Lecture 7:
Partial equilibrium analysis

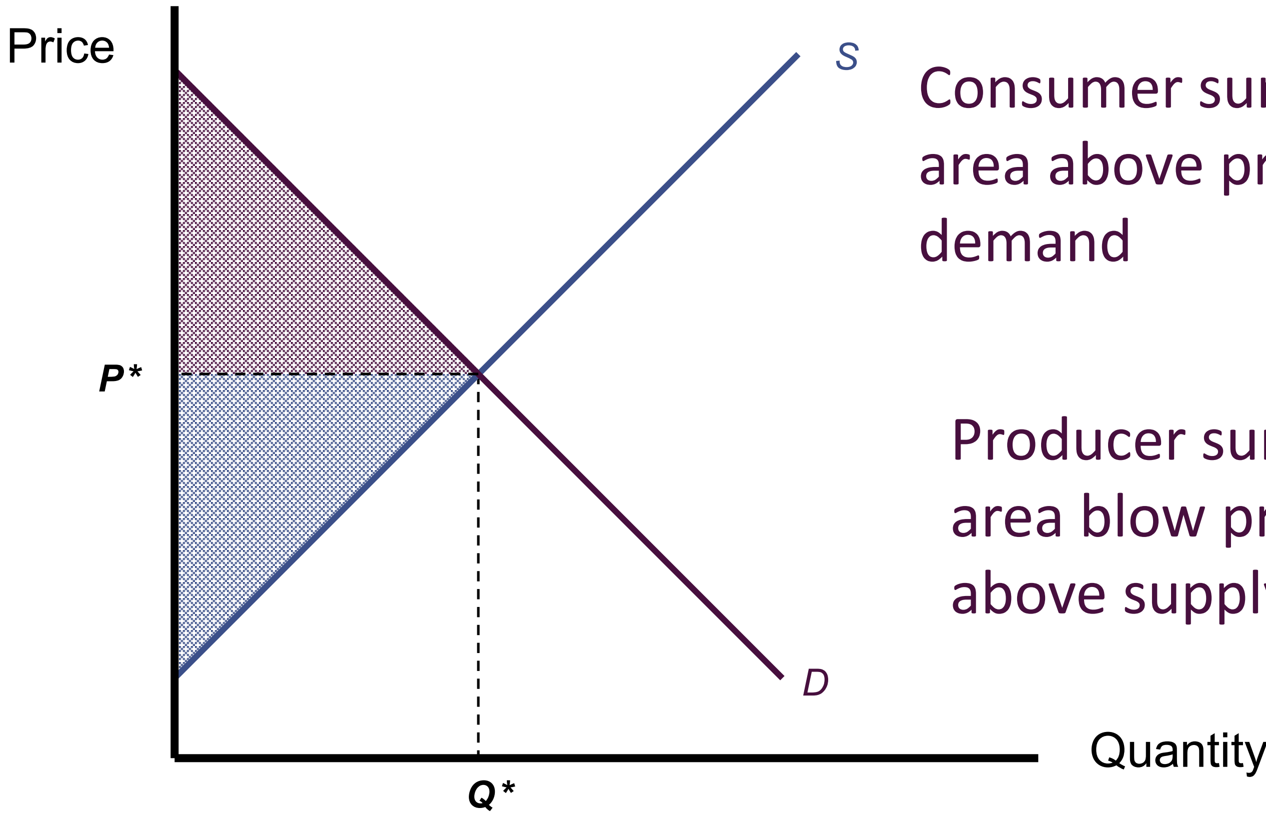
Long run competitive equilibria have the property of allocating resources *efficiently.*

Here we describe why this result holds in a context of partial equilibrium: i.e. we consider only one market in isolation.

Then we will analyze this result in a general equilibrium model where markets are interdependent each other

Economic Efficiency and Welfare Analysis

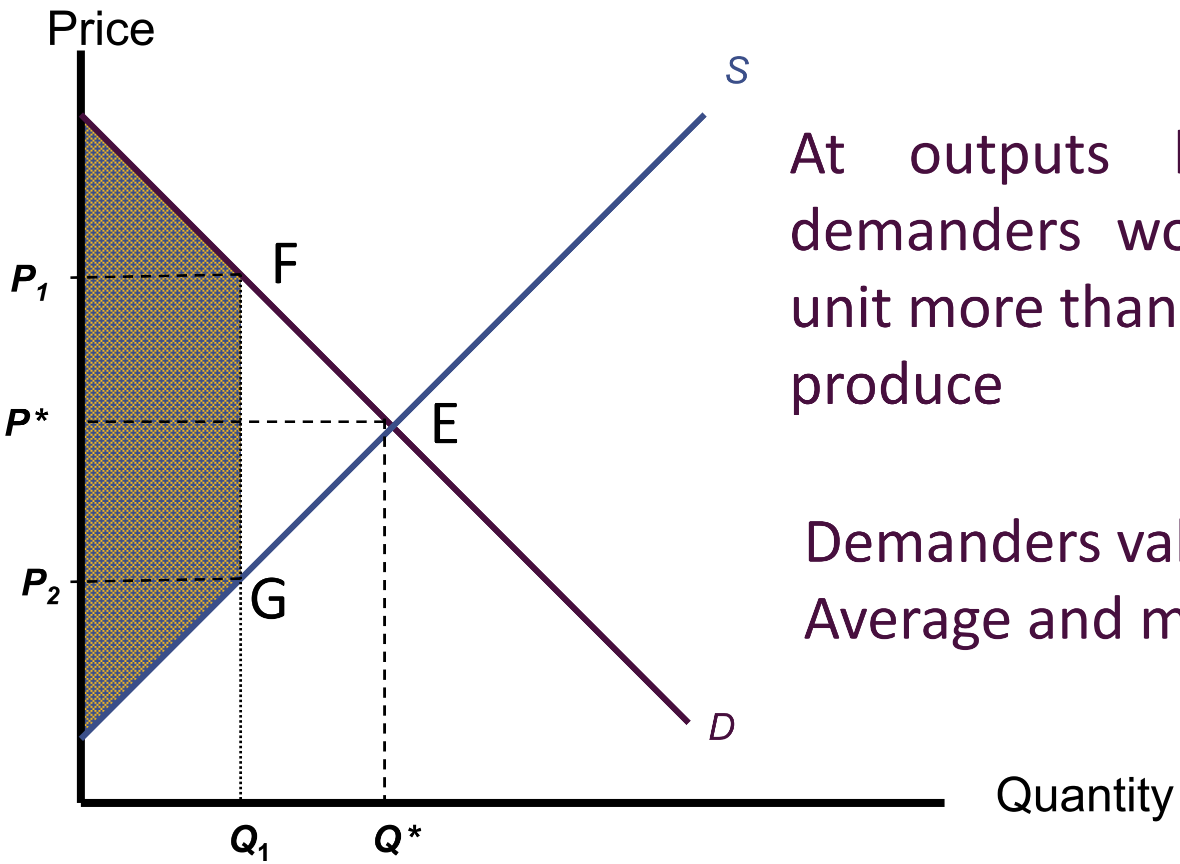
- The area between the demand and the supply curve represents the sum of consumer and producer surplus
- measures the total additional value obtained by market participants by being able to make market transactions
- This area is maximized at the competitive market equilibrium



Consumer surplus is the area above price and below demand

Producer surplus is the area blow price and above supply

For output levels less than Q^*, say Q_1, total surplus will be reduced



At outputs between Q_1 and Q^*, demanders would value an additional unit more than it would cost suppliers to produce

Demanders value P_1 additional units
Average and marginal costs are P_2

A mathematical proof

Mathematically, we wish to maximize

consumer surplus + producer surplus =

$$[U(Q) - PQ] + \left[PQ - \int_0^Q P(Q)dQ\right] = U(Q) - \int_0^Q P(Q)dQ$$

In long-run equilibria along the long-run supply curve,

$$P(Q) = AC = MC$$

Maximizing total surplus with respect to *Q* yields

$$U'(Q) = P(Q) = AC = MC$$

maximization occurs where the marginal value of *Q* to the representative consumer is equal to market price

This is the supply-demand equilibrium because the demand represents the consumer marginal utility whereas the supply curve represent the marginal cost.

Example 1: Welfare Loss Computations

Use of consumer and producer surplus notions makes possible the explicit calculation of welfare losses caused by restrictions on voluntary transactions

In the case of linear demand and supply curves, the calculation is simple because the areas of loss are often triangular

Suppose that the demand is given by

$$Q_D = 10 - P$$

and supply is given by

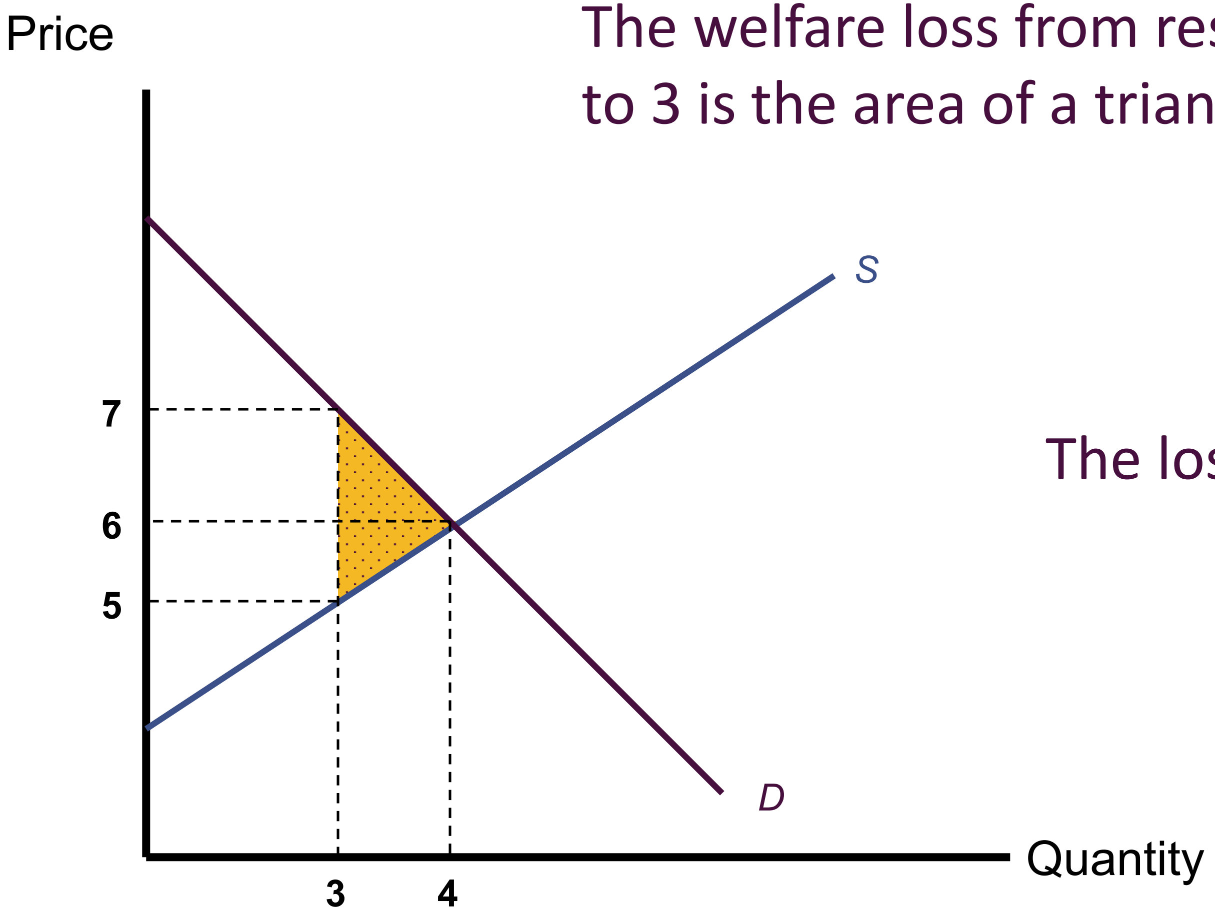
$$Q_S = P - 2$$

Market equilibrium occurs where $P^* = 6$ and $Q^* = 4$

Restriction of output to $Q_0 = 3$ would create a gap between what demanders are willing to pay (P_D) and what suppliers require (P_S)

$$P_D = 10 - 3 = 7$$
$$P_S = 2 + 3 = 5$$

The welfare loss from restricting output to 3 is the area of a triangle



The loss = (0.5)(2)(1) = 1

The welfare loss will be shared by producers and consumers

In general, it will depend on the price elasticity of demand and the price elasticity of supply to determine who bears the larger portion of the loss

- the side of the market with the smallest price elasticity (in absolute value)

Example 2: Welfare Loss Computations no linear curves

Suppose that the inverse demand is given by

$$P_D = 10Q^{-0.5}$$

and supply is given by

$$P_S = 0.1\,Q^{1.5}$$

Market equilibrium occurs where $P^* = 3.16$ and $Q^* = 10$

Restriction of output to Q_0 = 8 would create a gap between what demanders are willing to pay (P_D) and what suppliers require (P_S)

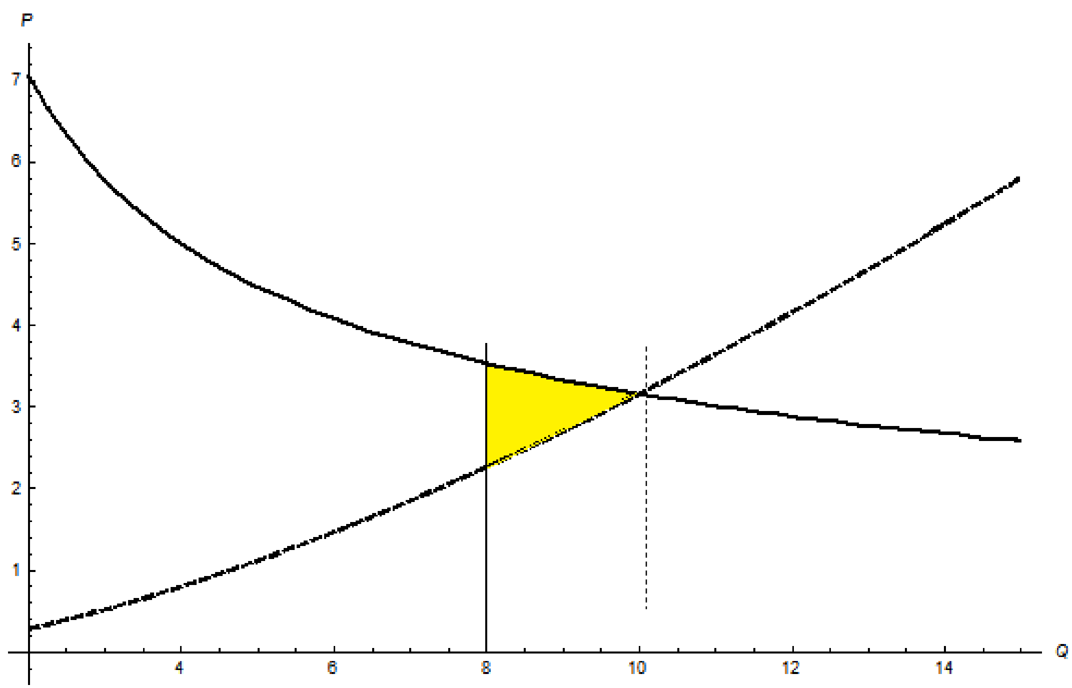
- $P_D = 10\;8^{-0.5} = 3.54$
- $P_S = 0.1\;8^{1.5} = 2.26$

The welfare loss is the yellow area between inverse demand and supply:

$$\int_8^{10} 10Q^{-0.5} - 0.1\,Q^{1.5} dQ =$$

$$= \left| 20\sqrt{Q} - \frac{Q^{2.5}}{25} \right|_8^{10} =$$

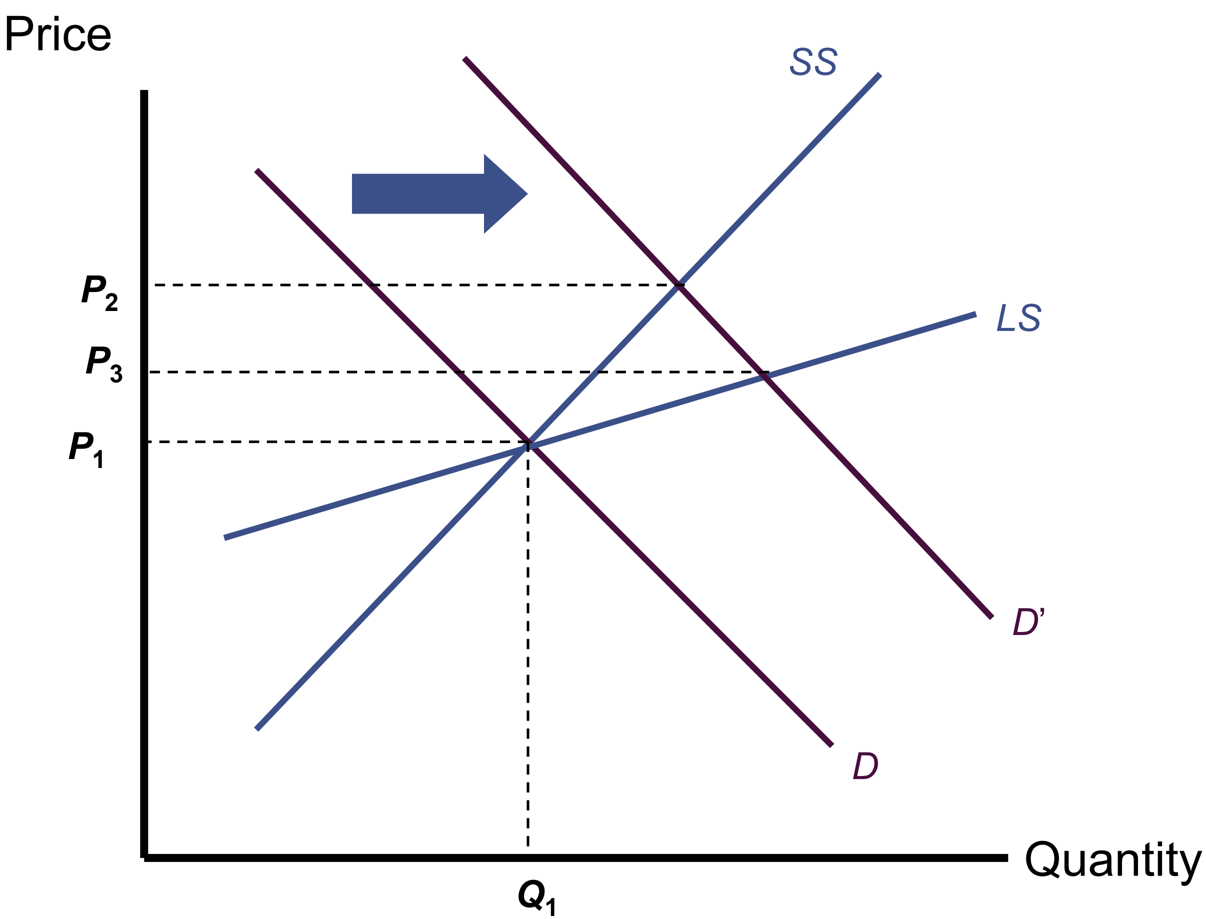
$$20\sqrt{10} - \frac{10^{2.5}}{25} - 20\sqrt{8} + \frac{8^{2.5}}{25} = 1.27$$



Price Controls and Shortages

Sometimes governments may seek to control prices at below equilibrium levels: this will lead to a shortage

We can look at the changes in producer and consumer surplus from this policy to analyze its impact on welfare



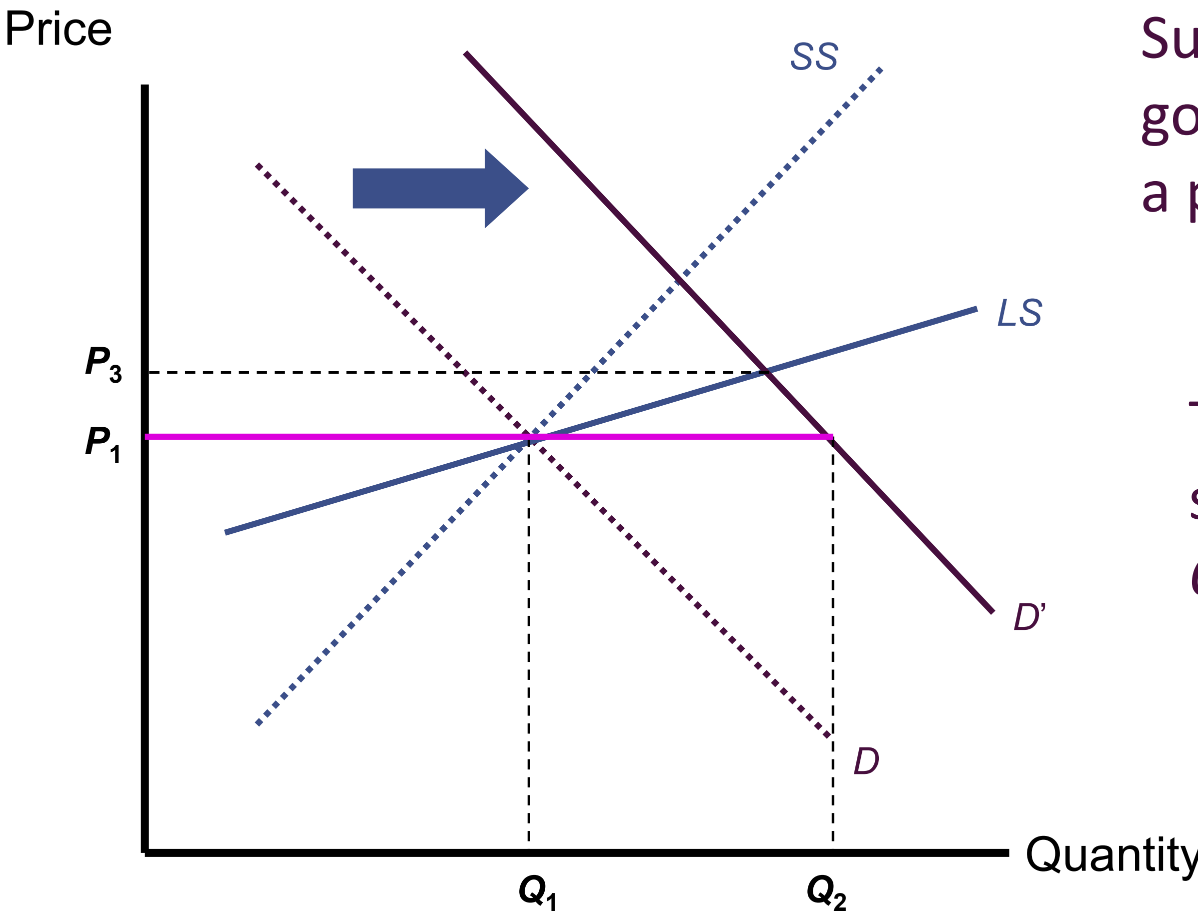
Initially, the market is in long-run equilibrium at P_1, Q_1

Demand increases to D'

In the short run, price rises to P_2

Firms would begin to enter the industry

The price would end up at P_3



Suppose that the government imposes a price ceiling at P_1

There will be a shortage equal to $Q_2 - Q_1$

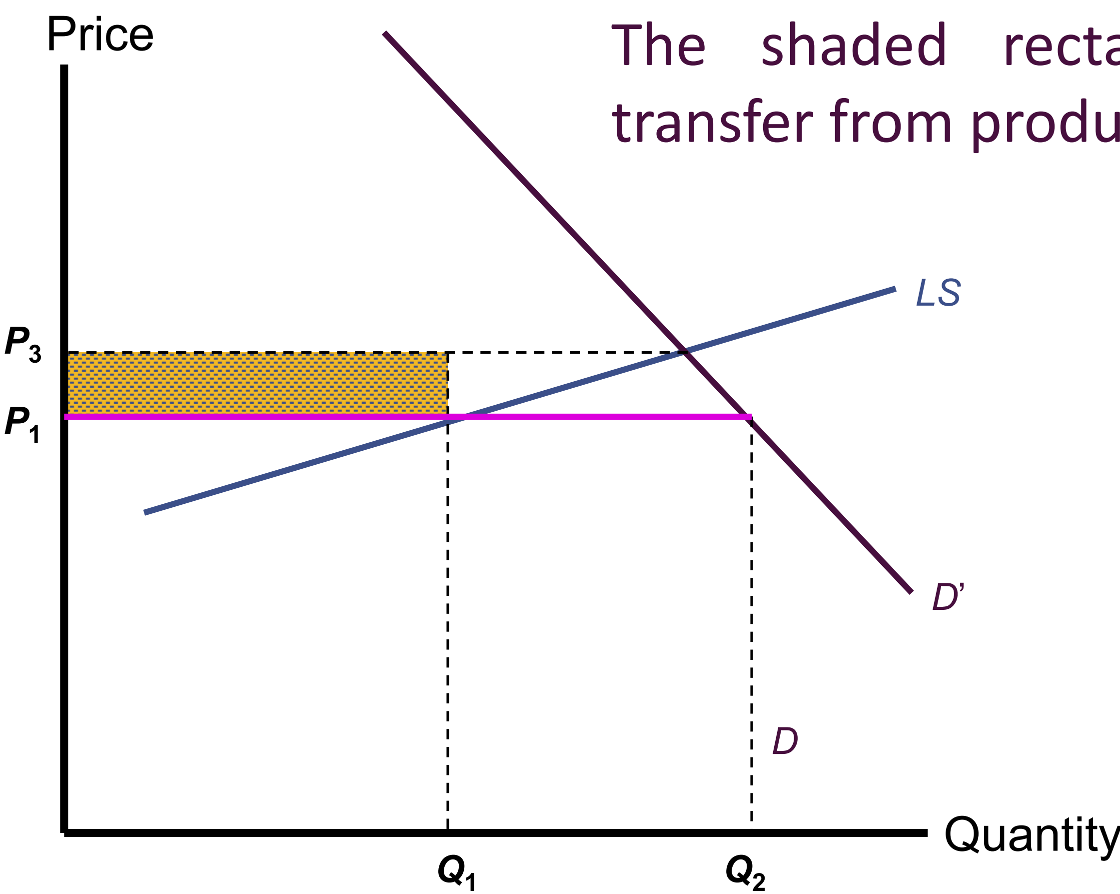
Some buyers will gain because they can purchase the good for a lower price

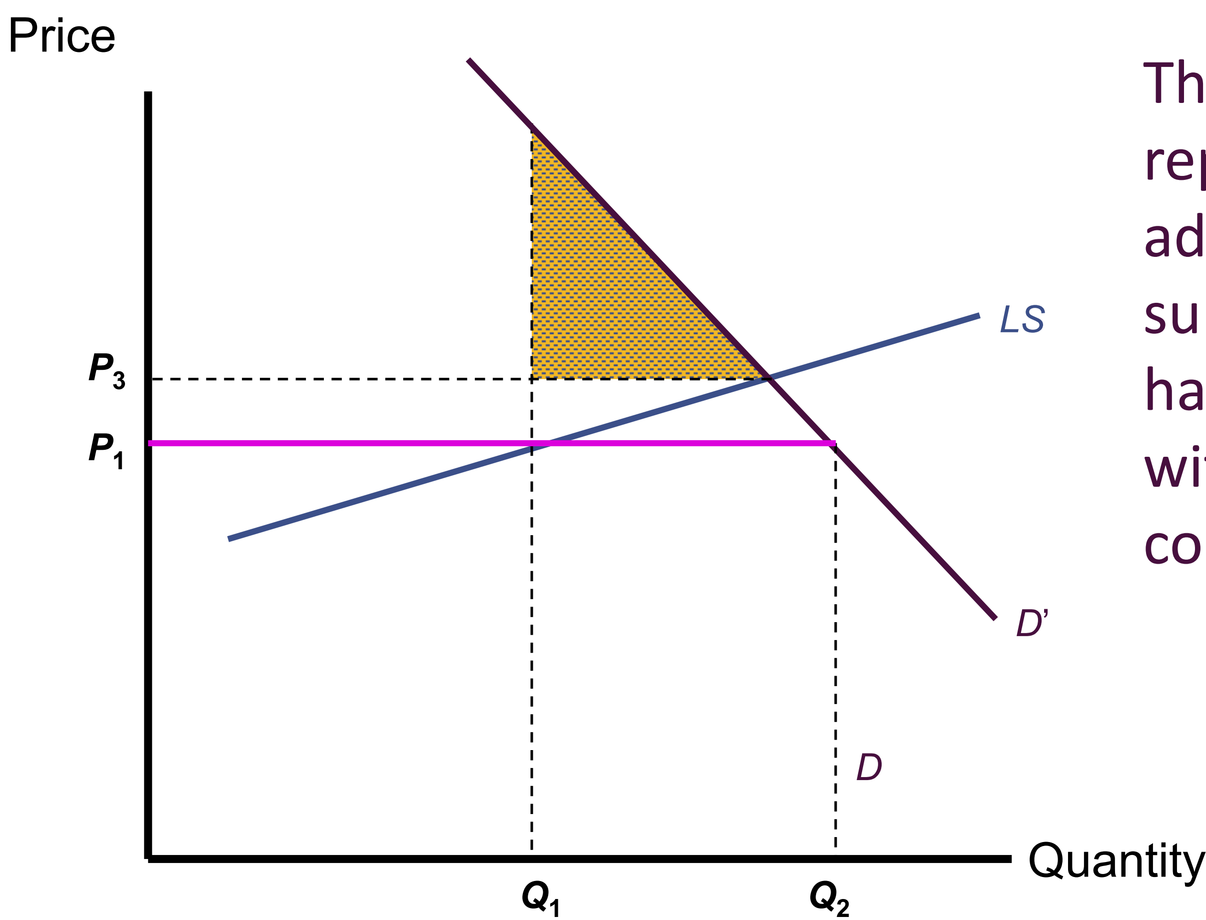
This gain in consumer surplus is the shaded rectangle

The gain to consumers is also a loss to producers who now receive a lower price

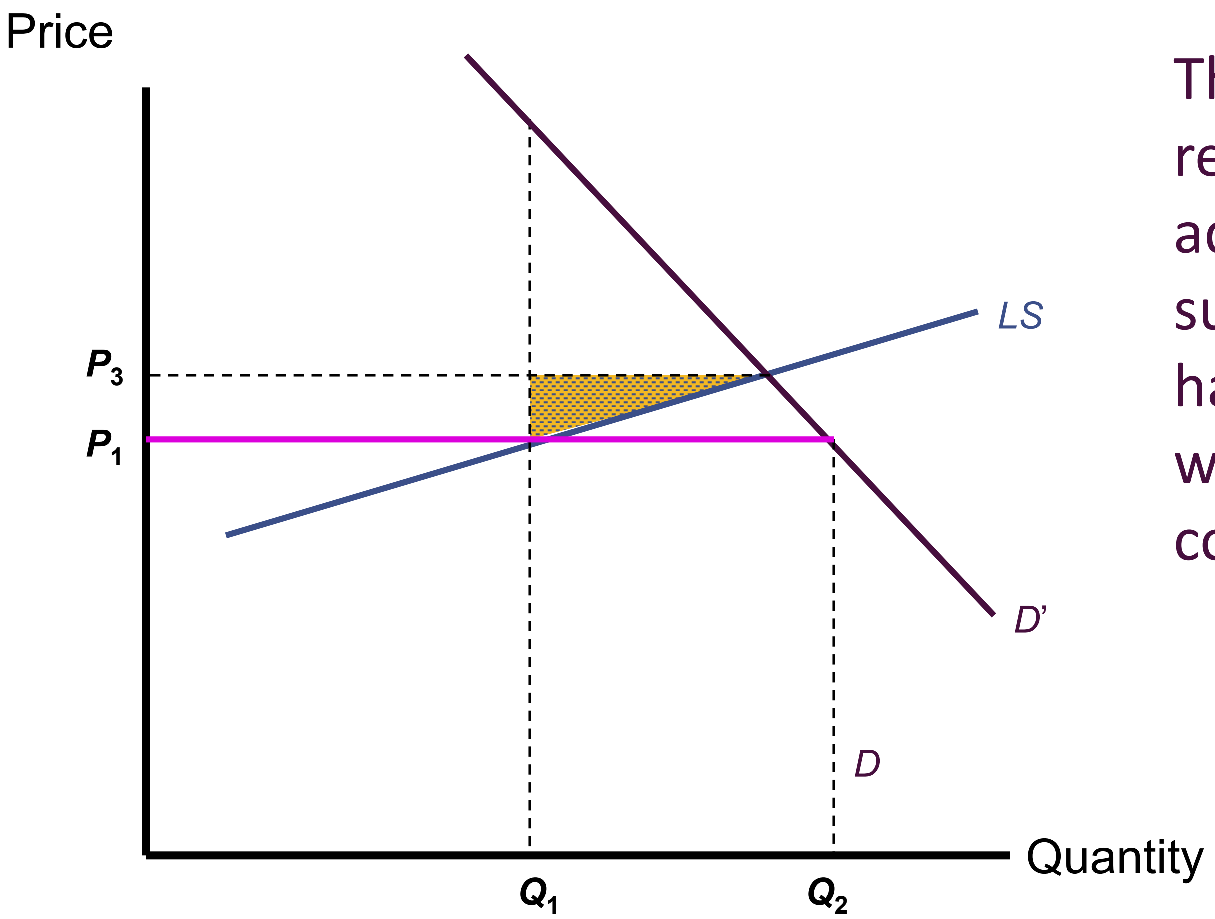
The shaded rectangle represents a pure transfer from producers to consumers

No welfare loss there

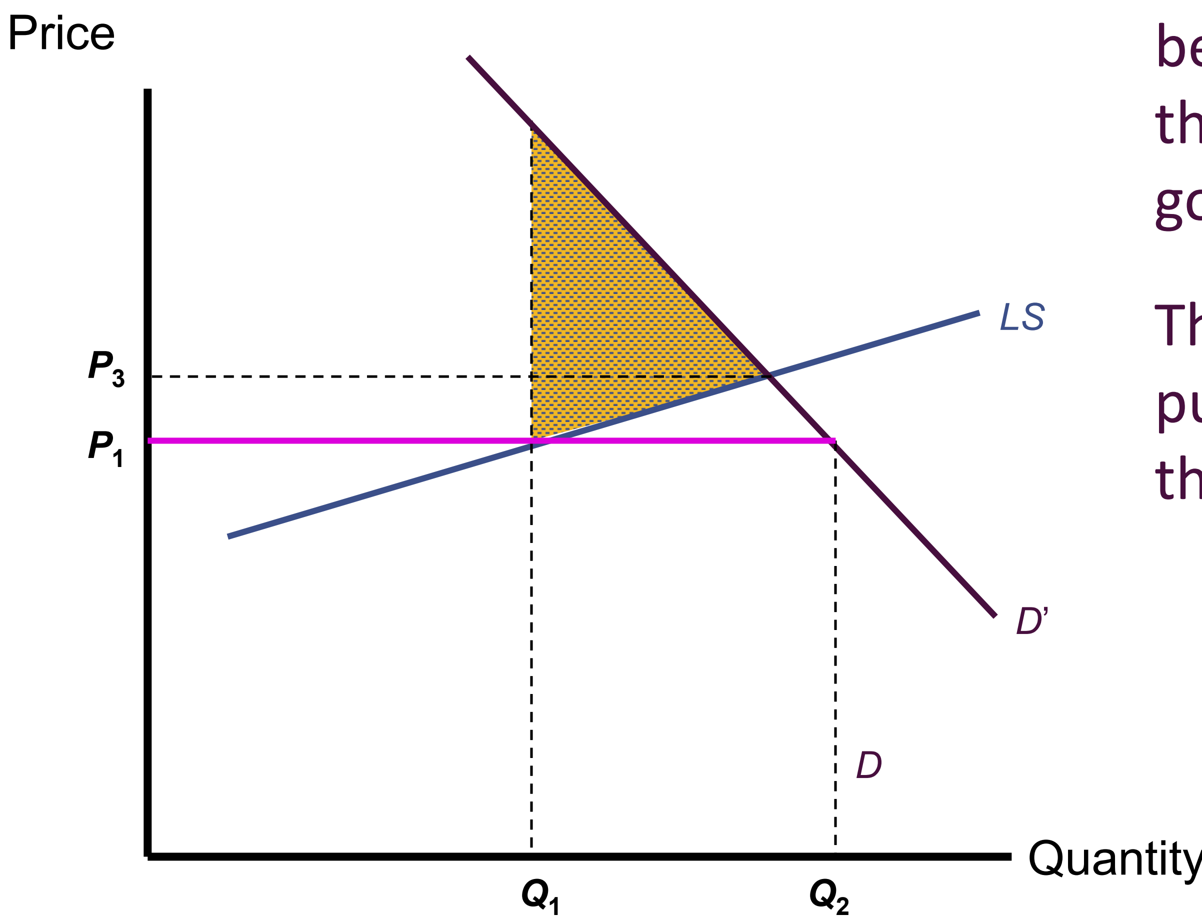




This shaded triangle represents the value of additional consumer surplus that would have been attained without the price control



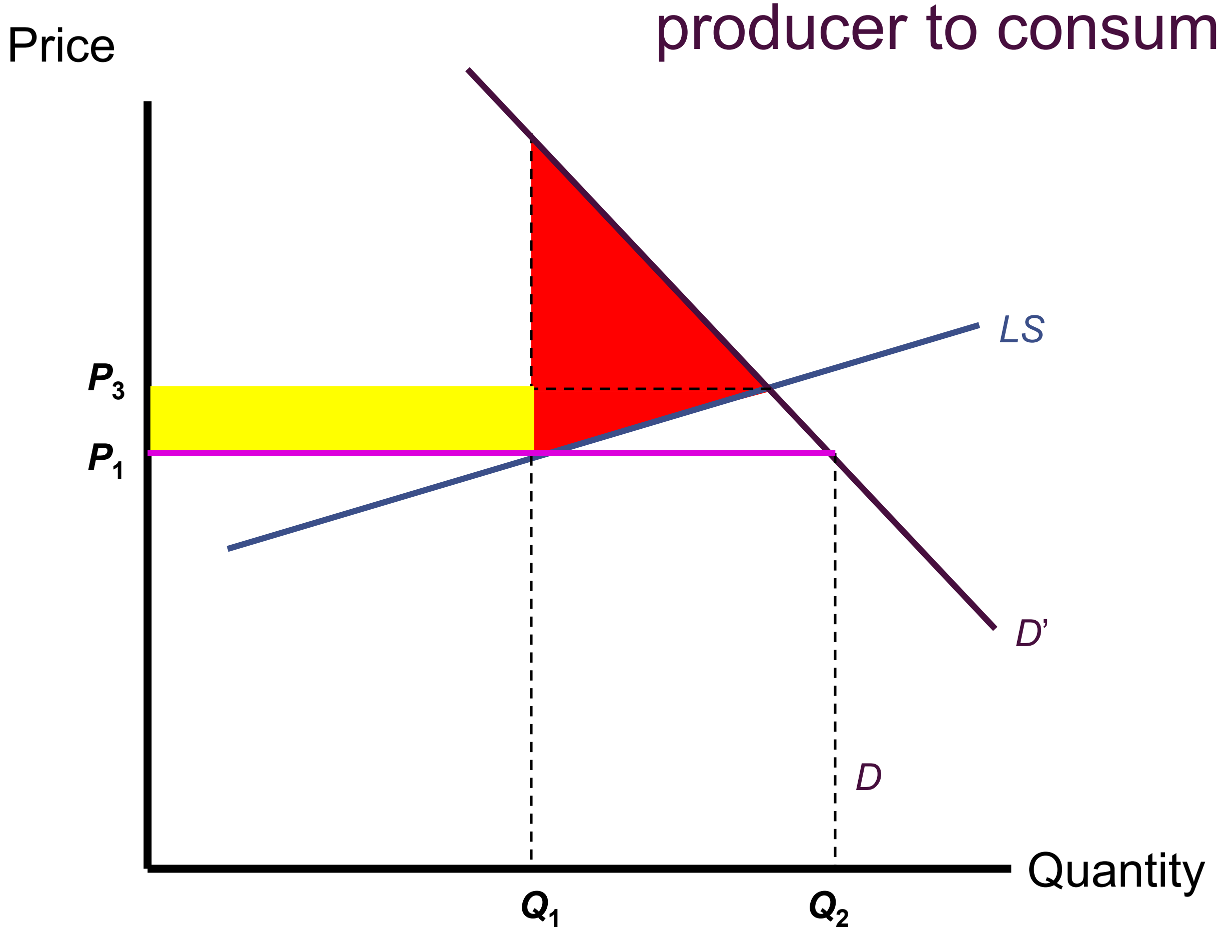
This shaded triangle represents the value of additional producer surplus that would have been attained without the price control



This shaded area represents the total value of mutually beneficial transactions that are prevented by the government

This is a measure of the pure welfare costs of this policy

The red area is the dead weight loss
The yellow area is the transfer from producer to consumers



Tax Incidence

To discuss the effects of a per-unit tax (t), we need to make a distinction between the price paid by buyers (P_D) and the price received by sellers (P_S)

$$P_D - P_S = t$$

Maintenance of equilibrium in the market requires

$$D(P_D) = S(P_S)$$

$$D(P_D) = S(P_D - t)$$

Differentiation with respect to t, yields:

$$D_P \frac{dP_D}{dt} = S_P \frac{dP_D}{dt} - S_P$$

where D_P denotes the derivative of $D(P_D)$ with respect to P_D and S_P denotes the derivative of $S(P_D - t)$ with respect to P_D

We can now solve for the effect of the tax on P_D:

$$\frac{dP_D}{dt} = \frac{S_P}{S_P - D_P}$$

Multiplying the numerator and the denominator of left hand side by $\frac{P_D}{S}$ and using the condition $D = S$ we can write

$$\frac{dP_D}{dt} = \frac{e_S}{e_S - e_D}$$

where e_S and e_D are, respectively, supply and demand elasticities

Starting from

$$D(P_s + t) = S(P_s)$$

we can get

$$\frac{dP_S}{dt} = \frac{D_P}{S_P - D_P} = \frac{e_D}{e_S - e_D}$$

From the previous computation we have

$\frac{dP_D}{dt} = \frac{e_S}{e_S - e_D}$ and $\frac{dP_S}{dt} = \frac{e_D}{e_S - e_D}$

Because $e_D \leq 0$ and $e_S \geq 0$, $dP_D/dt \geq 0$ and $dP_S/dt \leq 0$

If demand is perfectly inelastic ($e_D = 0$), the per-unit tax is completely paid by demanders

$$\frac{dP_D}{t} = \frac{e_S}{e_S - e_D} = 1$$

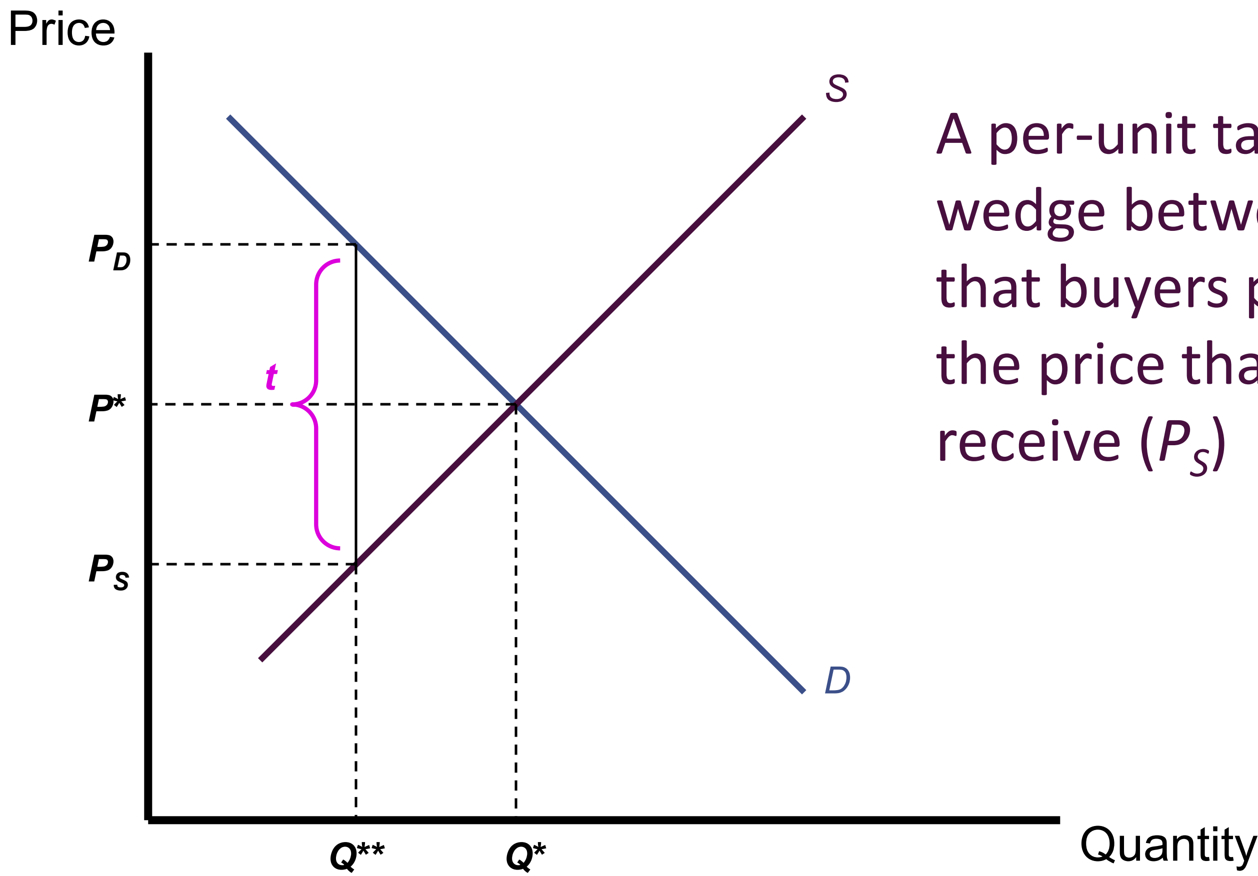
If demand is perfectly elastic ($e_D = -\infty$), the per-unit tax is completely paid by suppliers

$$\frac{dP_S}{t} = \frac{e_D}{e_S - e_D} = -1$$

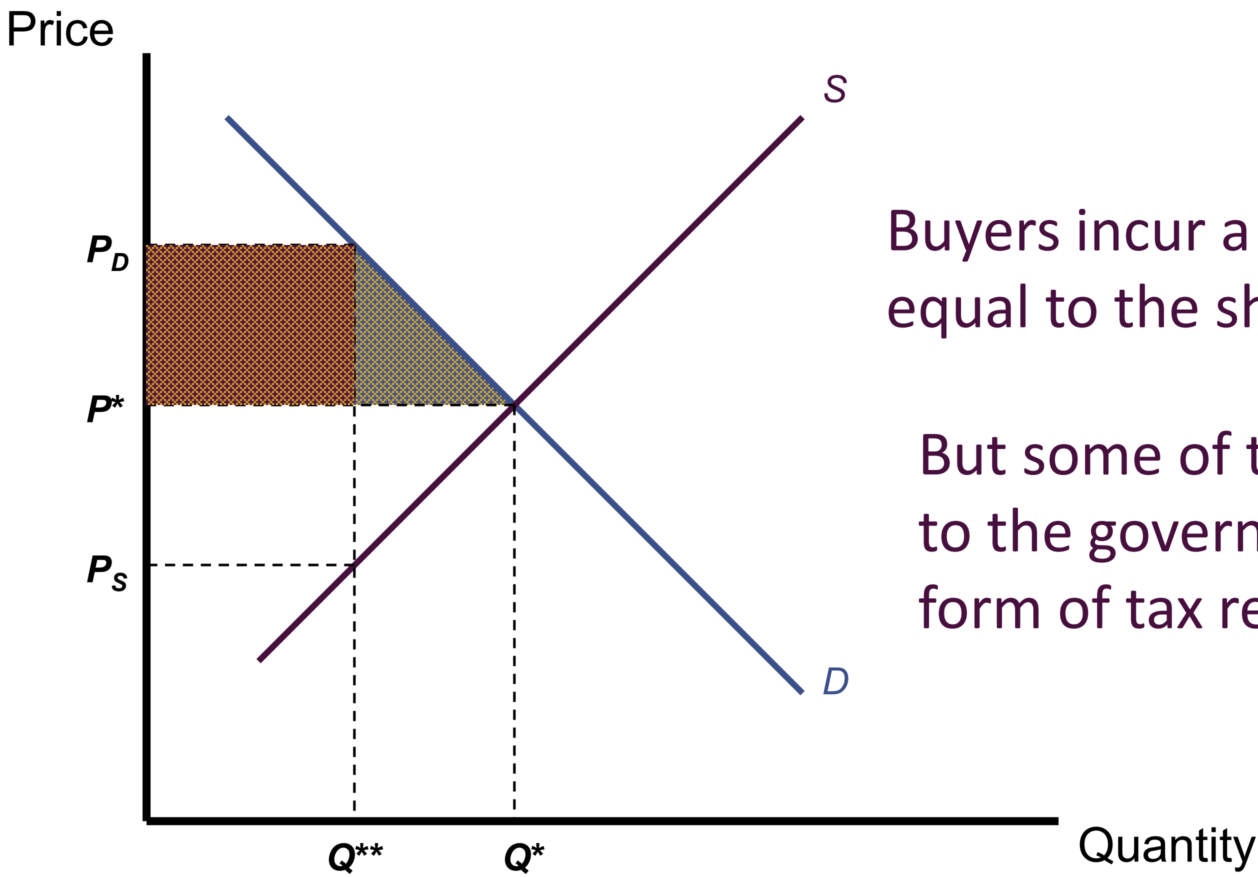
In general, the actor with the less elastic responses (in absolute value) will experience most of the price change caused by the tax

$$\left|\frac{\frac{dP_S}{dt}}{\frac{dP_D}{dt}}\right| = \left|\frac{e_D}{e_S}\right|$$

Welfare analysis: effect of a tax on welfare

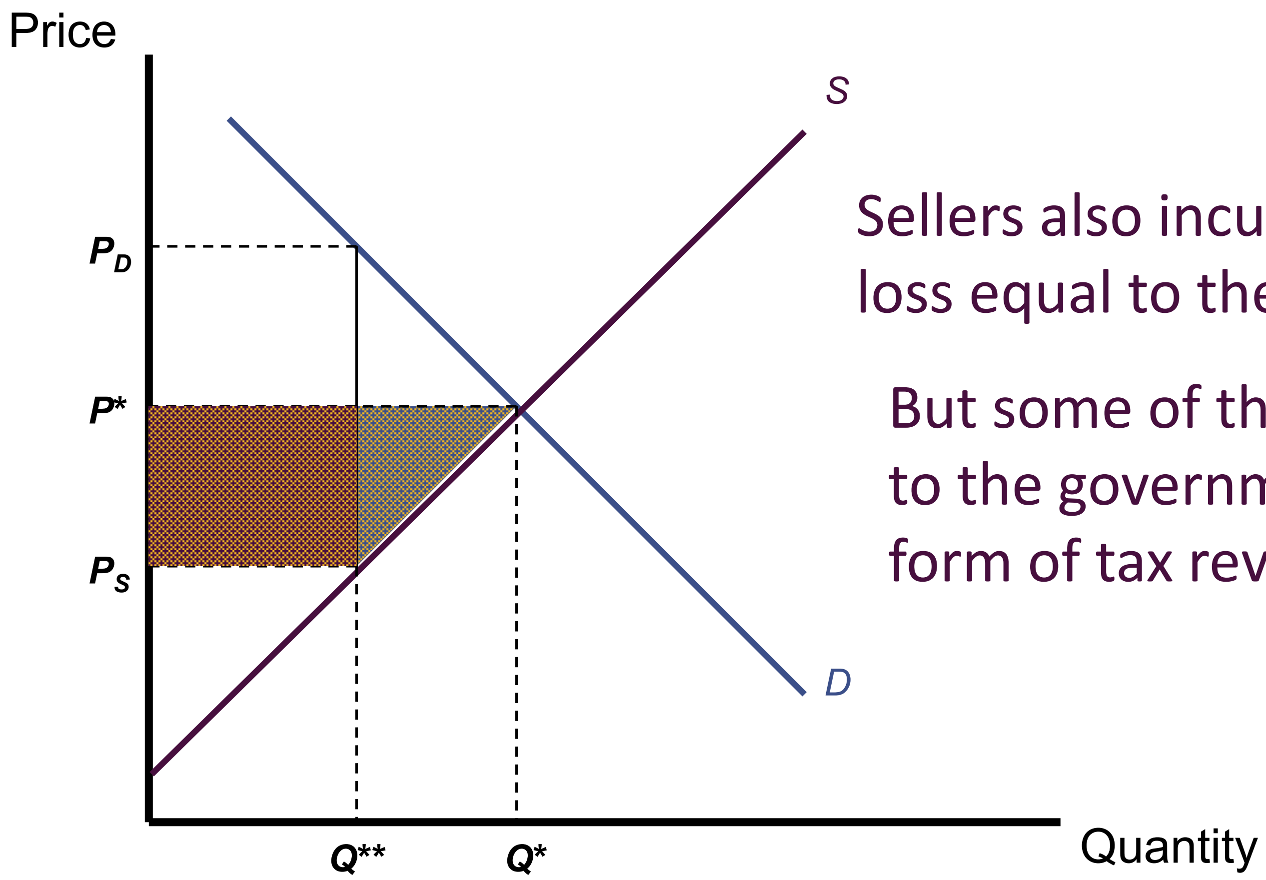


A per-unit tax creates a wedge between the price that buyers pay (P_D) and the price that sellers receive (P_S)



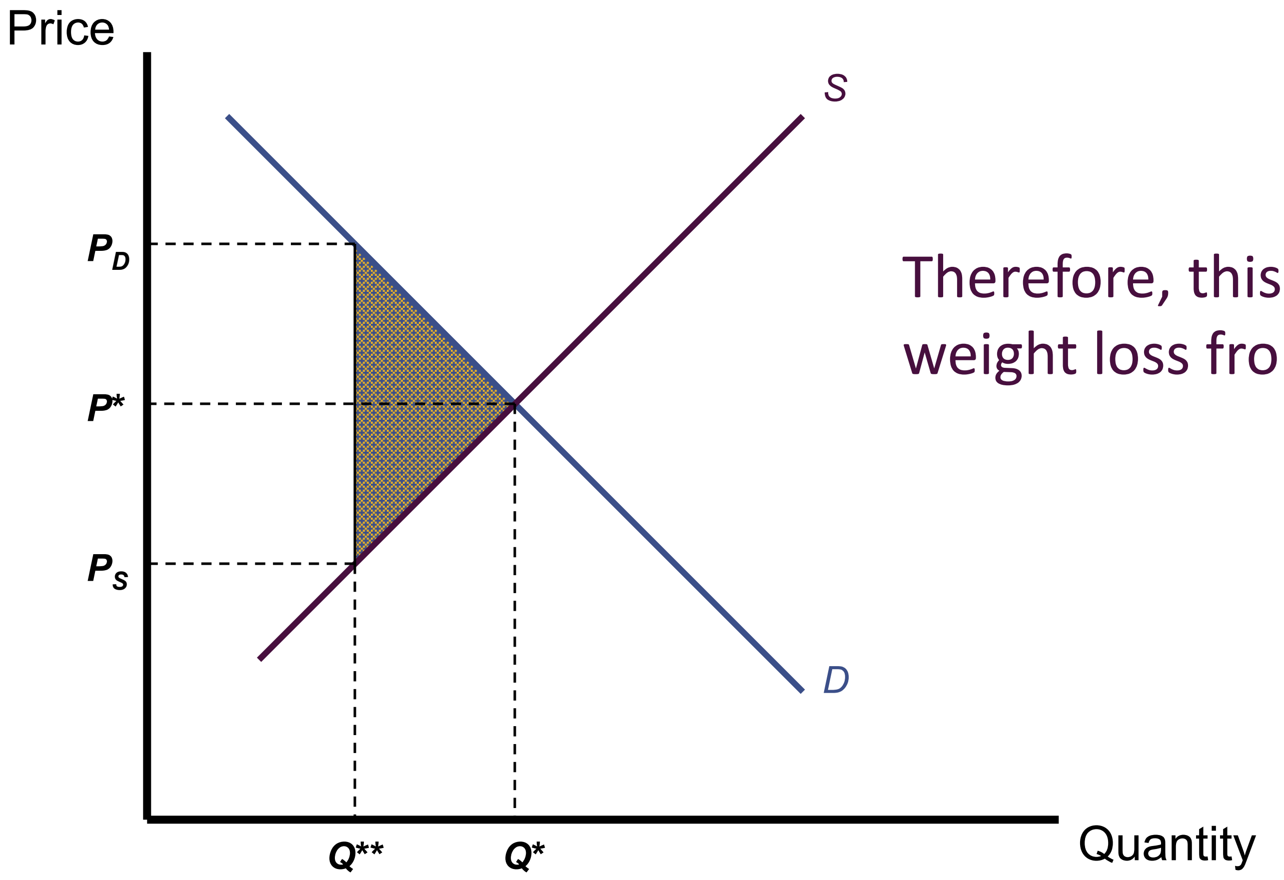
Buyers incur a welfare loss equal to the shaded area

But some of this loss goes to the government in the form of tax revenue



Sellers also incur a welfare loss equal to the shaded area

But some of this loss goes to the government in the form of tax revenue



Therefore, this is the dead-weight loss from the tax

Deadweight Loss and Elasticity

All nonlump-sum taxes involve deadweight losses

the size of the losses will depend on the elasticities of supply and demand

A linear approximation to the deadweight loss accompanying a small tax, *dt*, is given by

$$DW = -0.5\, t\, \frac{dQ}{dt} t = -0.5\, t^2 \frac{dQ}{dt}$$

From the definition of elasticity, we know that

$$\frac{dQ}{dt} = e_D \frac{dP}{dt} \frac{Q_0}{P_0}$$

This implies that

$$DW = -0.5\, t^2 \frac{e_D e_S}{e_S - e_D} \frac{Q_0}{P_0} = -0.5 \left(\frac{t}{P_0}\right)^2 \frac{e_D e_S}{e_S - e_D} P_0 Q_0$$

$$DW = -0.5\, t^2 \frac{e_D e_S}{e_S - e_D} \frac{Q_0}{P_0} = -0.5 \left(\frac{t}{P_0}\right)^2 \frac{e_D e_S}{e_S - e_D} P_0 Q_0$$

Deadweight losses are zero if either e_D or e_S are zero

the tax does not alter the quantity of the good that is traded

Deadweight losses are smaller in situations where e_D or e_S are small

Note: these are approximations that allow us to have some intuition.

The exact DW is computed as the areas. It implies the use of the integrals when curves are not linear

Transactions Costs

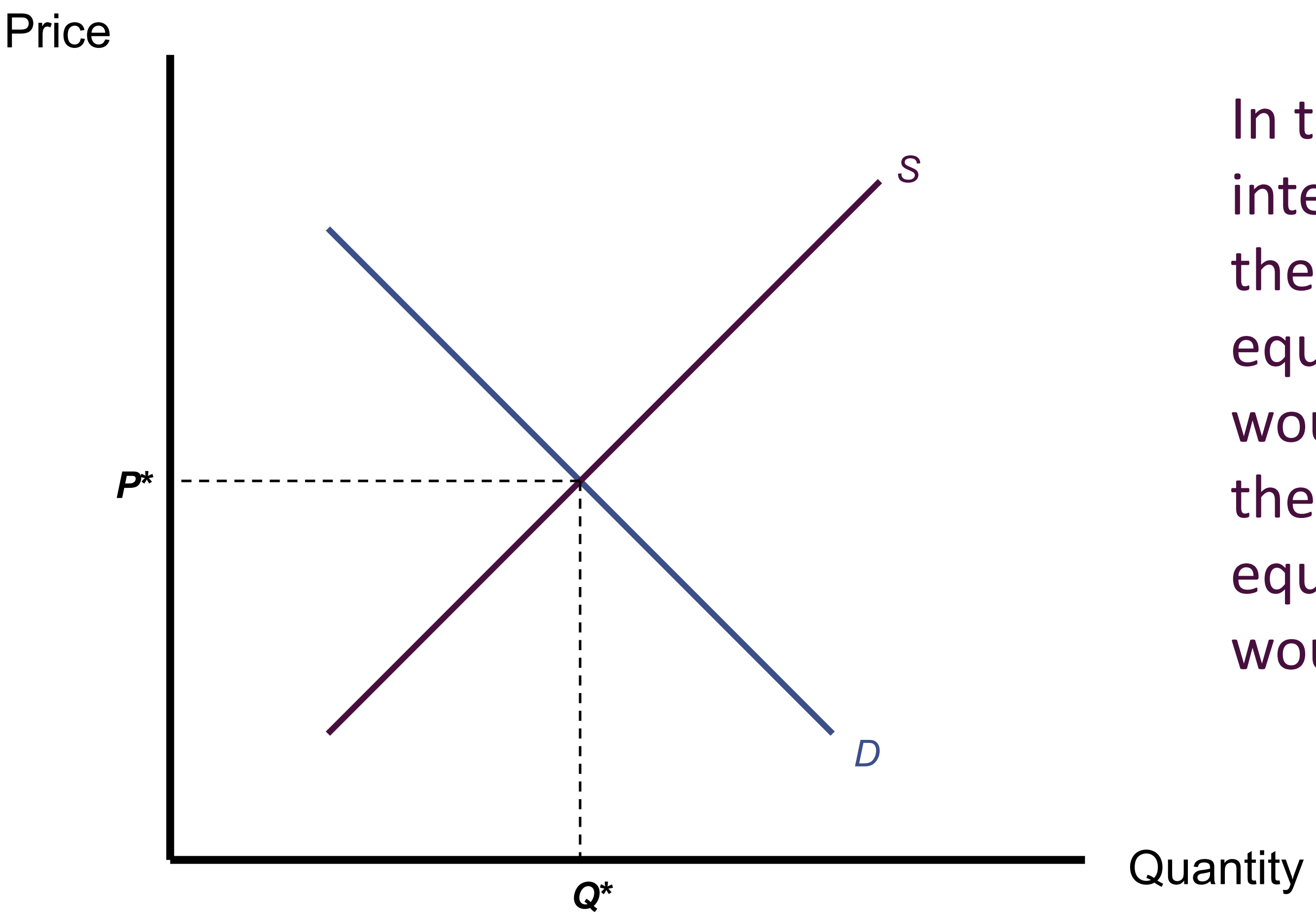
Transactions costs can also create a wedge between the price the buyer pays and the price the seller receives

- real estate agent fees
- broker fees for the sale of stocks

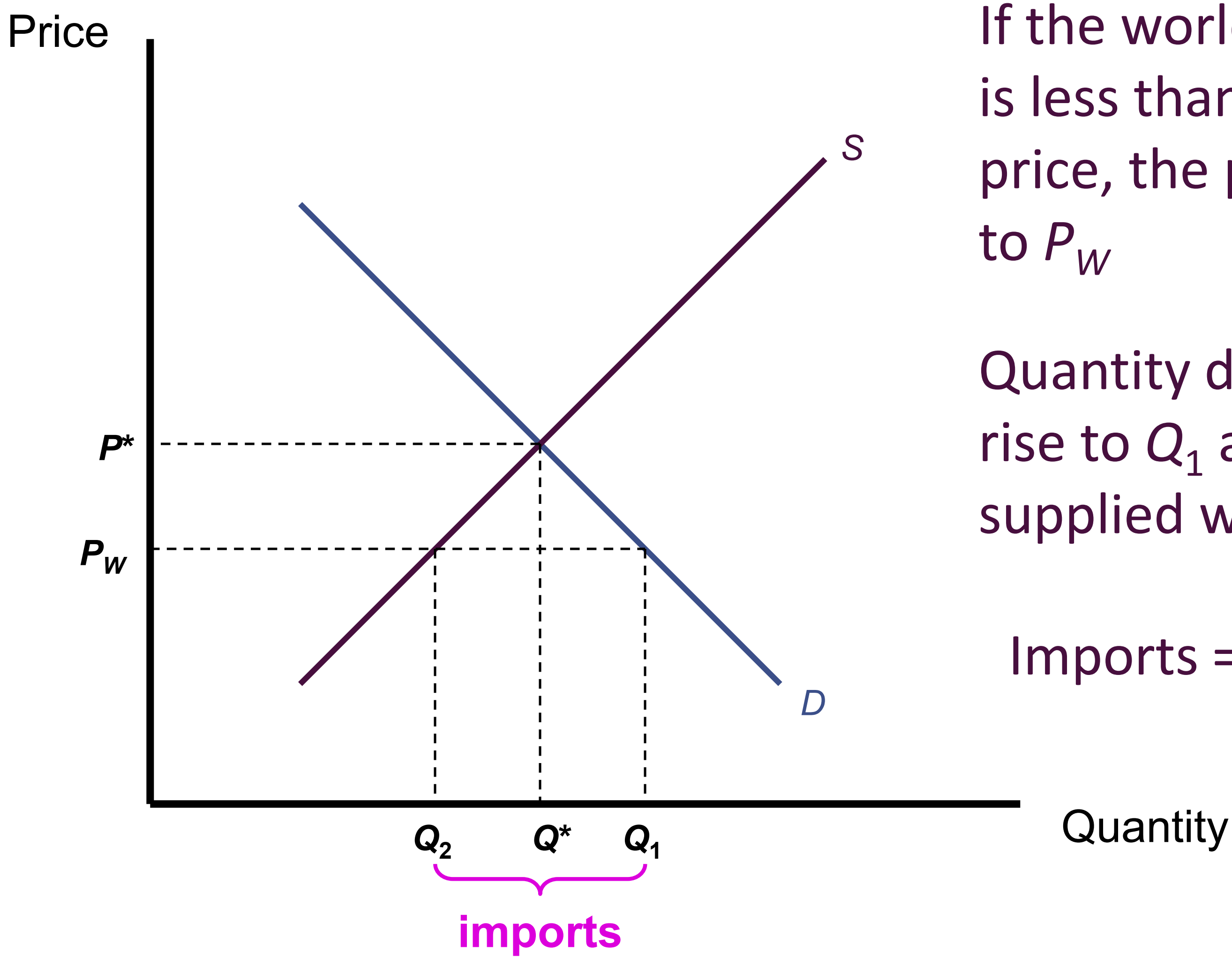
If the transactions costs are on a per-unit basis, these costs will be shared by the buyer and seller

- depends on the specific elasticities involved

Gains from International Trade



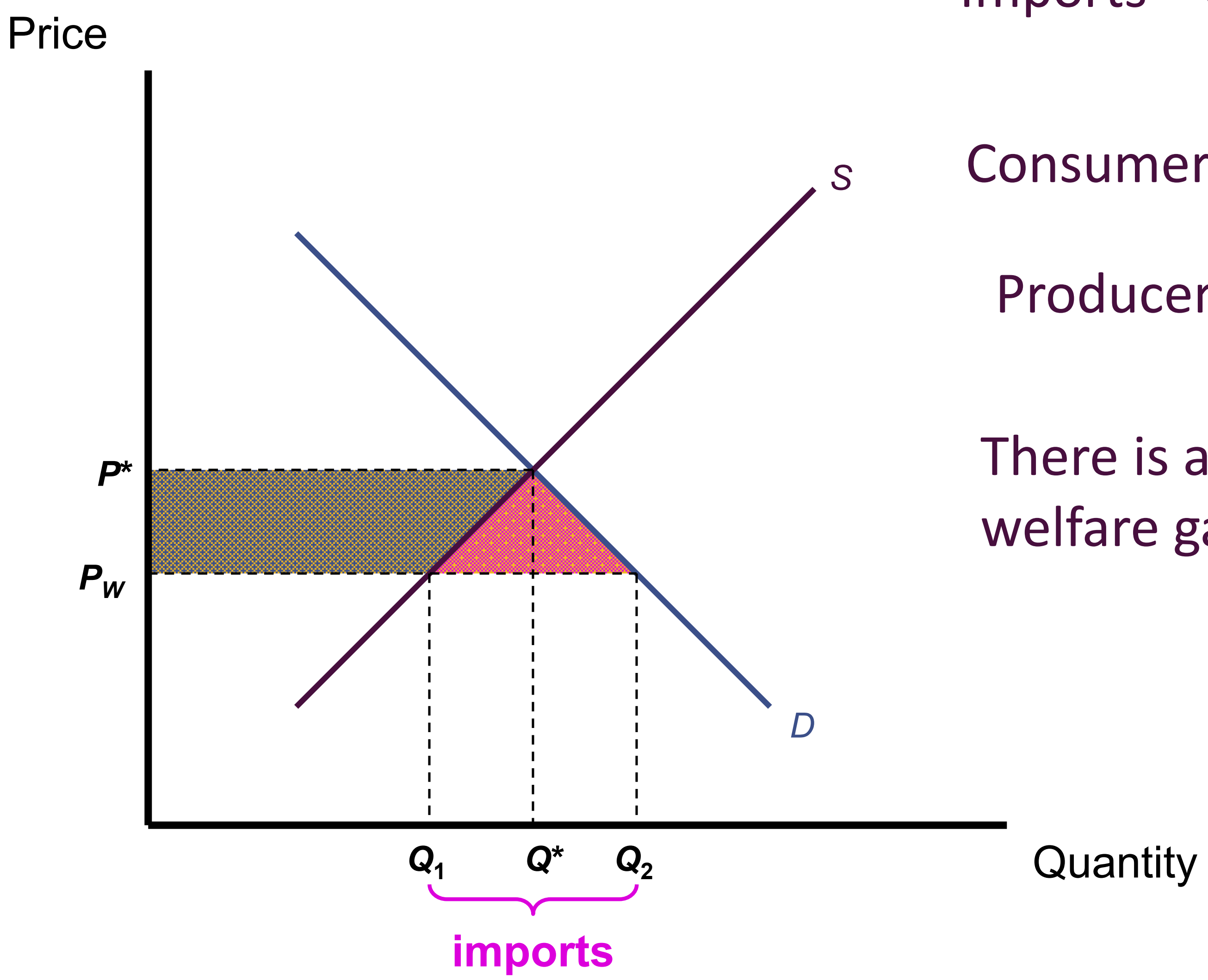
In the absence of international trade, the domestic equilibrium price would be *P** and the domestic equilibrium quantity would be *Q**



If the world price (P_W) is less than the domestic price, the price will fall to P_W

Quantity demanded will rise to Q_1 and quantity supplied will fall to Q_2

Imports = Q_1 - Q_2



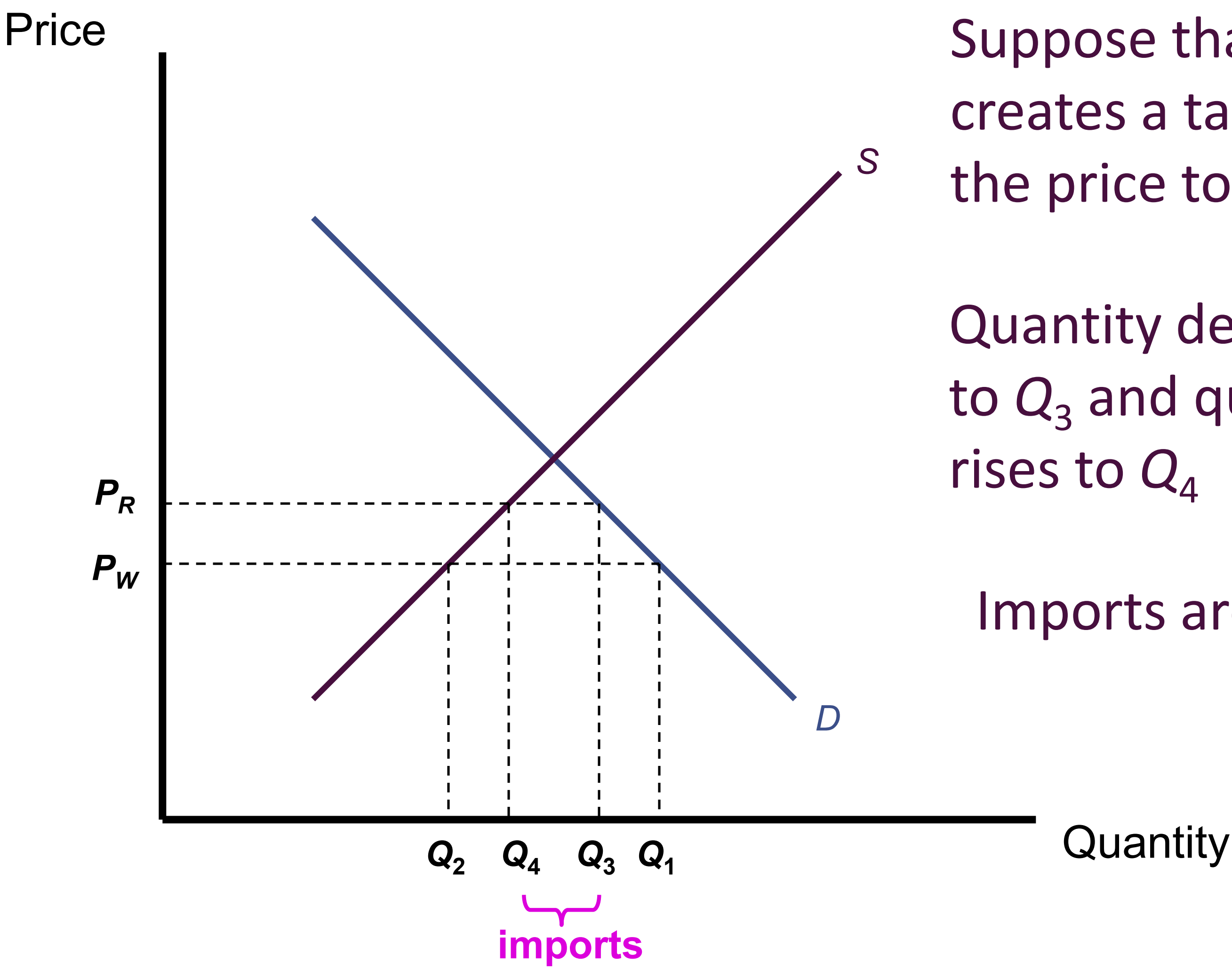
Imports = Q_1 - Q_2

Consumer surplus rises

Producer surplus falls

There is an unambiguous welfare gain

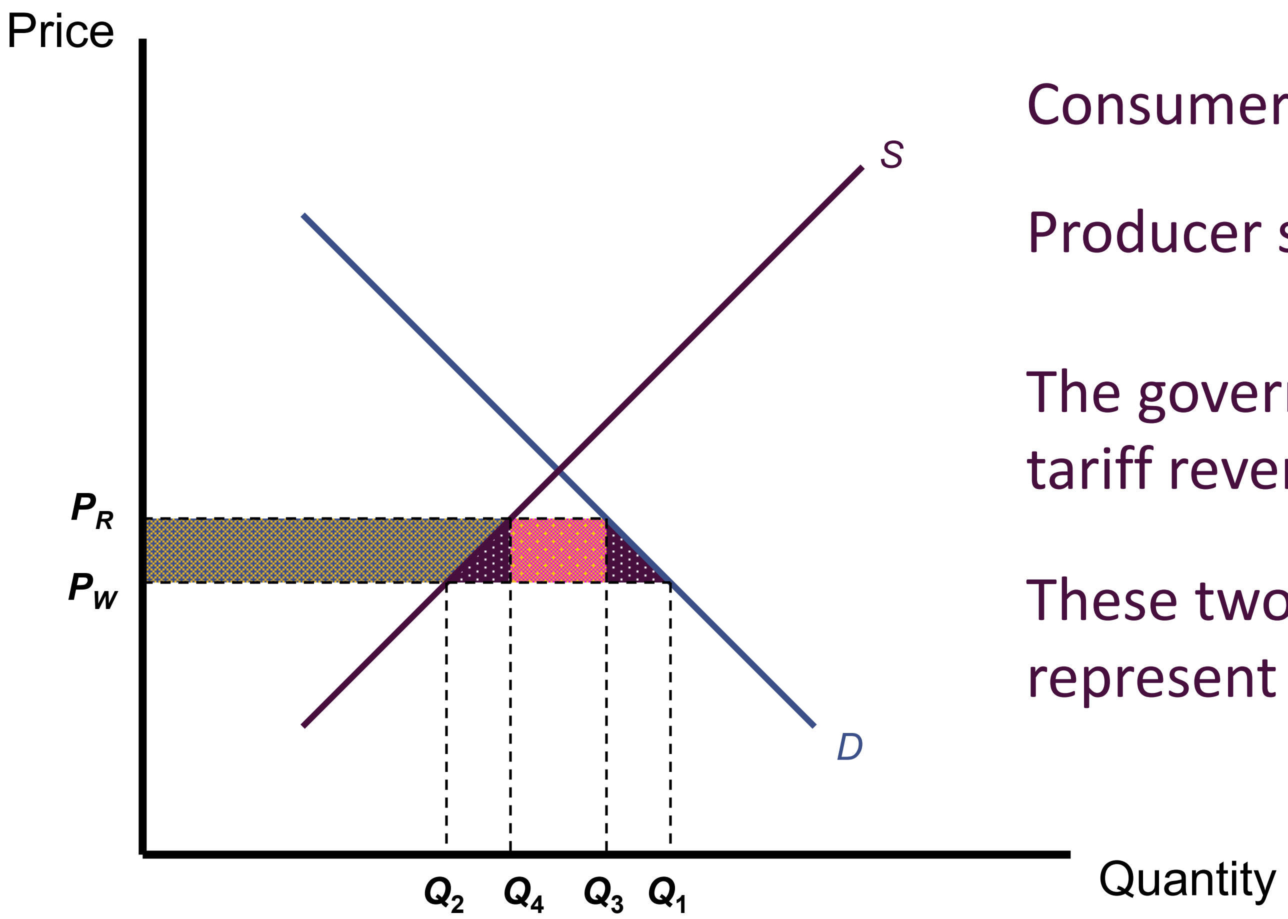
Effects of a Tariff



Suppose that the government creates a tariff that raises the price to P_R

Quantity demanded falls to Q_3 and quantity supplied rises to Q_4

Imports are now $Q_3 - Q_4$



Consumer surplus falls

Producer surplus rises

The government gets tariff revenue

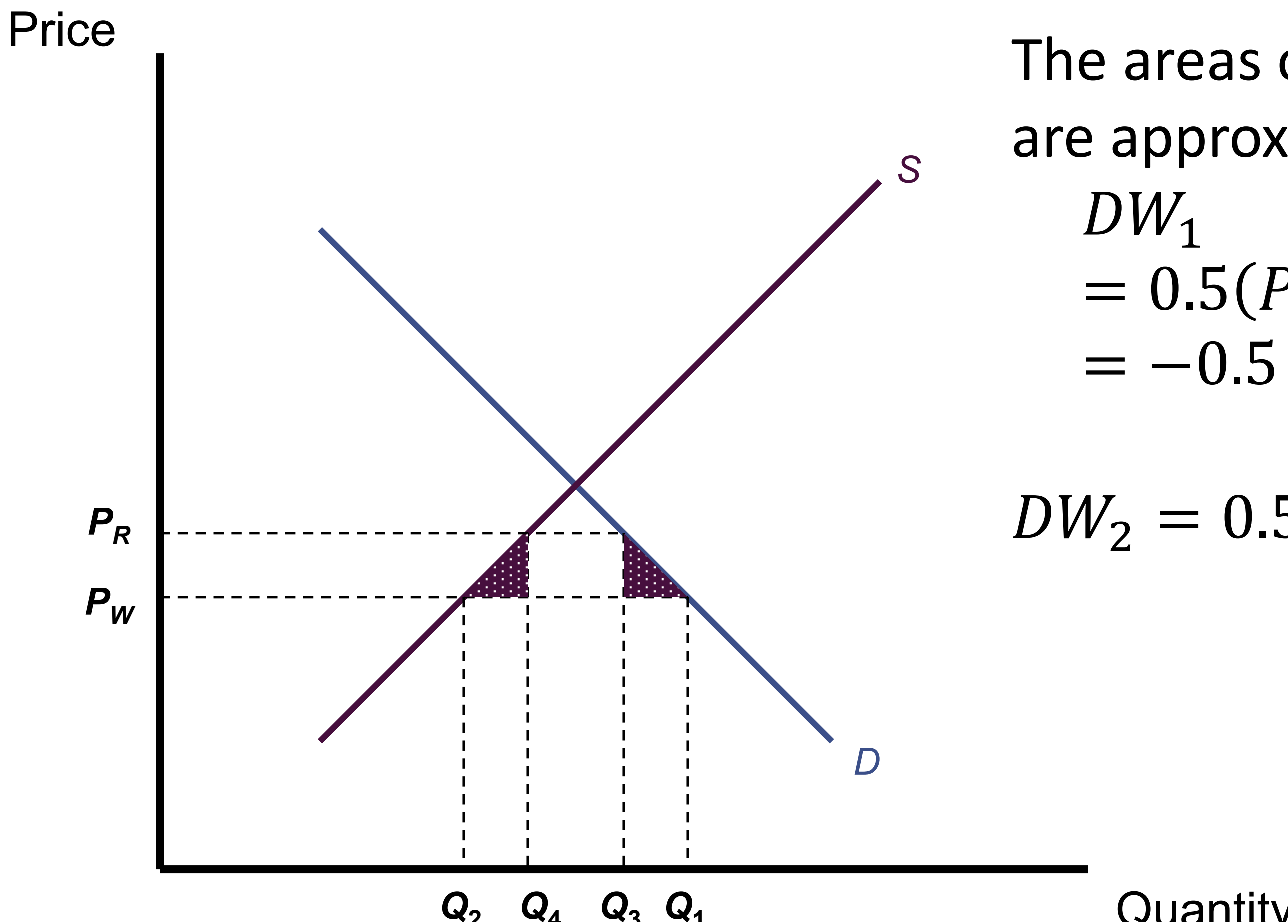
These two triangles represent deadweight loss

Quantitative Estimates of Deadweight Losses

Estimates of the sizes of the welfare loss triangle can be calculated

Because $P_R = PW + t$ the change in quantity demanded is

$$\frac{dQ}{Q} = e_d \frac{dP}{P} \rightarrow Q_1 - Q_3 \cong -e_d t$$



The areas of these two triangles are approximated as

$$DW_1 = 0.5(P_R - P_W)(Q_1 - Q_3) = -0.5\, t^2 e_d$$

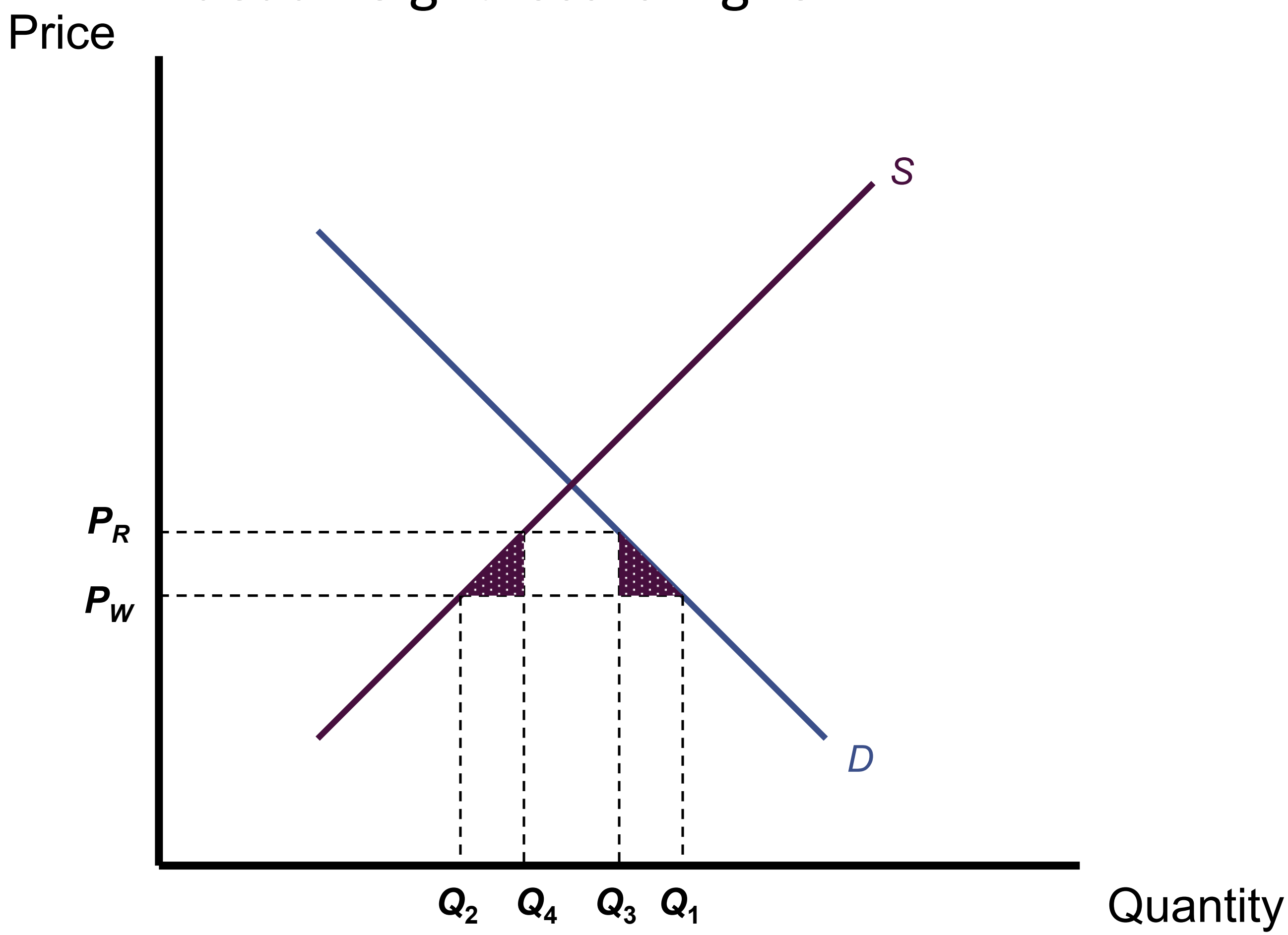
$$DW_2 = 0.5\, t^2 e_S$$

The areas of these two triangles are approximated by

$$DW_1 = -0.5\, t^2 e_d$$

$$DW_2 = 0.5\, t^2 e_S$$

Then when demand and supply are more elastic the deadweight loss is higher



Other Trade Restrictions

A quota that limits imports to Q_3 - Q_4 would have effects that are similar to those for the tariff

- same decline in consumer surplus

- same increase in producer surplus

One big difference is that the quota does not give the government any tariff revenue

- the deadweight loss will be larger

Trade and Tariffs

If the market demand curve is

$$Q_D = 200P^{-1.2}$$

and the market supply curve is

$$Q_S = 1.3P,$$

the domestic long-run equilibrium will occur where

$$P^* = 9.87 \text{ and } Q^* = 12.8$$

If the world price was $P_W = 9$, Q_D would be 14.3 and Q_S would be 11.7

imports will be 2.6

If the government placed a tariff of 0.5 on each unit sold, the world price will be $P_W = 9.5$

imports will fall to 1.0

because Q_D would be 13.4 and Q_S would be 12.4

The welfare effect of the tariff can be approximately calculated

$$DW_1 = 0.5(0.5)(14.3 - 13.4) = 0.225$$

$$DW_2 = 0.5(0.5)(12.4 - 11.7) = 0.175$$

Thus, total deadweight loss from the tariff is 0.225 + 0.175 = 0.4

Note that

DW_2 is exact because the supply is linear

DW_1 is an approximation because the demand is not linear.

The exact evaluation is

$$Q_D = 200P^{-1.2} \rightarrow P = \left(\frac{Q_D}{200}\right)^{-\frac{1}{1.2}}$$

$$DW_2 = \int_{13.4}^{14.3} \left(\frac{Q_D}{200}\right)^{-\frac{1}{1.2}} \mathrm{d}Q_D - 9\,(14.3 - 13.4) = 0.231$$

Important Points to Note:

The concepts of consumer and producer surplus provide useful ways of analyzing the effects of economic changes on the welfare of market participants

- changes in consumer surplus represent changes in the overall utility consumers receive from consuming a particular good
- changes in long-run producer surplus represent changes in the returns product inputs receive

Price controls involve both transfers between producers and consumers and losses of transactions that could benefit both consumers and producers

- Tax incidence analysis concerns the determination of which economic actor ultimately bears the burden of a tax
  - this incidence will fall mainly on the actors who exhibit inelastic responses to price changes
  - taxes also involve deadweight losses that constitute an excess burden in addition to the burden imposed by the actual tax revenues collected
- Transaction costs can sometimes be modeled as taxes
  - both taxes and transaction costs may affect the attributes of transactions depending on the basis on which the costs are incurred
- Trade restrictions such as tariffs or quotas create transfers between consumers and producers and deadweight losses of economic welfare
  - the effects of many types of trade restrictions can be modeled as being equivalent to a per-unit tariff

Exercise.

Demand: q= $1/p$ Supply: $\mathrm{q} = 8\sqrt{p}$

1. Find the equilibrium price and quantity
2. Suppose the government introduces a restriction on quantities: a maximum of q=2 is allowed on the market.
    1. Compute the reduction of the consumer surplus
    2. Compute the dead weight loss
3. Suppose is possible to import at price $1/10$
    1. Compute the quantity of import
    2. Compute the increase of consumer surplus
    3. Compute the decrease of producer surplus
    4. Compute the dead weight loss of a tariff of 1/10