  $S$ ?
- b) Find the solution(s) of this game by using the iterated elimination of strictly dominated strategies.

### 9) Bertrand Competition

Two firms, 1 and 2, compete by choosing prices. Each firm can choose among four possible prices from the set  $P_i = \{1, 2, 3, 4\}$ . The cost to produce one unit is 2 for each firm, and there is unit demand (i.e. one consumer asking one unit). The firm charging the lowest price gets to serve the unit demand. If there is a tie, each firm will serve half of the demand. Please answer the following questions:

- a) What is the payoff function for each player in this game?
- b) Express this situation using the normal form representation.
- c) Find the set of strategies that survive the iterated elimination of strictly dominated strategies. Which strategies were eliminated? Why does it make sense that they were eliminated?