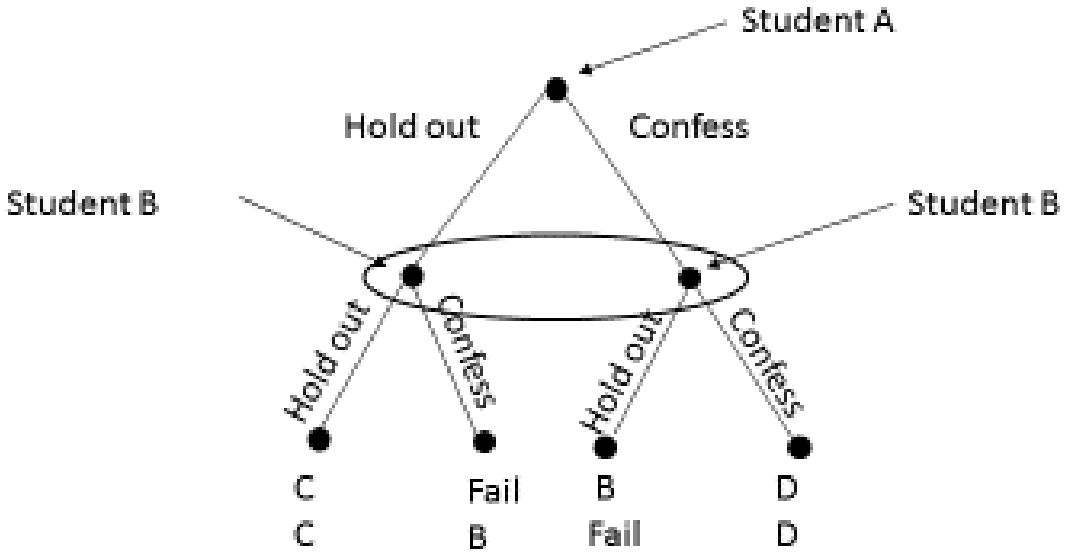
Solutions problem set 3

1) Represent using the extensive form the two examples in slides 9 and 10 (Prisoner dilemma and meeting for dinner)

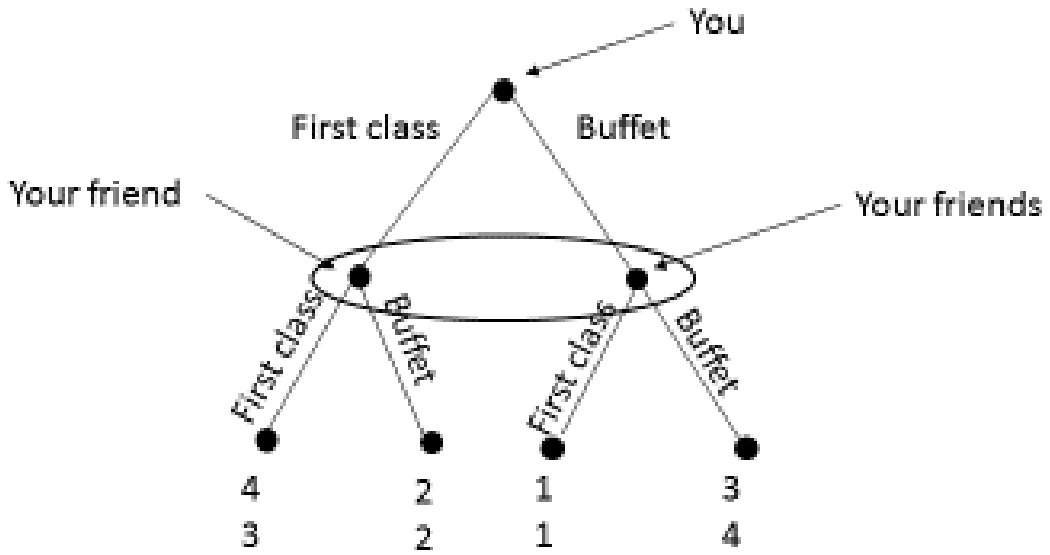
Solutions

Prisoner dilemma



Meeting for dinner

Assume that the preferred outcome gives an utility of 4, the second preferred outcome gives an utility of 3, the third and the fourth referred outcomes give, respectively utilities of 2 and 1.

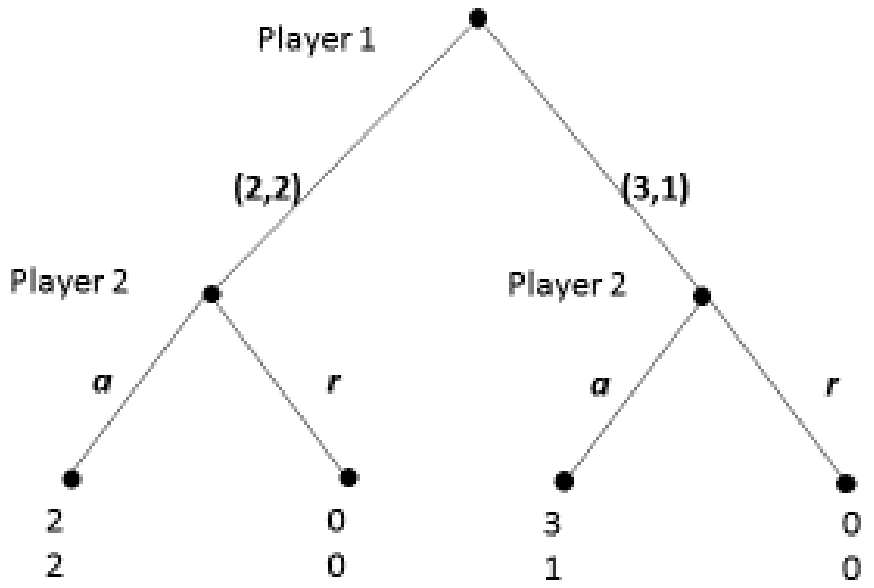


2) Two individuals have to agree on how to divide 4 dollars. Two divisions are being considered: an even split that would give 2 dollars to each of them, and an asymmetric division that would leave 3 dollars with one of the players (labelled player 1) and one dollar with the other (player 2). The following allocation procedure is considered. First, player 1 has to make a proposal (i.e. one of the previous two possibilities), to which player 2 then has to respond with acceptance or rejection. If the proposal is accepted the four dollars are divided accordingly, whereas in the alternative case neither of them receives any money at all.

   a. Represent this situation using the extensive form
   b. Write all possible strategies of player 1
   c. Write all possible strategies of player 2

   Solutions

   a.



b. Player 1's strategies: (2, 2) and (3, 1)
c. Player 2's strategies:
a. Accept (2, 2) and accept (3, 1)
b. Accept (2, 2) and reject (3, 1)
c. Reject (2, 2) and accept (3, 1)
d. Reject (2, 2) and Reject (3, 1)

3) A three-man board, composed of ***A***, ***B***, and ***C***, has held hearings on a personnel case involving an officer of the company. This officer was scheduled for promotion but, prior to final action on his promotion, he made a decision that cost the company a good deal of money. The question is whether he should be (1) promoted anyway, (2) denied the promotion, or (3) fired. The board has discussed the matter at length and is unable to reach unanimous agreement. In the course of the discussion it has become clear to all three of them that their separate opinions are as follows:

A considers the officer to have been a victim of bad luck, not bad judgment, and wants to go ahead and promote him but, failing that, would keep him rather than fire him.

B considers the mistake serious enough to bar promotion altogether; he'd prefer to keep the officer, denying promotion, but would rather fire than promote him.

C thinks the man ought to be fired but, in terms of personnel policy and morale, believes the man ought not to be kept unless he is promoted, i.e., that keeping an officer who has been declared unfit for promotion is even worse than promoting him.

These preferences are summarized in the following table

	Promote	Keep	Fire
A:	Best	Middle	Worst
B:	Worst	Best	Middle
C:	Middle	Worst	Best

Assume that everyone's preferences among the three outcomes are fully evident as a result of the discussion. The three must proceed to a vote.

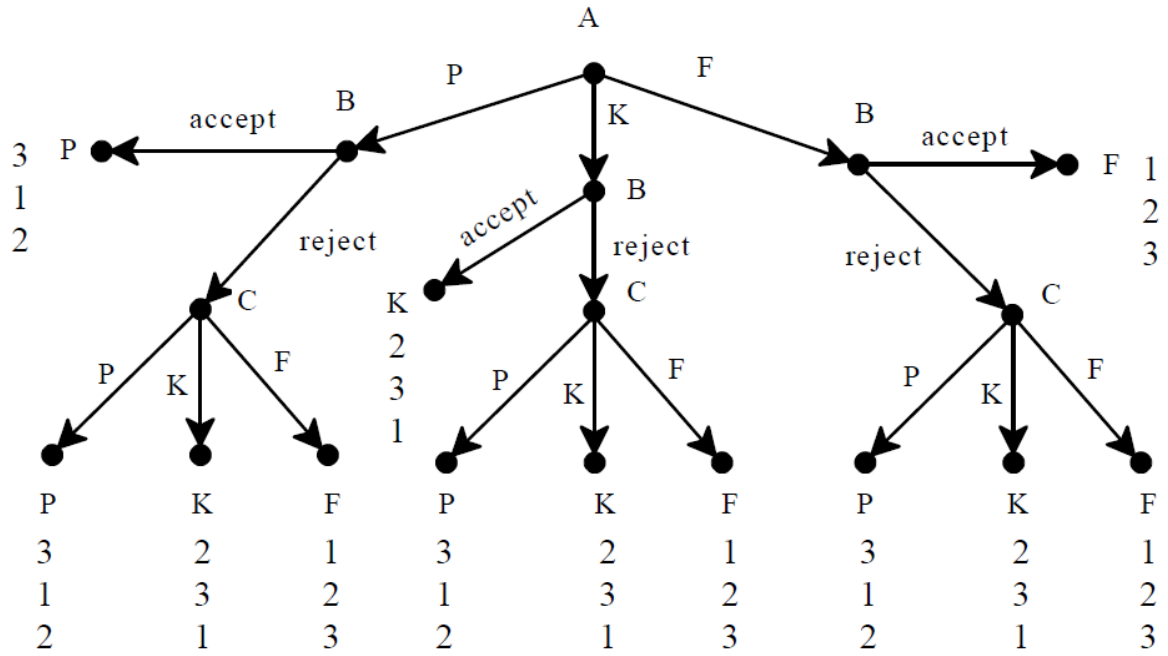
Consider the following voting procedure. First A proposes an action (either promote or keep or fire). Then it is B's turn. If B accepts A's proposal, then this becomes the final decision. If B disagrees with A'a proposal, then C makes the final decision (which may be any of the three: promote, keep or fire).

Assume that the best outcome gives an utility equal to 3, the middle outcome gives an utility equal 2 and the worst outcome gives an utility equal 1.

a. Represent this situation using the extensive form
b. Write all possible strategies of player A
c. Write all possible strategies of player B
d. Write all possible strategies of player C

Solution

a. P stands for “Promote”, F for “Fire”, K for “Keep”



b. The three player A’s strategies are: P or K or F

c. The eight player B’s strategies are:

   a. Accept P, Accept K, Accept F
   b. Reject P, Accept K, Accept F
   c. Accept P, Reject K, Accept F

d. Accept P, Accept K, Reject F
e. Accept P, Reject K, Reject F
f. Reject P, Accept, K, Reject F
g. Reject P, Reject K, Accept F
h. Accept P, Accept K, Accept F

d. The 27 player C's strategies are (action in the left node, in the central node, in the right node):

1	P, P, P	10	K, P, P	19	F, P, P
2	P, P, K	11	K, P, K	20	F, P, K
3	P, P, F	12	K, P, F	21	F, P, F
4	P, K, P	13	K, K, P	22	F, K, P
5	P, K, K	14	K, K, K	23	F, K, K
6	P, K, F	15	K, K, F	24	F, K, F
7	P, F, P	16	K, F, P	25	F, F, P
8	P, F, K	17	K, F, K	26	F, F, K
9	P, F, F	18	K, F, F	27	F, F, F

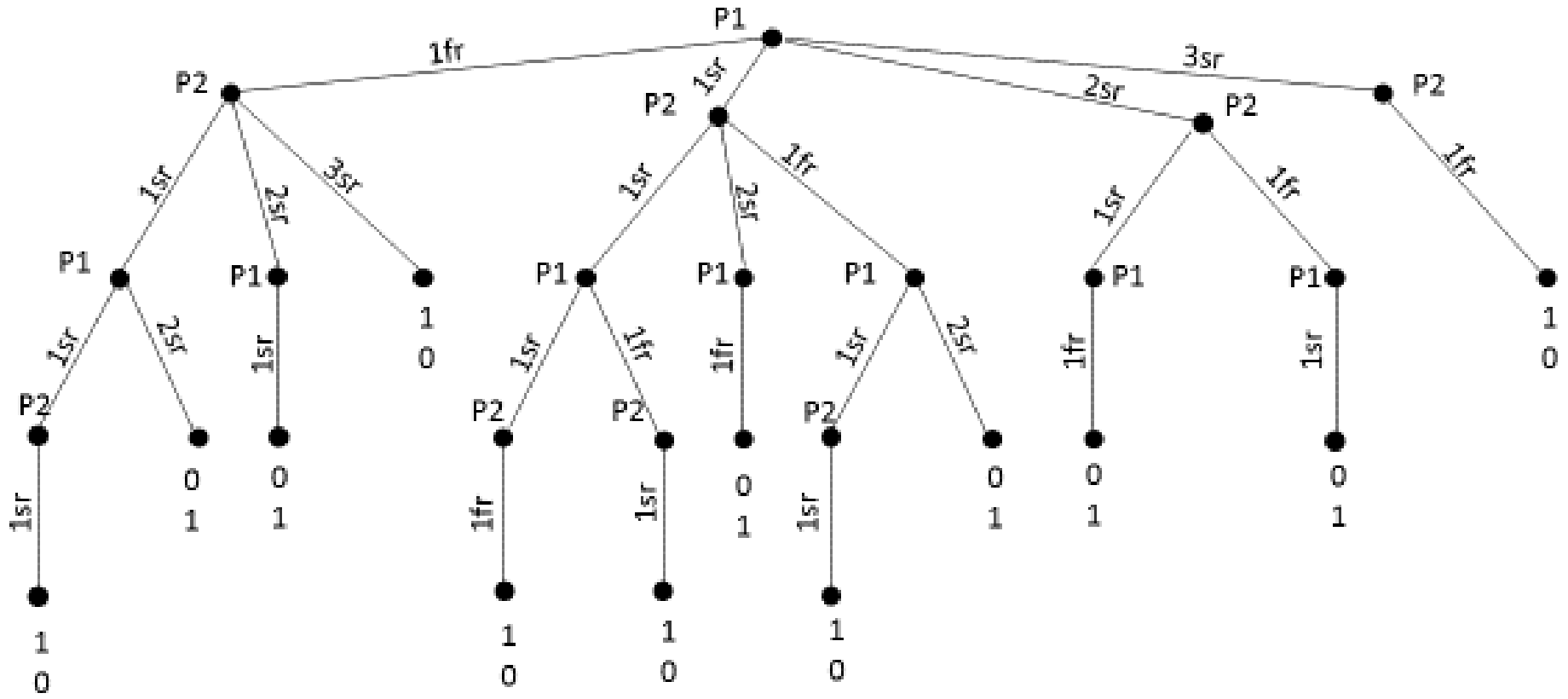
4) A pack of matchsticks are arranged on the table in a triangular shape, one in the first row, three in the second row.

The player who makes the first move may pick as many matchsticks as he likes from one, but only one, row. Then the other player picks as many matchsticks as he likes from one row. This goes on until one of the players has to pick the last matchstick, and loses; Answer the following questions:

a. Is it a game of perfect or imperfect information
b. Represent it using the extensive form
c. How many subgames?
d. How many information sets has each player?
e. How many strategies has each player?

Solutions

a. It is of perfect information because, by the rules of the game, each player can observe all actions of the other player
b. Player 1 is denoted by **P1**, player 2 is denoted by **P2**
   1fr means taking 1 matchstick from the first row
   1sr means taking 1 matchstick from the second row
   2sr means taking 2 matchsticks from the second row
   3sr means taking 3 matchsticks from the second row
   Winner has a payoff of 1, loser has a payoff of 0



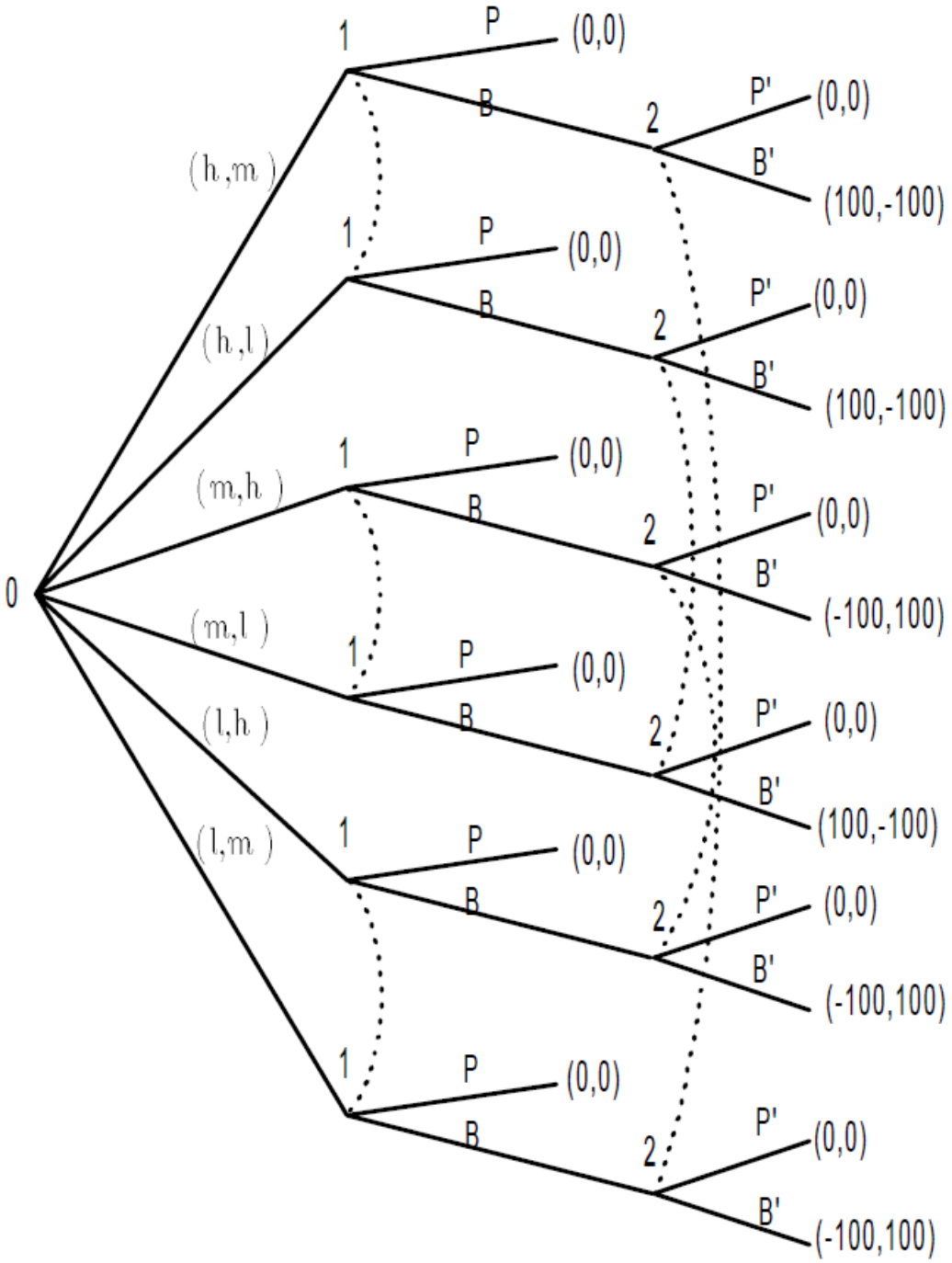
c. 16 subgames
d. P1 has 8 information sets, P2 has 8 information sets
e. P1 has 32 strategies, P2 has 18 strategies

5) Two players use a ‘‘pack’’ of three distinct cards, $C \equiv \{h(high), m(medium), l(low)\}$, to participate in the following game. First, player 1 picks a card, sees it, and then decides to either ‘‘Bet’’ (B) or ‘‘Pass’’ (P). If player 1 bets, then player 2 picks a card out of the two remaining ones, sees it, and chooses as well to either ‘‘Bet’’ (B’, or ‘‘Pass’’ (P’). If both players bet, the player who has the highest card (no ties are possible) receives a hundred dollars from the opponent. On the other hand, if at least one of the players does not bet, no payments at all are made.

    a. Is it a game of perfect or imperfect information
    b. Represent it using the extensive form
    c. How many subgames?
    d. How many information sets has each player?
    e. How many strategies has each player?

Solutions

a. Is a game of imperfect information
b. See the picture in the next page
c. 1 subgame
d. Each player has 3 information sets
e. Each player has 8 strategies


0
(h,m)
(h,l)
(m,h)
(m,l)
(l,h)
(l,m)
1
2
P
B
P'
B'
(0,0)
(100,-100)
(-100,100)

6) Consider the following normal form game :

		Player 2		
		L	M	R
Player 1	T	3, 0	2, 1	1, 0
	C	2, 1	1, 1	1, 0
	B	0, 1	0, 1	0, 0

.

a) What is the set of pure strategies S_i for each player $i = 1, 2$? What is the set of all pure strategy profiles S?
b) Which player has a strategy that is strictly dominated? State the strategy that is dominated and the strategy that dominates it, explain why this is an example of a strictly dominated strategy
c) Find the solution(s) of this game by applying the iterated elimination of strictly dominated strategies.

Solution

a) strategy set of Player 1 is $S_1 = (\mathrm{T}, \mathrm{C}, \mathrm{B})$,
strategy set of Player 1 is $S_2 = (\mathrm{L}, \mathrm{M}, \mathrm{R})$
the set of all pure strategy profiles is $S = \{(T, L), (T, M), (T, R), (C, L), (C, M), (C, R), (B, L), (B, M), (B, R), \}$

b) For Player 1 strategy B is dominated by strategy C,
$U(C, L) > U(B, L)$ and $U(C, M) > U(B, M)$ and $U(C, R) > U(B, R)$
strategy B is also dominated by strategy T
$U(T, L) > U(B, L)$ and $U(T, M) > U(B, M)$ and $U(T, R) > U(B, R)$

For player 2 strategy R is dominated by strategy M
$U(T, M) > U(T, R)$ and $U(C, M) > U(C, R)$ and $U(B, M) > U(B, R)$

		Player 2		
		L	M	R
Player 1	T	3, 0	2, 1	1, 0
	C	2, 1	1, 1	1, 0
	B	0, 1	0, 1	0, 0

3

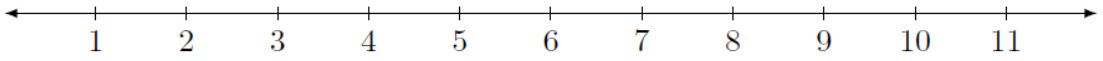
1

4

2

7) Location Game

Assume that the players are two vendors who simultaneously choose a location. Following the vendors choice of location, the customers then choose the vendor that they will purchase from - customers will always buy from the vendor that they are closest to. The profit for each vendor equals the number of customers it has attracted (if customers are indifferent between vendors assume that each vendor receives 0.5 of a customer each). To be more specific, assume that vendors choose a location from the following subset of the natural numbers: $A = \{1, 2, \ldots\ldots, 11\}$Further, let these locations be viewed as points on the real line, and that at each location there is exactly one customer. Since we have 11 locations this can be visualised as below:



a) What is the set of pure strategies, S_i, for each vendor $i = 1, 2$ What is the set of all pure strategy profiles S?
b) If vendor 1 chooses to locate at 4, $s_1 = 4$, and vendor 2 chooses to locate at 8, $s_2 = 8$, what are their respective payoffs? What is the general formula for vendors' payoffs in this game?
c) Find the solution(s) of the location game by using iterated elimination of strictly dominated

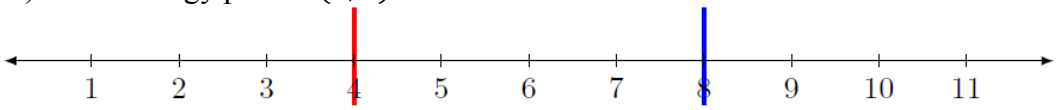
Solution

a) The strategy sets S_i are: $S_i = \{1, 2, 3, \ldots\ldots, 11\}$ for $i = 1, 2$.
The set of all pure strategy profiles is the cartesian product of the two sets S_i of pure strategies, i.e. $S = S_1 x S_2$

$$S = \{(1,1), (1,2), (1,3) \ldots\ldots (2,1), (2,2), (2,3), \ldots\ldots (11,10), (11,11)\ \}$$

The set S has 121 strategy profiles

b) The strategy profile $(4, 8)$ is shown on the real line below



$\pi_1 = 5.5 \quad \pi_2 = 5.5$

$$\pi_i(s_i, s_{-i}) = \begin{cases} \dfrac{s_i + s_{-i} - 1}{2} & if\ s_i < s_{-i} \\ 11 - \dfrac{s_i + s_{-i} - 1}{2} & if\ s_i > s_{-i} \\ \dfrac{11}{2} & if\ s_i = s_{-i} \end{cases}$$

c) Consider Player 1.
Strategy 1 is dominated by strategy 2. To verify this claim you need to verify that $U(2, s_2) > U(1, s_2)$ for all strategies s_2 of player 2.

s_2	1	2	3	4	5	6	7	8	9	10	11
$U(1, s_2)$	5.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5
$U(2, s_2)$	10	5.5	2	2.5	3	3.5	4	4.5	5	6.5	7

By similar reasoning we can prove that:

- strategy 1 is dominated by strategy 2 for player 2
- strategy 11 is dominated by strategy 10 for both players
- strategy 2 is dominated by 3 for both players
- strategy 10 is dominated by 9 for both players
- strategy 3 is dominated by 4 for both players
- strategy 9 is dominated by 8 for both players
- strategy 4 is dominated by 5 for both players
- strategy 8 is dominated by 7 for both players
- strategy 5 is dominated by 6 for both players
- strategy 7 is dominated by 6 for both players

Finally remain only strategy 6
Then the solution of the game is represented by the strategy profile $(6\ ,6)$