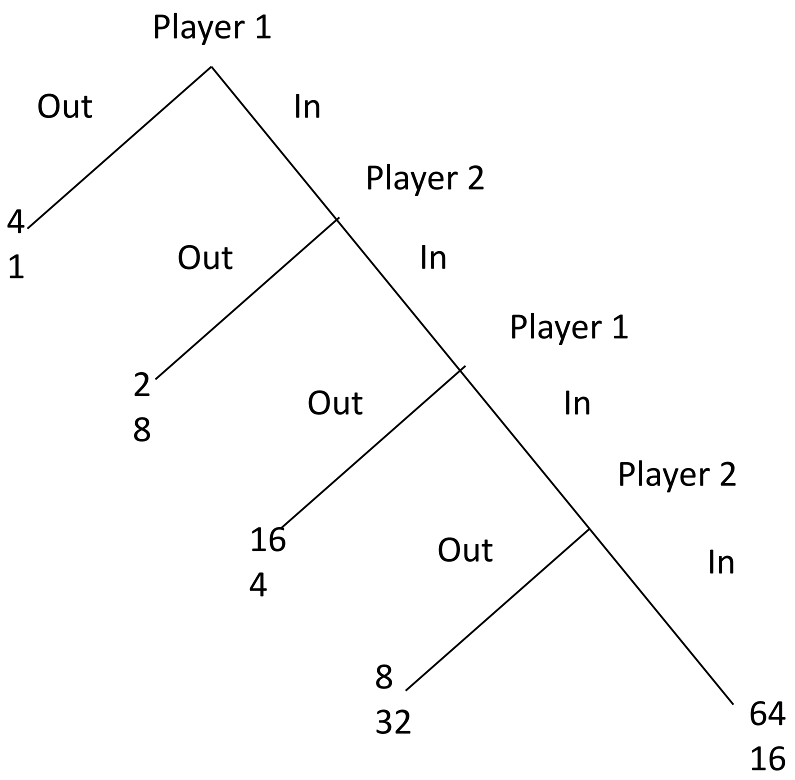
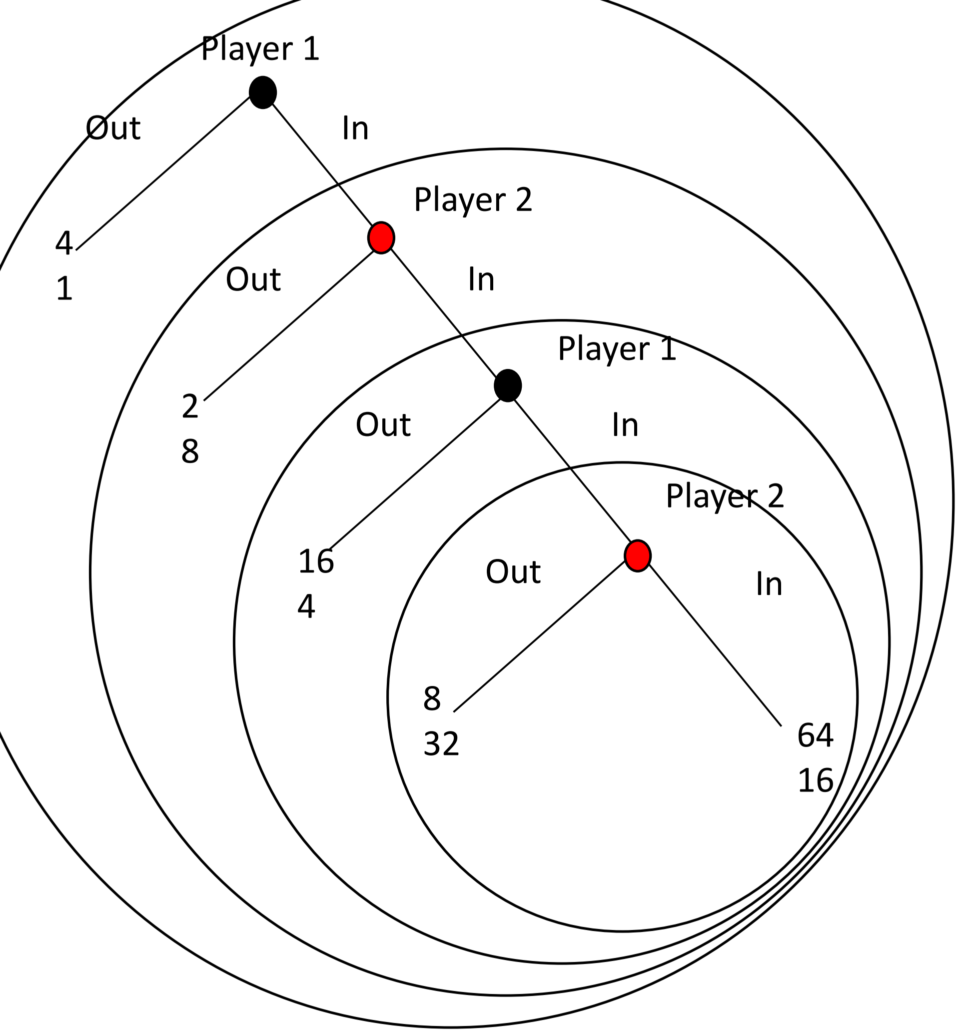
Solution problem set 6

Ex 1.



In each decision node players have two possible actions, *In* or *Out*

a) How many information sets for each player?
b) How many subgames?
c) Write all possible strategies for both players
d) Represent this game in normal form and find all Nash equilibria.



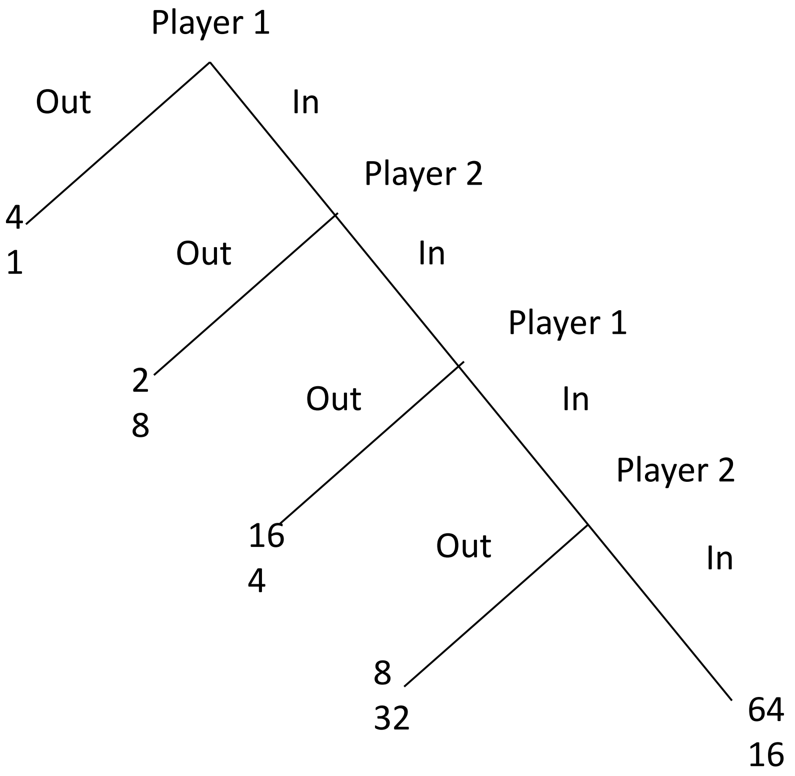
a) Each player has two information sets

- Two for player 1 (black nodes)
- Two for player 2 (red nodes)

b) There are 4 subgames

c) Player 1: {(out, out), (out, in), (in, out), (in, in)}

Player 2: {(out, out), (out, in), (in, out), (in, in)}



Strategies

Player 1: {(out, out), (out, in), (in, out), (in, in)}

Player 2: {(out, out), (out, in), (in, out), (in, in)}

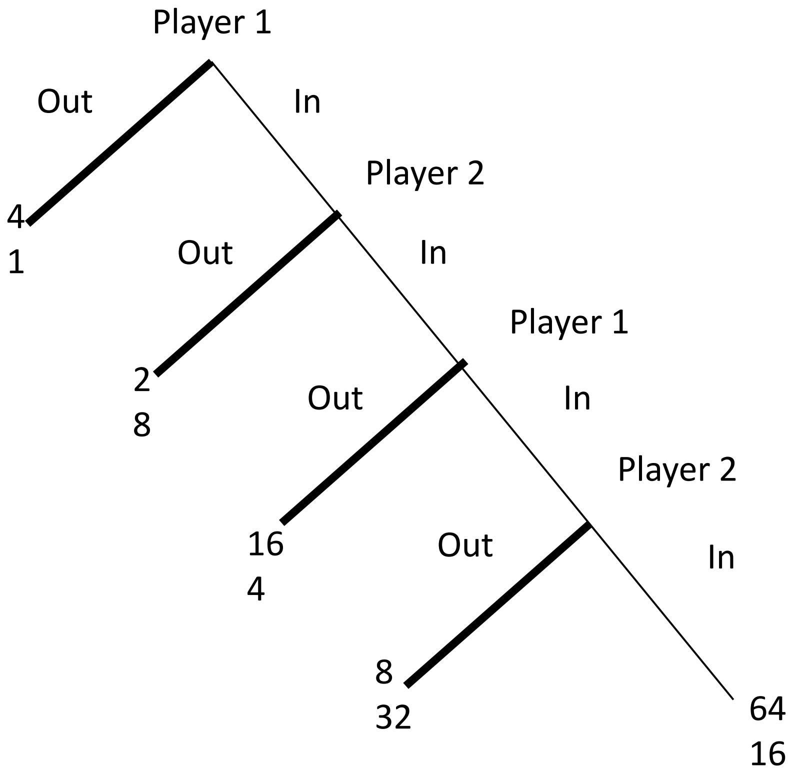
		Player 2			
		Out Out	Out In	In Out	In In
Player 1	Out Out	$\underline{4}$, $\underline{1}$	$\underline{4}$, $\underline{1}$	4, $\underline{1}$	4, $\underline{1}$
	Out In	$\underline{4}$, $\underline{1}$	$\underline{4}$, $\underline{1}$	4, $\underline{1}$	4, $\underline{1}$
	In Out	2, $\underline{8}$	2, $\underline{8}$	$\underline{16}$, 4	16, 4
	In In	2, 8	2, 8	8, $\underline{32}$	$\underline{64}$, 16

Nash equilibria:

{(Out, Out), (Out, Out)} {(Out, Out), (Out, In)}

{(Out, In), (Out, Out)} {(Out, In), (Out, In)}

e) Starting from the last subgame, the branches in bold are the best responses



The unique subgame perfect Nash equilibrium is

{(Out, Out), (Out, Out)}

BIO is Player 1 plays Out

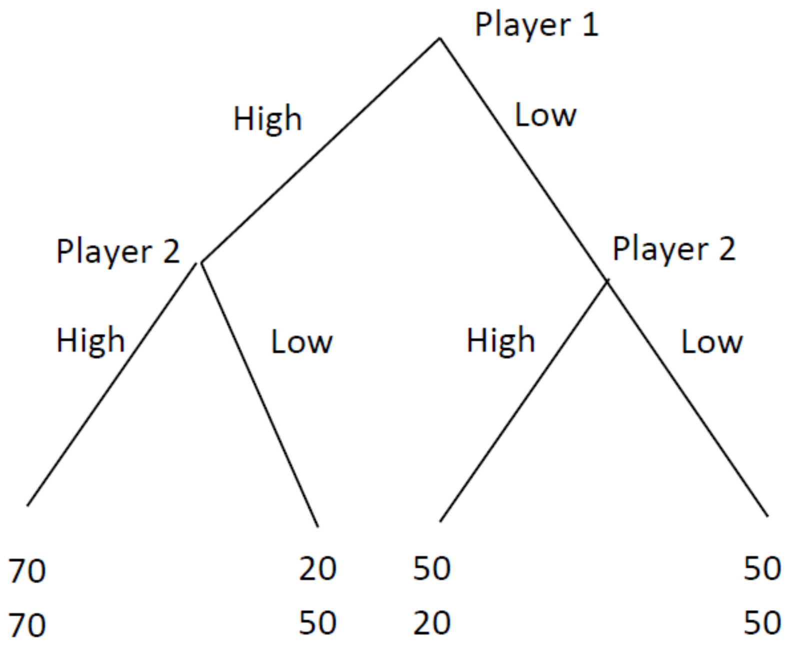
Ex 2. Two individuals, A and B, are working on a join project. They can devote it either high effort or low effort. If both players devote high effort, the outcome of the project is of high quality and each one receives 100$. If one or both devote low effort, the outcome of the project is of low quality and each one receives 50$. The opportunity cost to provide high effort is 30. The opportunity cost to provide low effort is 0. Individual A moves first, individual B observes the action of A and then moves.

Represent this situation using the extensive form representation

for both players write all possible strategies

Using the normal form, find all Nash equilibriums

Find all Subgame Perfect Nash Equilibria a)



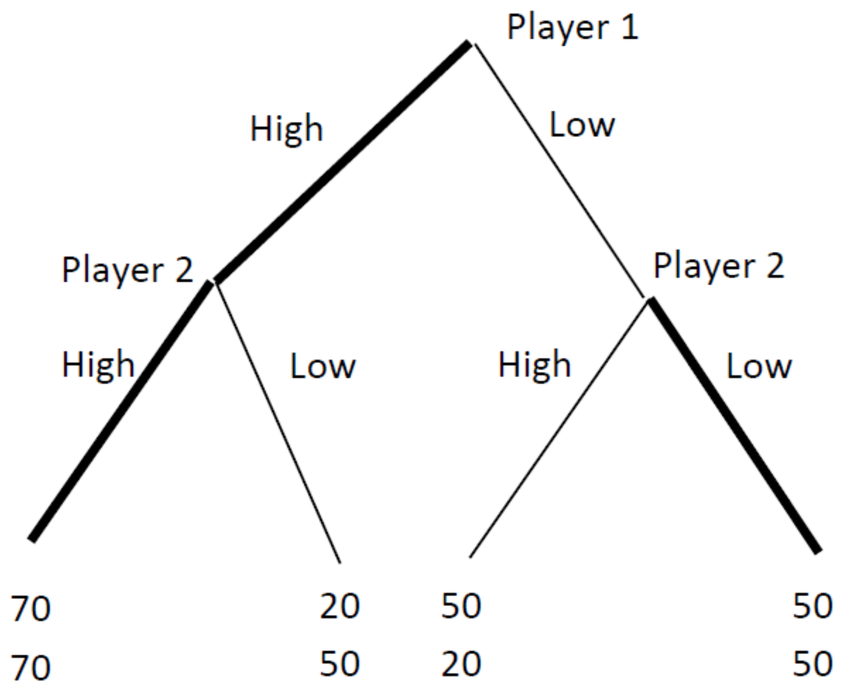
b) Player 1’s strategies: {High, Low}

Player 2’s strategies: {(Low, Low), (Low, High), (High, Low), (High, High)} where by (x, y) we denote action x after High, action y after Low c)

		Player 2			
		High High	High Low	Low High	Low Low
Player 1	High	$\underline{70}$, $\underline{70}$	$\underline{70}$, $\underline{70}$	20, 50	20, 50
	Low	50, 20	50, $\underline{50}$	$\underline{50}$, 20	$\underline{50}$, $\underline{50}$

- we underline the best responses.
- Nash equilibria:
- {(High), (High, High)}   {(High), (High, Low)}
- {(Low), (Low, Low)}

d)

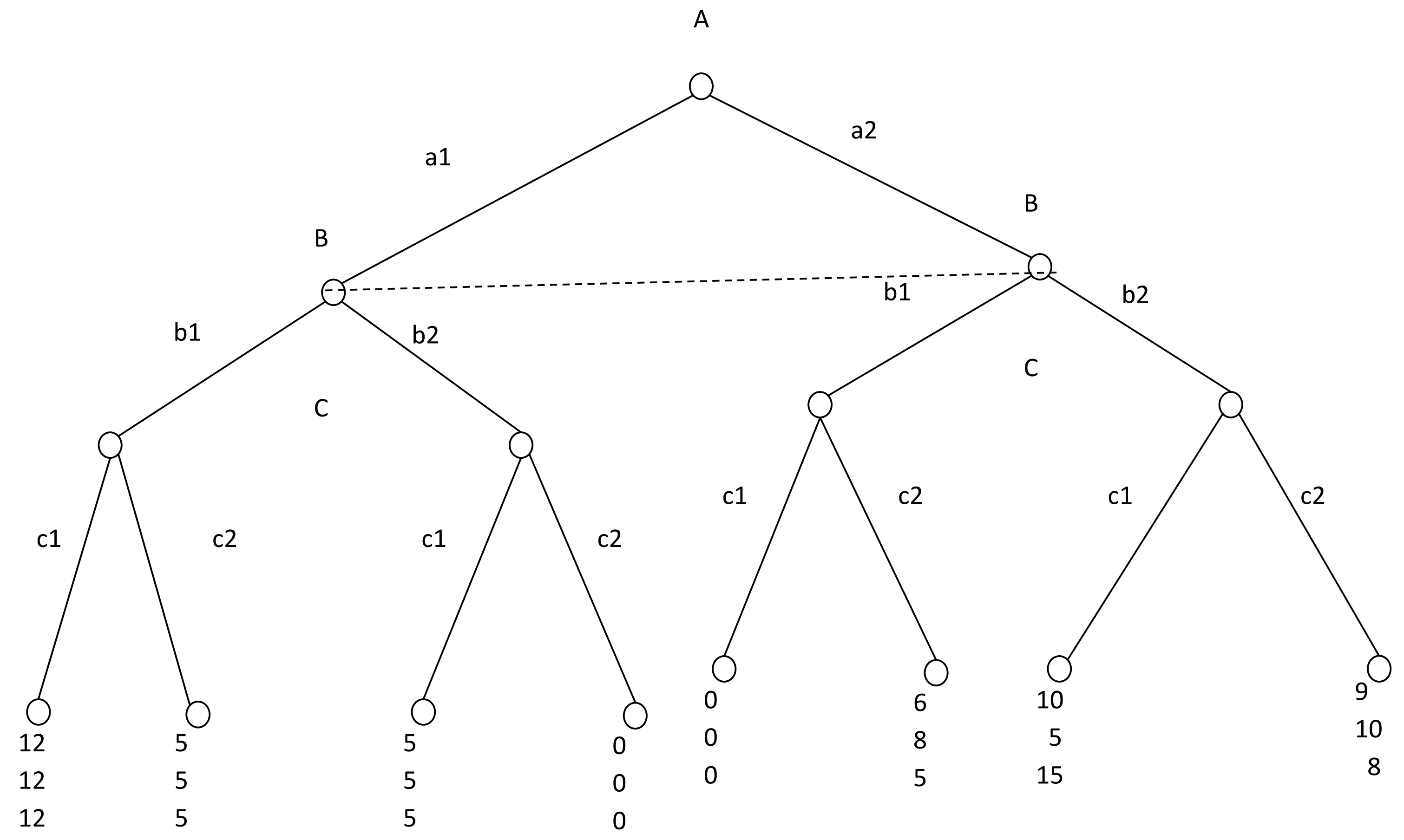


Starting from the last subgame, the branches in bold are the best responses

The unique subgame perfect Nash equilibrium is {(High), (High, Low)}

The BIO is Player 1 plays High, Player 2 plays High

Ex. 3



a) How many information sets has player A?and player B? And player C? How many subgames?

b) For each player describe all possible strategies

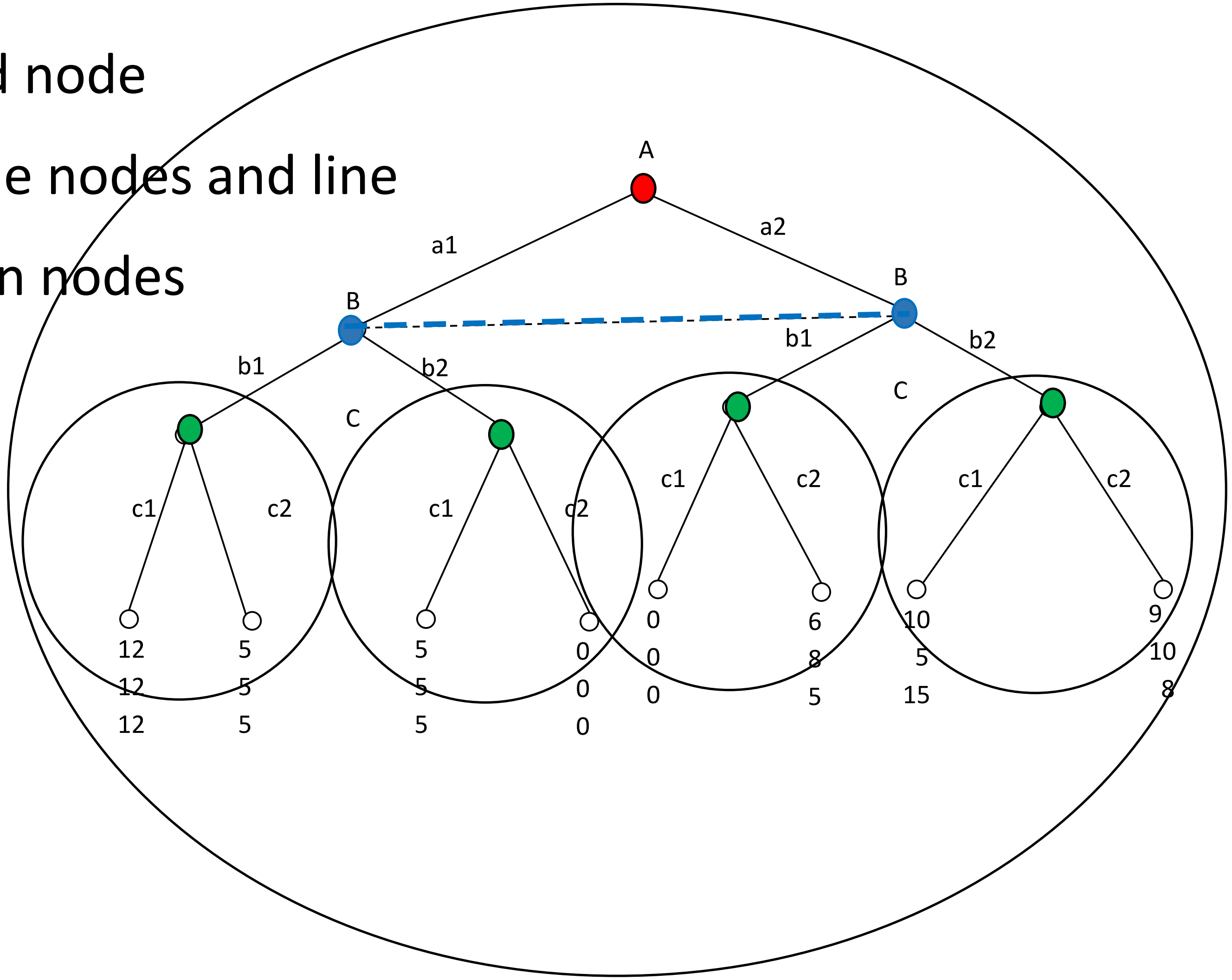
c) Find all Subgame Perfect Nash Equilibria a)

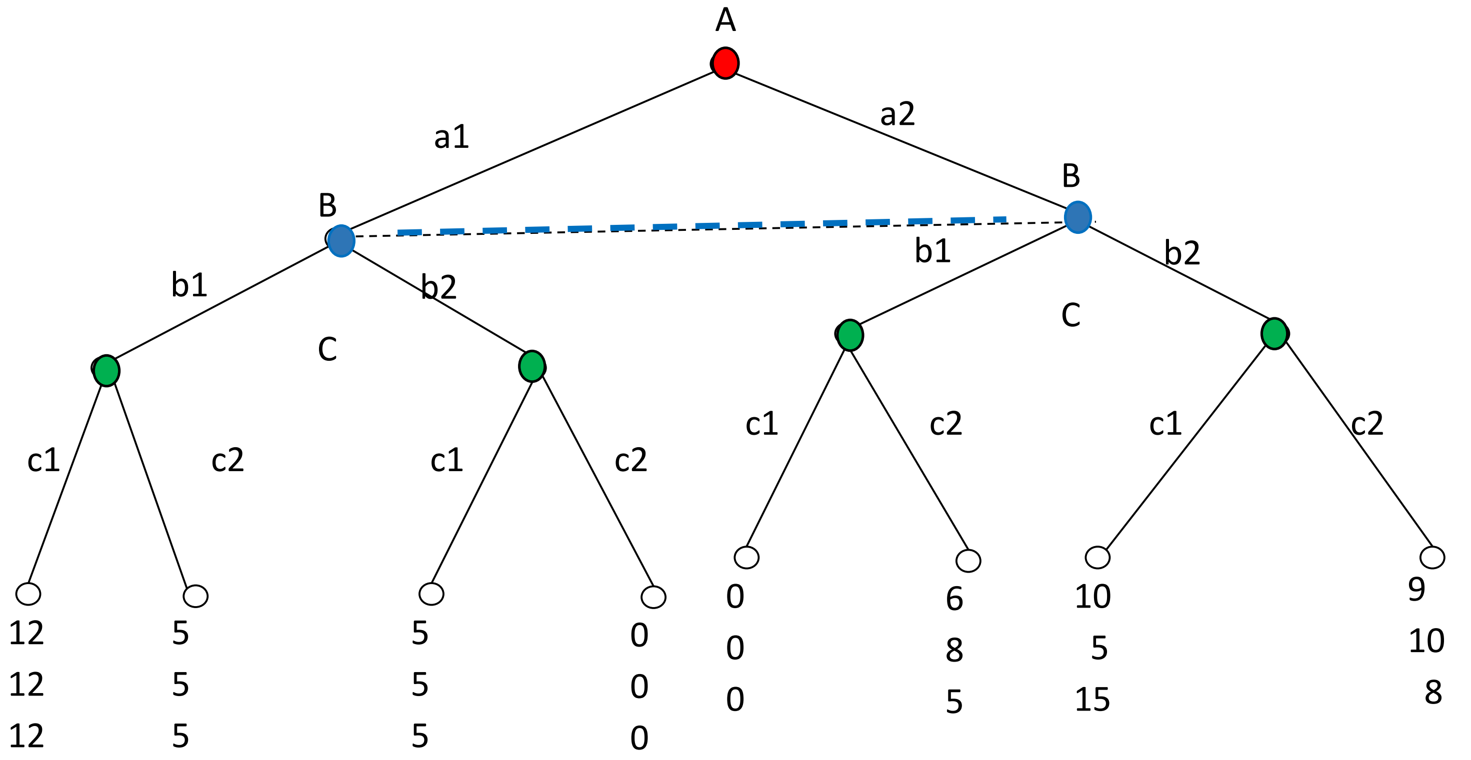
Player A has one information set, red node

Player B has one information set, blue nodes and line

Player C has 4 information sets. green nodes

There are 5 subgames



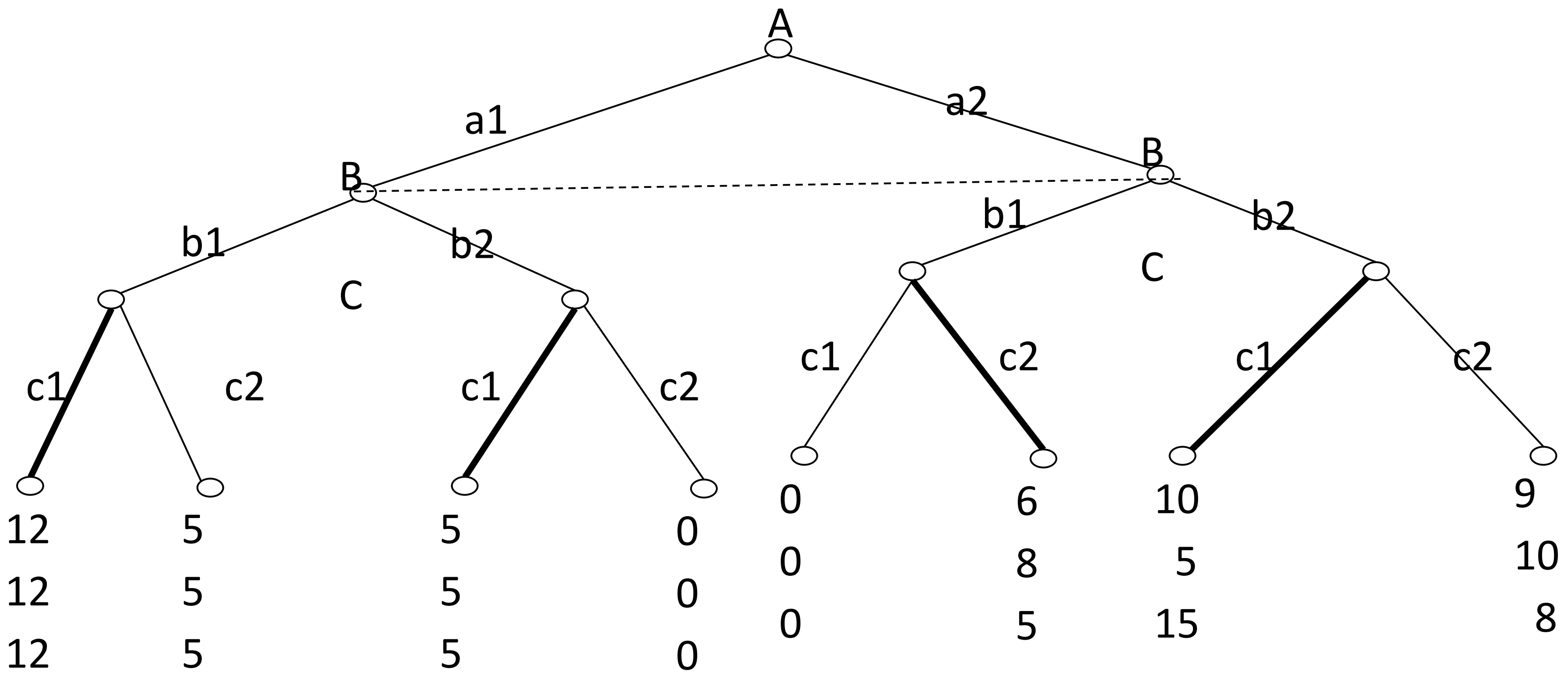


b) Players' strategies are

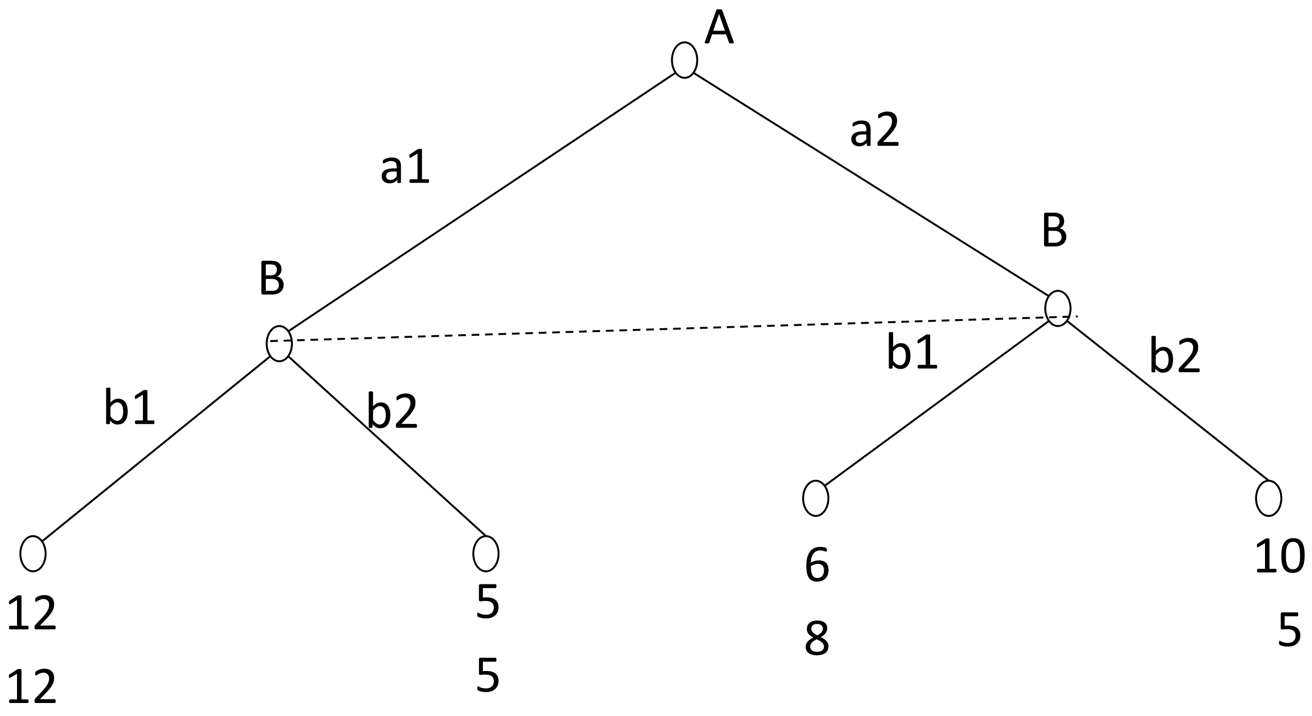
- Player A: $\{(a_1),(a_2)\}$
- Player B: $\{(b_1),(b_2)\}$
- Player C: $\left\{\begin{matrix}(c_1,c_1,c_1,c_1),(c_2,c_1,c_1,c_1),(c_1,c_2,c_1,c_1),(c_1,c_1,c_2,c_1),, \\ (c_1,c_1,c_1,c_2),(c_2,c_2,c_1,c_1),(c_2,c_1,c_2,c_1)(c_2,c_1,c_1,c_2), \\ (c_1,c_2,c_2,c_1),(c_1,c_2,c_1,c_2),(c_1,c_1,c_2,c_2),(c_1,c_2,c_2,c_2), \\ (c_2,c_1,c_2,c_2),(c_2,c_2,c_1,c_2),(c_2,c_2,c_2,c_1),(c_2,c_2,c_2,c_2)\end{matrix}\right\}$

c)

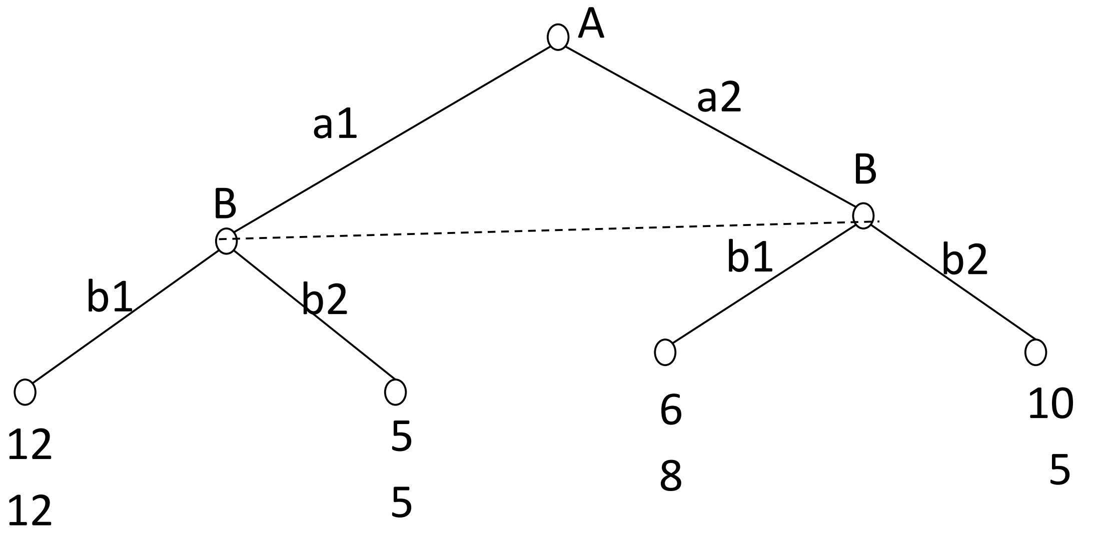
In the first step of backward induction we find the optimal action in each node of Player 3 (the last to move). Then the best response strategy of Player C is $s_c = (c1, c1, c2, c1)$ (from the left node to the right node). In the figure below player 3's optimal actions are marked by thicker branches.



The reduced game played by Player A and Player B is



This is a game of imperfect information with only one subgame. Then we use the normal form to find the Nash equilibria



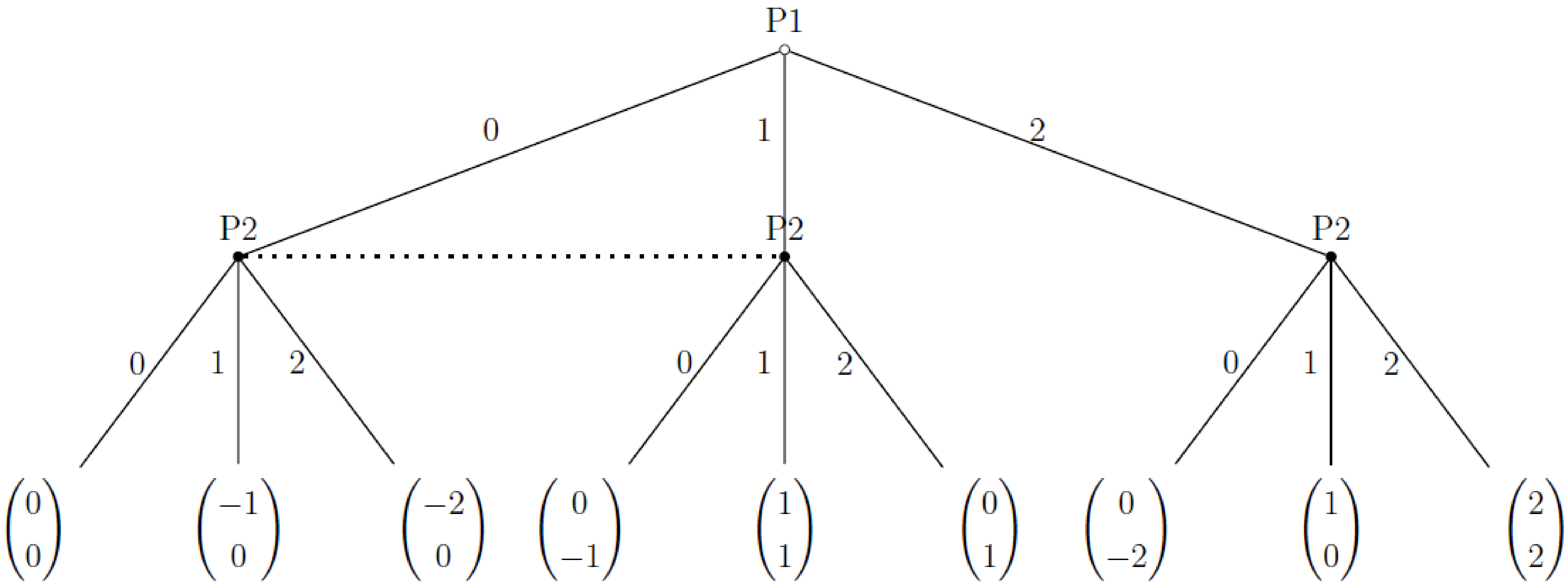
		Player B	
		b1	b2
Player A	a1	12, 12	5, 5
	a2	6, 8	10, 5

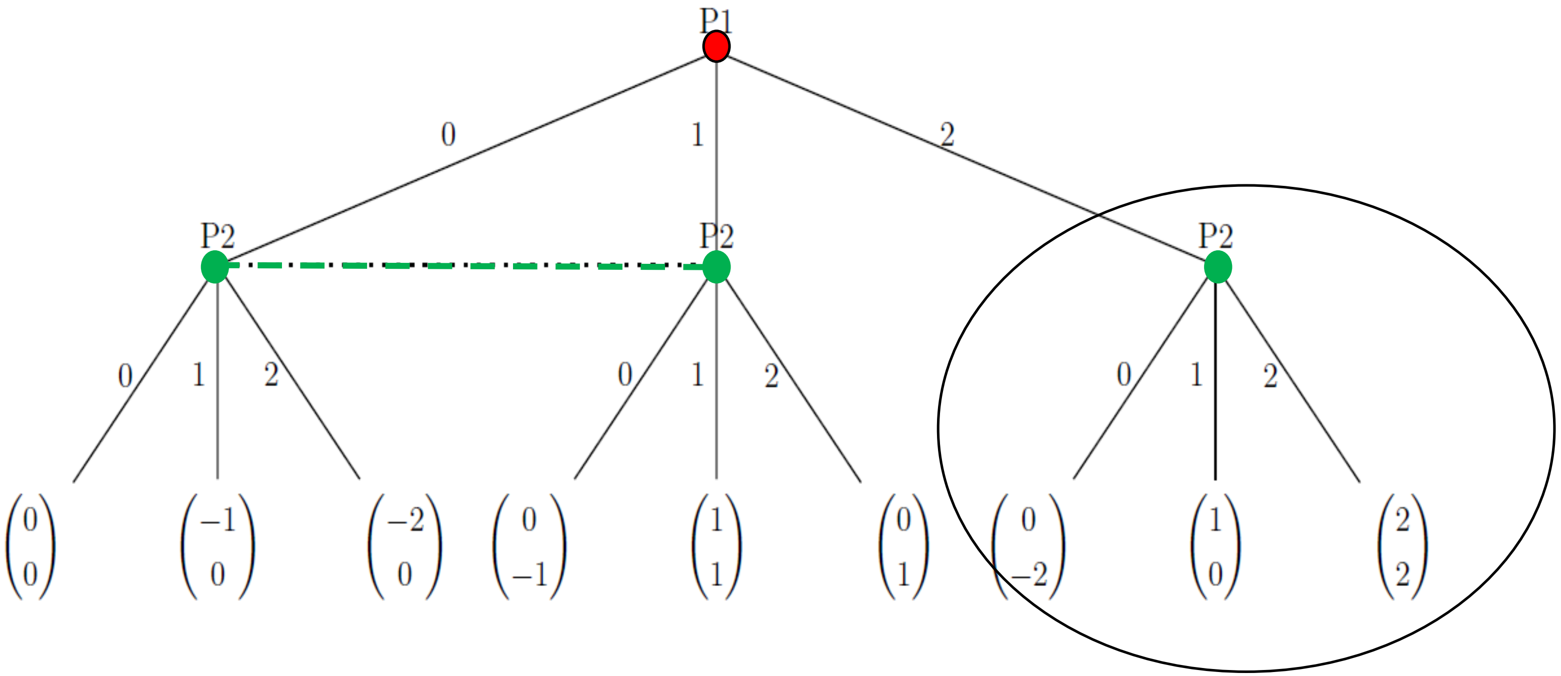
There is only one Nash equilibrium: Player A plays strategy $s_A = a1$ and Player B plays strategy $s_B = \ b1$.

Then we conclude that there is only one subgame perfect Nash equilibrium where:

Player A plays strategy $s_A = a1$, Player B plays strategy $s_B = \ b1$ and Player C plays strategy $s_c = (c1, c1, c2, c1)$

- **Ex. 4** There are 2 players that must state one number from the set {0, 1, 2). The payoff of each player is given by the stated number minus the absolute difference between his stated number and the number stated by the other player. Players move in a sequence: Player 1 moves first then player 2. When player 2 has to move he is only partially informed about the choice of player 1: he can see if player 1 chosen 2 but he cannot discriminate if player 1 chosen 0 or 1
- Represent this situation using the extensive form
- How many information sets and subgames has this game
- Describe all strategies of players 1 and 2
- using the normal form find all NE
- Find all Subgame Perfect Nash Equilibria


P1
0
1
2
P2
P2
P2
0
1
2
0
1
2
0
1
2
(0, 0)
(−1, 0)
(−2, 0)
(0, −1)
(1, 1)
(0, 1)
(0, −2)
(1, 0)
(2, 2)



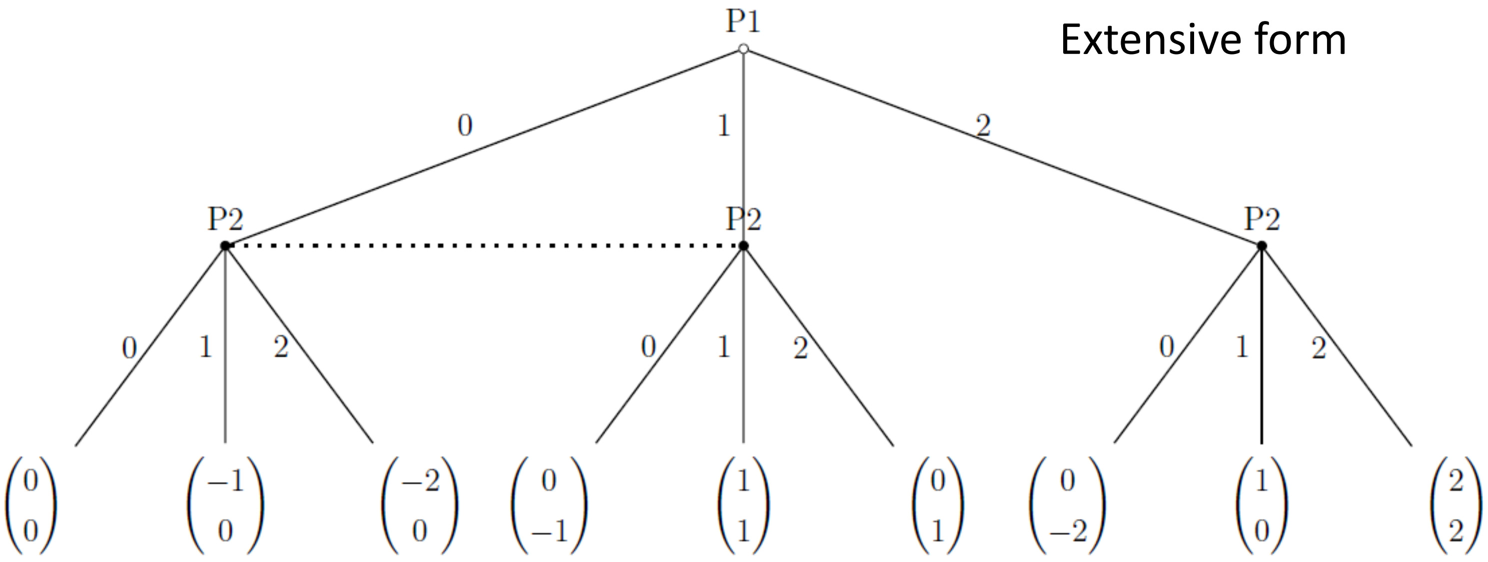
a) This game has three information set, one for player 1 (red) and two for player 2 (green). There are two subgames: the subgame starting after player 2's action (2) and the whole game.

b) The Player 1'set of strategy is $S_1 = \{0, 1, 2\}$

The Player 2'set of strategy is

$$S_2 = \{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2)\}$$

The normal form is



Extensive form

		P2								
		00	01	02	10	11	12	20	21	22
P1	0	$\underline{0}$, $\underline{0}$	0, $\underline{0}$	0, $\underline{0}$	-1, $\underline{0}$	-1, $\underline{0}$	-1, $\underline{0}$	-2, $\underline{0}$	-2, $\underline{0}$	-2, $\underline{0}$
	1	$\underline{0}$, -1	0, -1	0, -1	$\underline{1}$, $\underline{1}$	$\underline{1}$, $\underline{1}$	1, $\underline{1}$	$\underline{0}$, $\underline{1}$	0, $\underline{1}$	0, $\underline{1}$
	2	$\underline{0}$. -2	$\underline{1}$, 0	$\underline{2}$, $\underline{2}$	0. -2	$\underline{1}$, 0	$\underline{2}$, $\underline{2}$	$\underline{0}$. -2	$\underline{1}$, 0	$\underline{2}$, $\underline{2}$

Therefore, the 7 Nash equilibria in this game are:

$\{(0),(0;\ 0)\},,\{(1),(1;\ 0)\},\{(1),(1;\ 1)\},\{(1),(2;\ 0)\},\{(2),(0;\ 2)\ \},\{(2),(1;\ 2)\ \},\{(2),(2;\ 2)\}$

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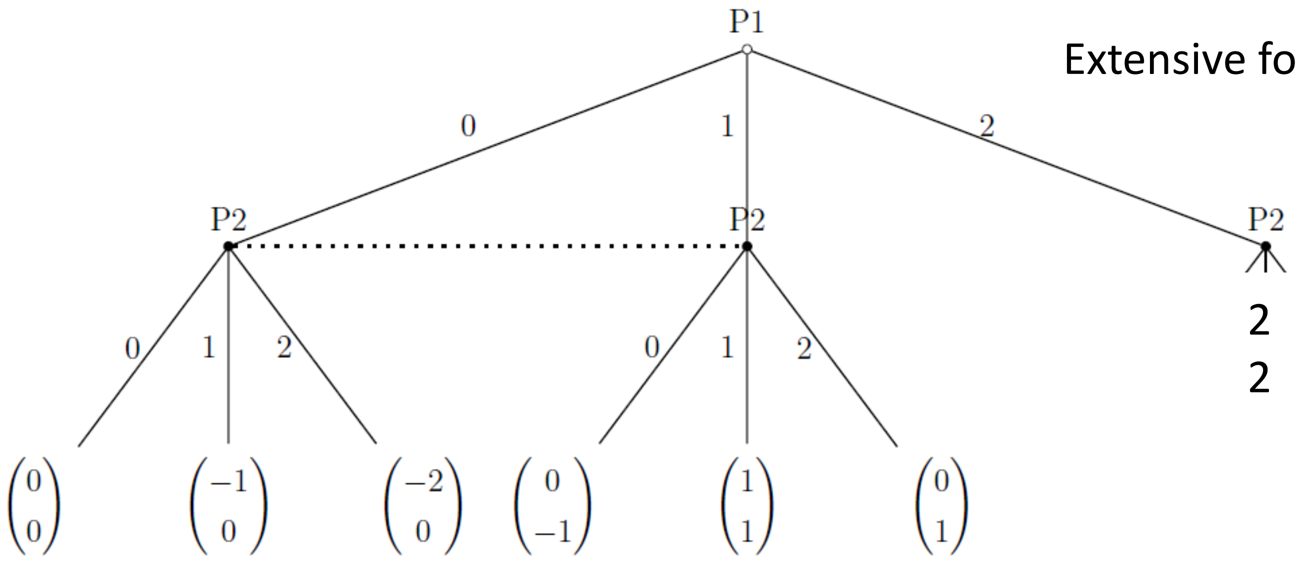
The optimal decision of P2 in the second info set is 2

Then the reduced extensive form is:

Extensive form



This is reduced game has only one subgame, so we need the normal form to find the Nash equilibria

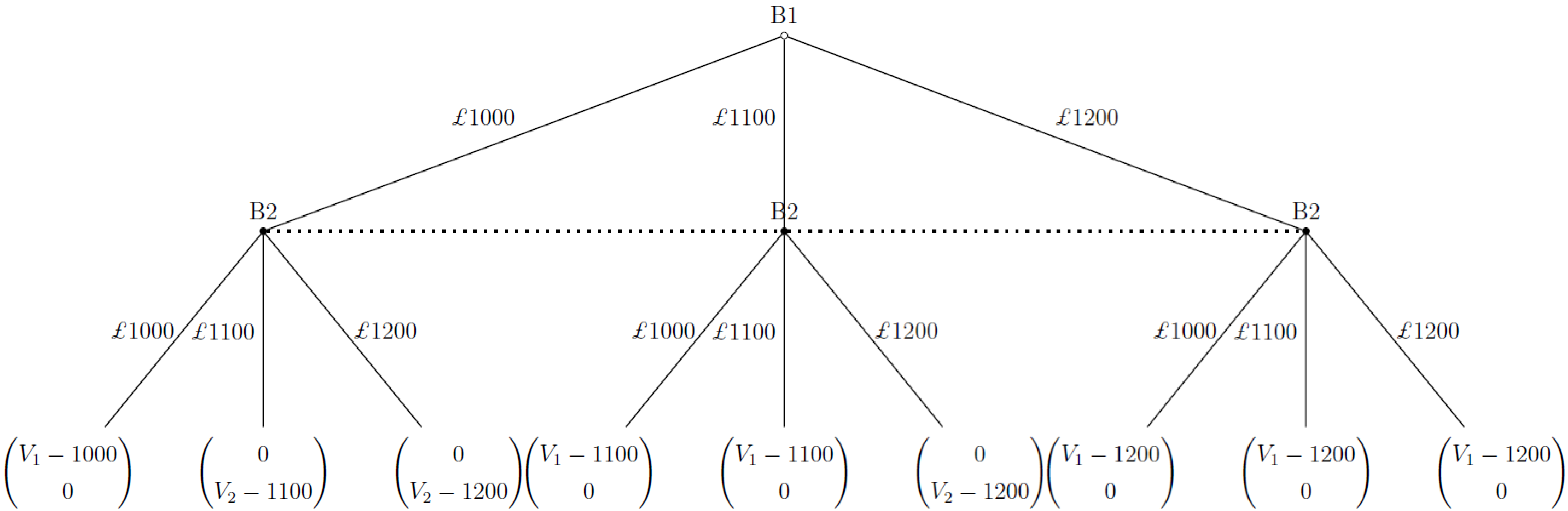
		P2		
		02	12	22
P1	0	0, 0	-1, 0	-2, 0
	1	0, -1	1, 1	0, 1
	2	2, 2	2, 2	2, 2

There are only 3 NE that represent the subgame perfect Nash equilibria

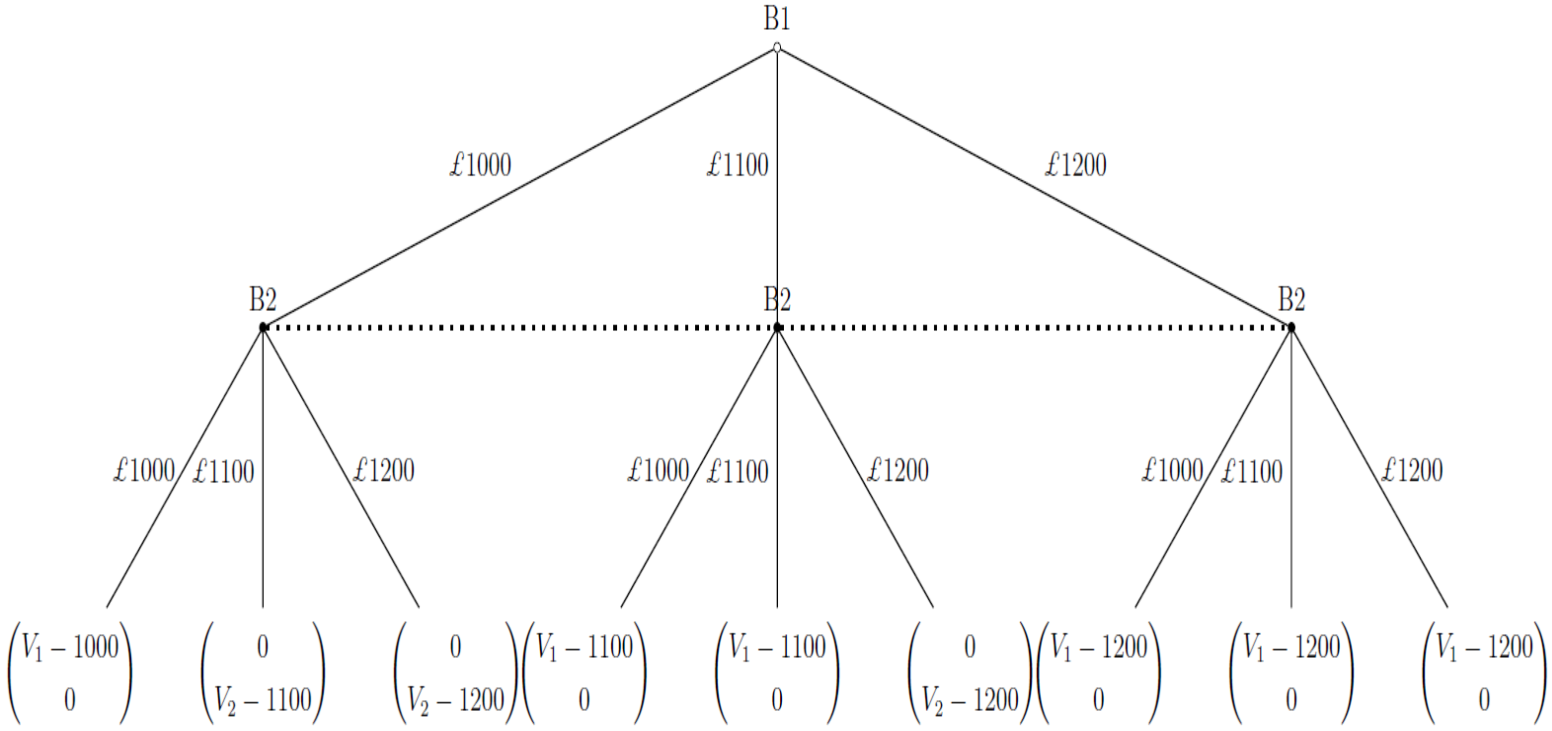
Therefore, the 3 Subgame perfect Nash equilibria in this game are:

$\{(2),(0;\ 2)\},\{(2),(1;\ 2)\},\{(2),(2;\ 2)\}.$

- **Ex. 5** An individual want to sell a car at a price no lower than £ 1.000. Two buyers, 1 and 2, simultaneously send to the car’s seller their offers. Car’s seller chooses to sell the car to the buyer that sent the best offer. If the two offers are equal, the car’s seller sells the car to buyer 1
    - Represent this situation using the extensive form assuming that buyers can send only three offers: 1000, 1100, 1200.
    - Represent the game in point a) using the normal form and find all NE.
- Find all Subgame Perfect Nash Equilibria



B1
£1000
£1100
£1200
B2
B2
B2
£1000
£1100
£1200
£1000
£1100
£1200
£1000
£1100
£1200
$\begin{pmatrix} V_1 - 1000 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ V_2 - 1100 \end{pmatrix}$
$\begin{pmatrix} 0 \\ V_2 - 1200 \end{pmatrix}$
$\begin{pmatrix} V_1 - 1100 \\ 0 \end{pmatrix}$
$\begin{pmatrix} V_1 - 1100 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ V_2 - 1200 \end{pmatrix}$
$\begin{pmatrix} V_1 - 1200 \\ 0 \end{pmatrix}$
$\begin{pmatrix} V_1 - 1200 \\ 0 \end{pmatrix}$
$\begin{pmatrix} V_1 - 1200 \\ 0 \end{pmatrix}$



		Buyer 2		
		£1000	£1100	£1200
	£1000	$\underline{V_1 - 1000}, 0$	$0, \underline{V_2 - 1100}$	$0, V_2 - 1200$
Buyer 1	£1100	$V_1 - 1100, 0$	$\underline{V_1 - 1100}, 0$	$0, \underline{V_2 - 1200}$
	£1200	$V_1 - 1200, \underline{0}$	$V_1 - 1200, \underline{0}$	$\underline{V_1 - 1200}, \underline{0}$

		Buyer 2		
		£1000	£1100	£1200
	£1000	$\underline{V_1 - 1000}, 0$	$0, \underline{V_2 - 1100}$	$0, V_2 - 1200$
Buyer 1	£1100	$V_1 - 1100, 0$	$\underline{V_1 - 1100}, 0$	$0, \underline{V_2 - 1200}$
	£1200	$V_1 - 1200, \underline{0}$	$V_1 - 1200, \underline{0}$	$\underline{V_1 - 1200}, \underline{0}$

There is only one Nash equilibrium: Buyer 1 offers 1200 and Buyer 2 offer 1200

c) Given that there is only one subgame, by definition the Nash equilibrium is subgame perfect