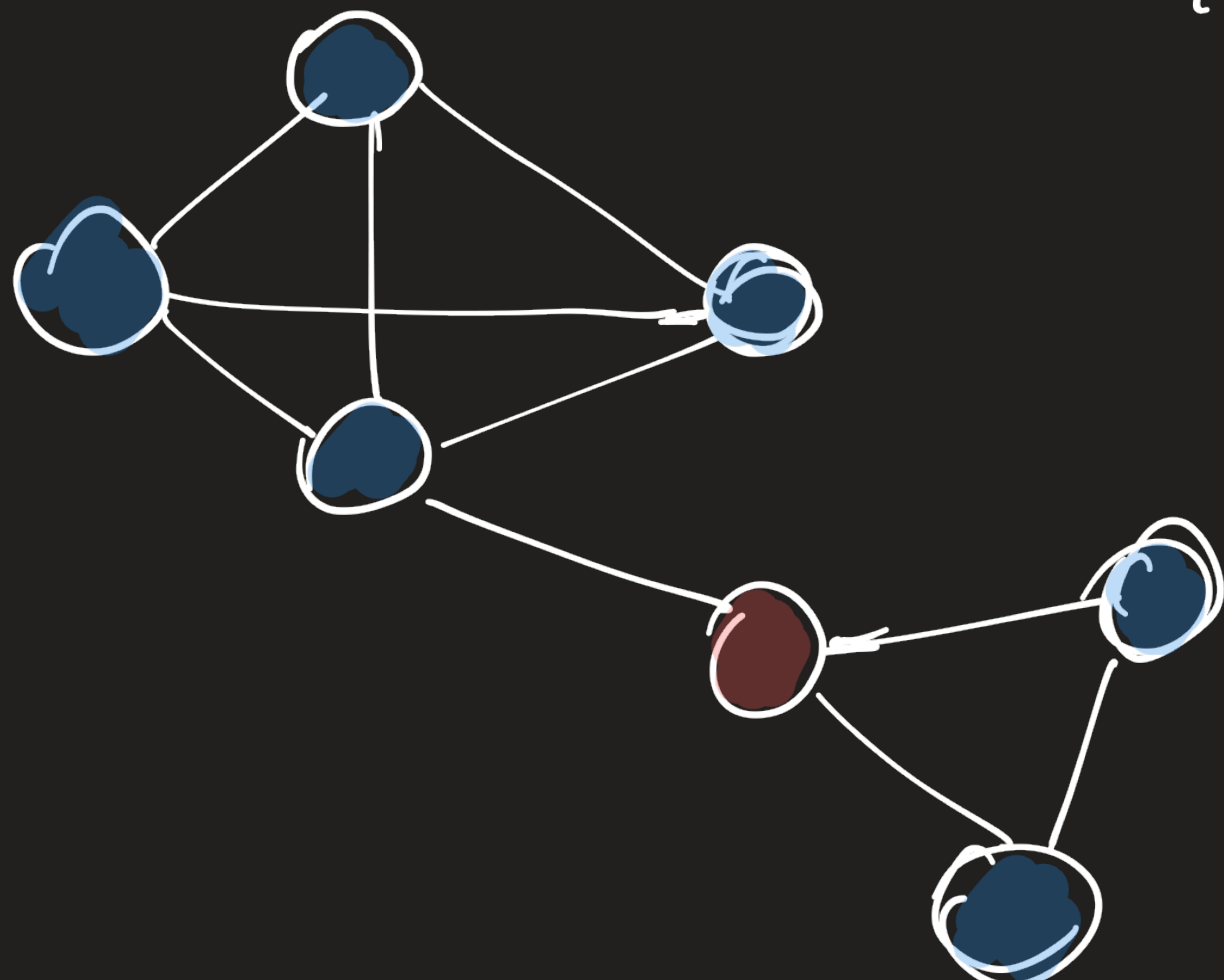
Dynamical Processes on Networks

NETWORKS OF AGENTS

nodes = agents

edges / undirect links = contact / connection



nodes have states

$n \in$ Nodes

$X_n \in S$ - agent states

$\{S, I, R\}$

$\boxed{n.\text{state} \in S}$

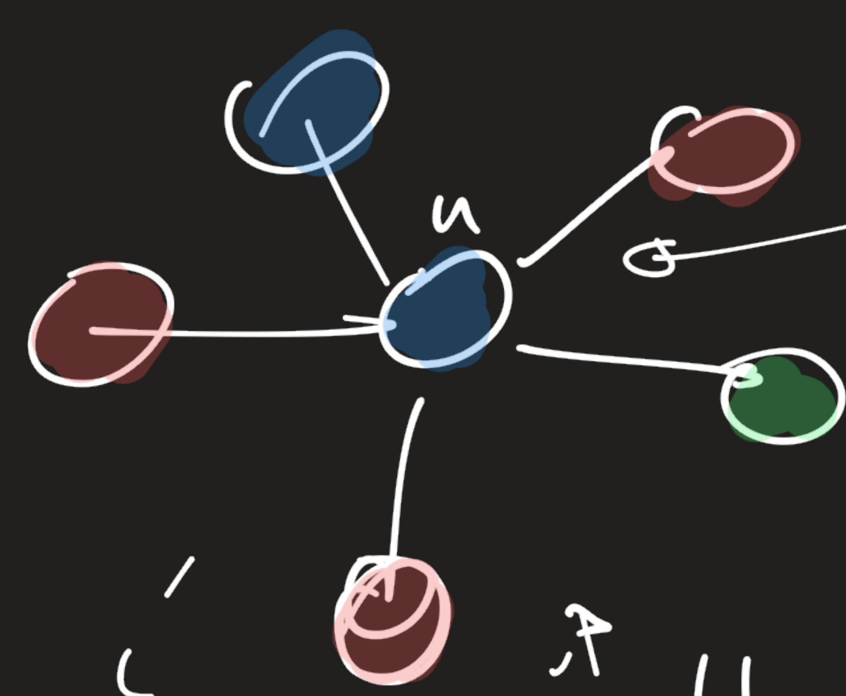
$n.\text{battery\_level} \in [0, 1]$

CONTACT PROCESSES: AN AGENT IS INFLUENCED ONLY BY ITS NEIGHBOURS!

(1) contacts happen on a single edge

λ := rate of an exp. distribution.

② state changes depend from NEIGHBOURHOOD



what is the rate at which u changes state from S to I?

$$3\lambda = \lambda \cdot \left|\{u' \mid u' \in \mathcal{N}(u),\ x_{u'} = I\}\right|$$

neighbourhood vector

rate $S \to I$:

$f(m[u])$

$m[u]$ = # of neigh. nodes in each state

$= (1, 3, 1)$

S I R

We get a CTMC, with state space $\mathcal{S} = \{ \nu : \text{NODE} \to S \}$

- node n changes state from s_1 to s_2 with rate $f(s_1, s_2, m[n]) \geq 0$
- we have a rule/reaction for each $n \in N$ and $(s_1, s_2) \in S^2$

dynamics is given by race condition on these rules.

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so we can simulate network dynamics.

SIMULATION.STEP(Network, t)

- FOR each $n \in$ NODES, FOR EACH $s \in S$,
  $r(n,s) = f(x_n, s, m[n])$
- choose n, s according to its $r(n,s)$
- Then we change $x_n = s$ and advance time accordingly to $\text{EXP}\left(\sum_{n,s} r(n,s) \right)$

s - state in S

X_s - mean field

K - degree (size of neighbourhood)

$X_{s,k}$ - degree-based m.f.

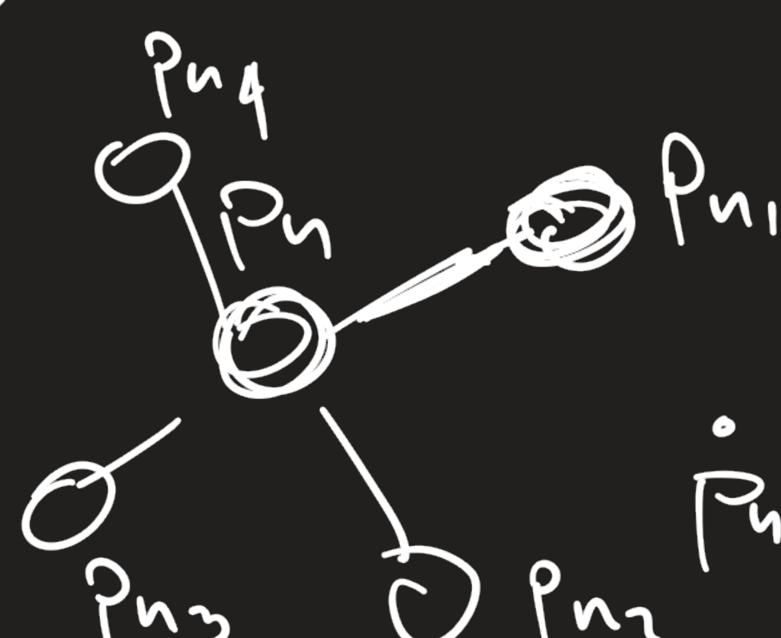
$X_{s,m}$ - approximate master equation.

→ co[illegible] variable

n

$p_n = (p_{n,s_1}, \dots, p_{n,s_k})$, $\sum p_{n,s_i} = 1$

$p_n(t)$



$P = (p_n)_{n \in NODES}$

$\dot{p}_n[I] = \lambda \cdot p_n[S] \cdot p_{n_1}[I]$ ← one term per neighbour

$\dot{p}_n[S] = -\lambda p_n[S] \cdot p_{n_2}[I]$