# Lecture 8:
# General Equilibrium And Welfare

## Assumptions

We will assume that all markets are perfectly competitive:

- there is some large number of homogeneous goods in the economy
  - both consumption goods and factors of production
- each good has an equilibrium price
- there are no transaction or transportation costs
- individuals and firms have perfect information

## Law of One Price

A homogeneous good trades at the same price no matter who buys it or who sells it

- if one good traded at two different prices, demanders would rush to buy the good where it was cheaper and firms would try to sell their output where the price was higher
- these actions would tend to equalize the price of the good

**Behavioral assumptions**

There are a large number of people buying any one good

- each person takes all prices as given and seeks to maximize utility given his budget constraint

There are a large number of firms producing each good

- each firm takes all prices as given and attempts to maximize profits

# General Equilibrium with two goods

Assume that there are only two goods, *x* and *y*

All individuals are assumed to have identical preferences

represented by an indifference map

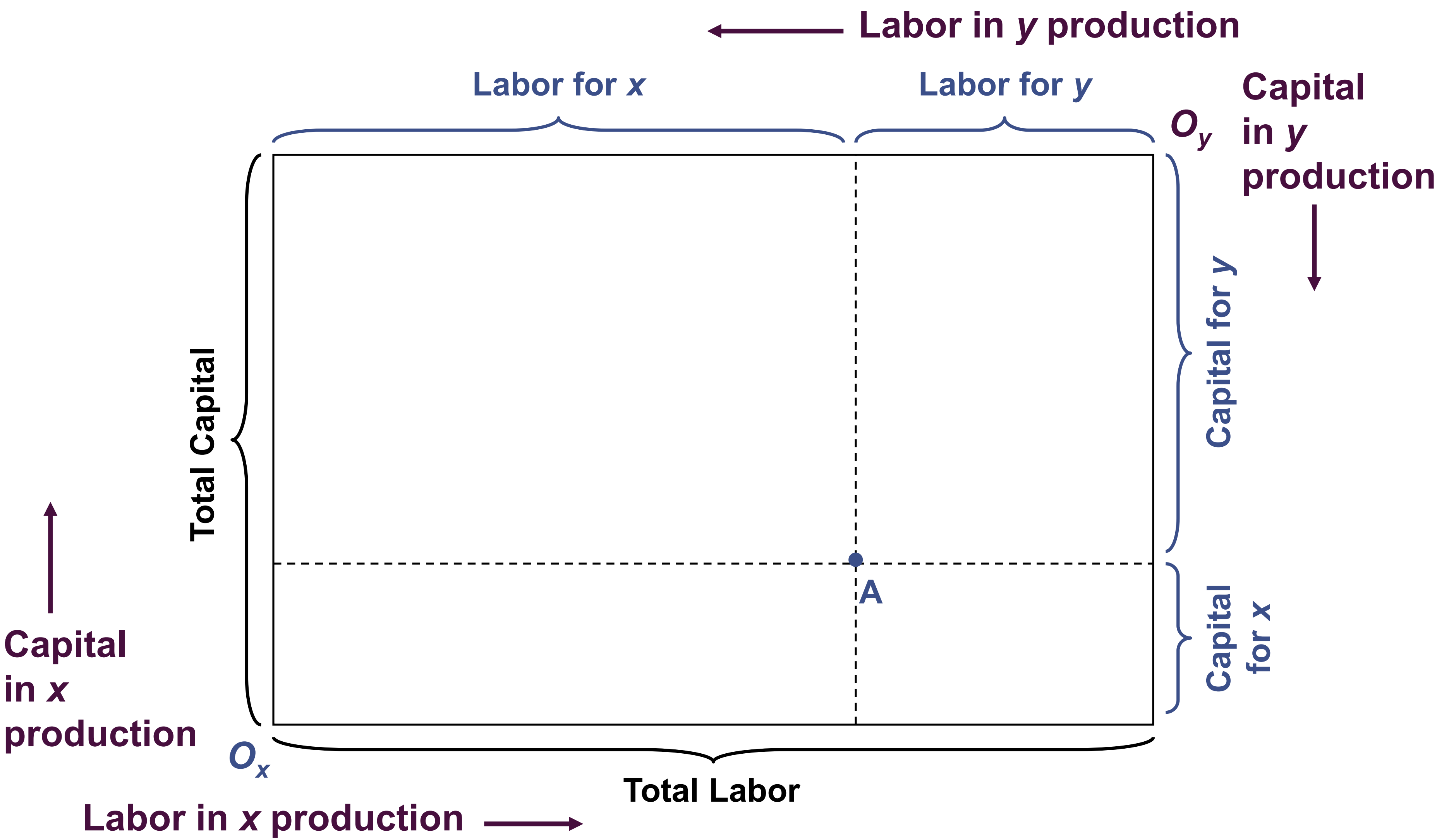
The production possibility curve can be used to show how outputs and inputs are related

# Edgeworth Box Diagram for production

Construction of the production possibility curve for $x$ and $y$ starts with the assumption that the amounts of $k$ and $l$ are fixed

An Edgeworth box shows every possible way the existing $k$ and $l$ might be used to produce $x$ and $y$

- any point in the box represents a fully employed allocation of the available resources to $x$ and $y$


Labor in y production
Labor for x
Labor for y
$O_y$
Capital in y production
Total Capital
Capital for y
A
Capital for x
Capital in x production
$O_x$
Total Labor
Labor in x production

Many of the allocations in the Edgeworth box are technically inefficient

- it is possible to produce more $x$ and more $y$ by shifting capital and labor around

We will assume that competitive markets will not exhibit inefficient input choices

We want to find the efficient allocations

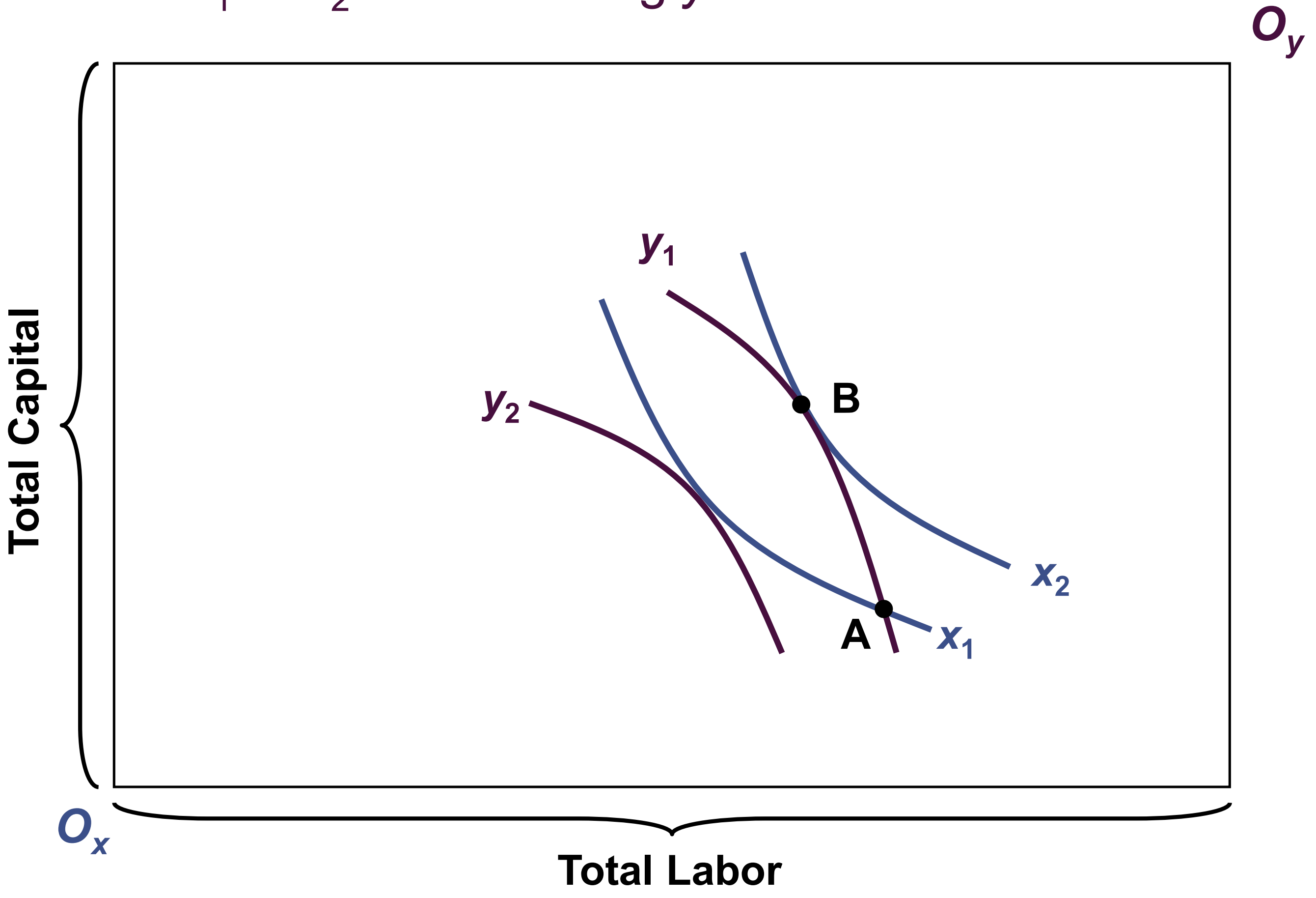
- they illustrate the actual production outcomes

We will use isoquant maps for the two goods

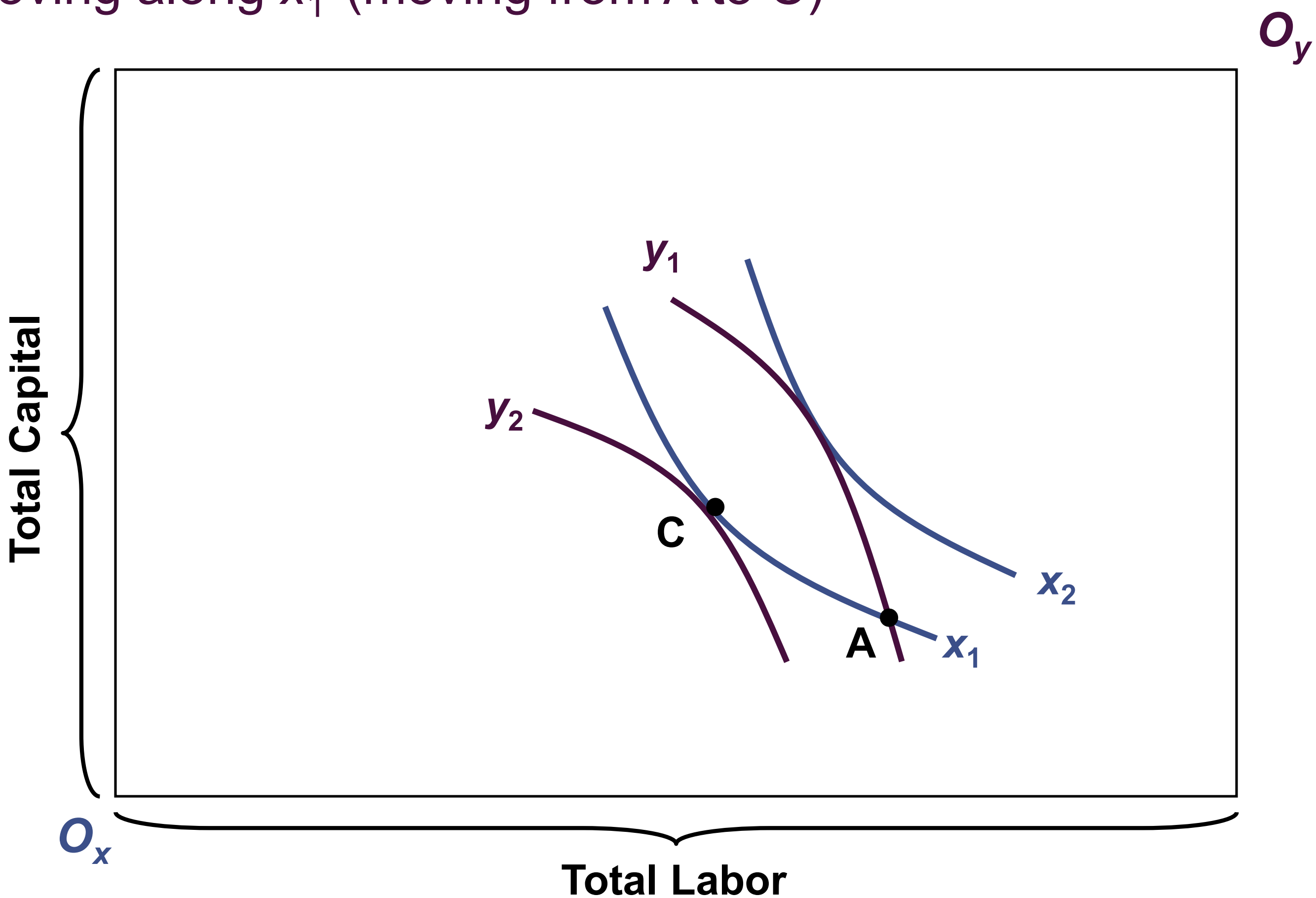
- the isoquant map for good $x$ uses $O_x$ as the origin
- the isoquant map for good $y$ uses $O_y$ as the origin

The efficient allocations will occur where the isoquants are tangent to one another

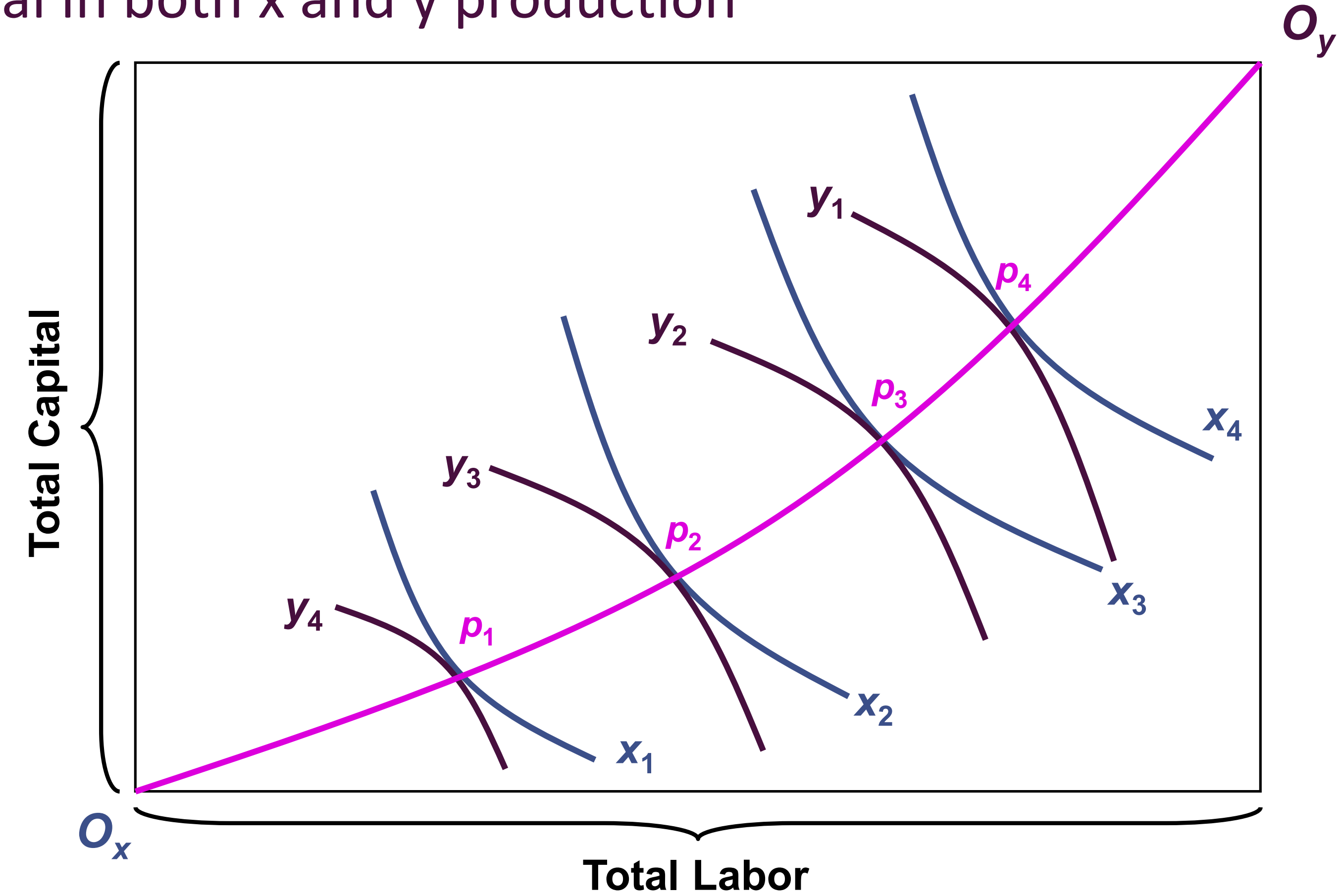
Point *A* is inefficient because, by moving along $y_1$ to point *B*, we can increase *x* from $x_1$ to $x_2$ while holding *y* constant



We could also increase $y$ from $y_1$ to $y_2$ while holding $x$ constant by moving along $x_1$ (moving from A to C)



At each efficient point, the rate of technical substitution (RTS of k for l) is equal in both x and y production

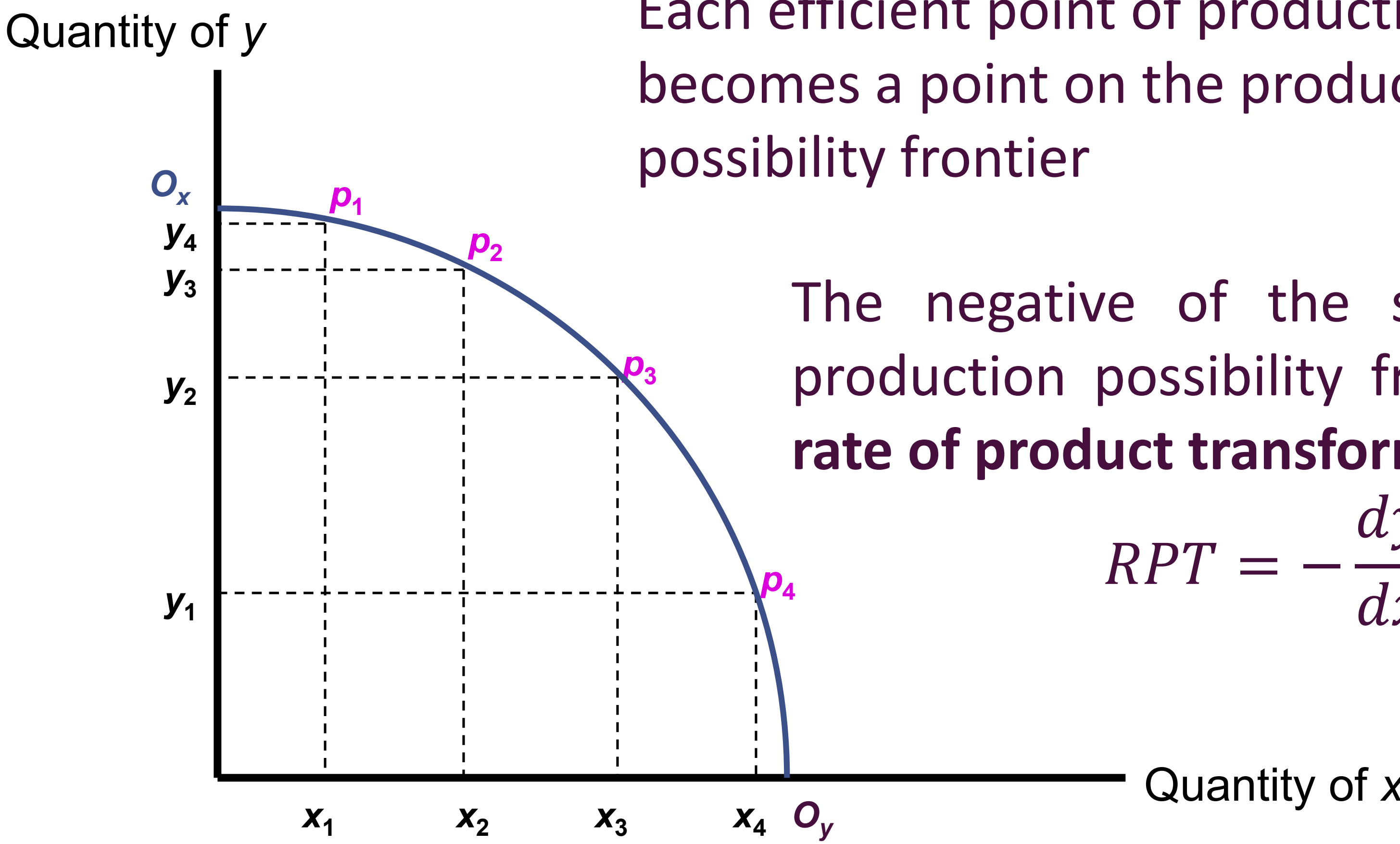


# Production Possibility Frontier

The locus of efficient points shows the maximum output of $y$ that can be produced for any level of $x$

we can use this information to construct a production possibility frontier:

> It shows the alternative outputs of $x$ and $y$ that can be produced with the fixed capital and labor inputs that are employed efficiently




Each efficient point of production becomes a point on the production possibility frontier

The negative of the slope of the production possibility frontier is **the rate of product transformation (*RPT*)**

$$RPT = -\frac{dy}{dx}$$

The rate of product transformation shows how *x* can be technically traded for *y* while continuing to keep the available productive inputs efficiently employed

## Shape of the Production Possibility Frontier

The production possibility frontier shown earlier exhibited an increasing *RPT*

this concave shape will characterize most production situations

*RPT* is equal to the ratio of $MC_x$ to $Mc_y$

Suppose that the costs of any output combination are $C(x,y)$

along the production possibility frontier, $C(x,y)$ is constant because inputs are constant.

We can write the total differential of the cost function as

$$dC = \frac{dC}{dx}dx + \frac{dC}{dy}dy = 0$$

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Rewriting, we get

$$RPT = -\frac{dy}{dx} = \frac{\frac{dC}{dx}}{\frac{dC}{dy}} = \frac{MC_x}{MC_y}$$

The *RPT* is a measure of the relative marginal costs of the two goods

As production of $x$ rises and production of $y$ falls, the ratio of $MC_x$ to $MC_y$ rises

- this occurs if both goods are produced under diminishing returns
  - increasing the production of $x$ raises $MC_x$, while reducing the production of $y$ lowers $MC_y$
- this could also occur if some inputs were more suited for $x$ production than for $y$ production

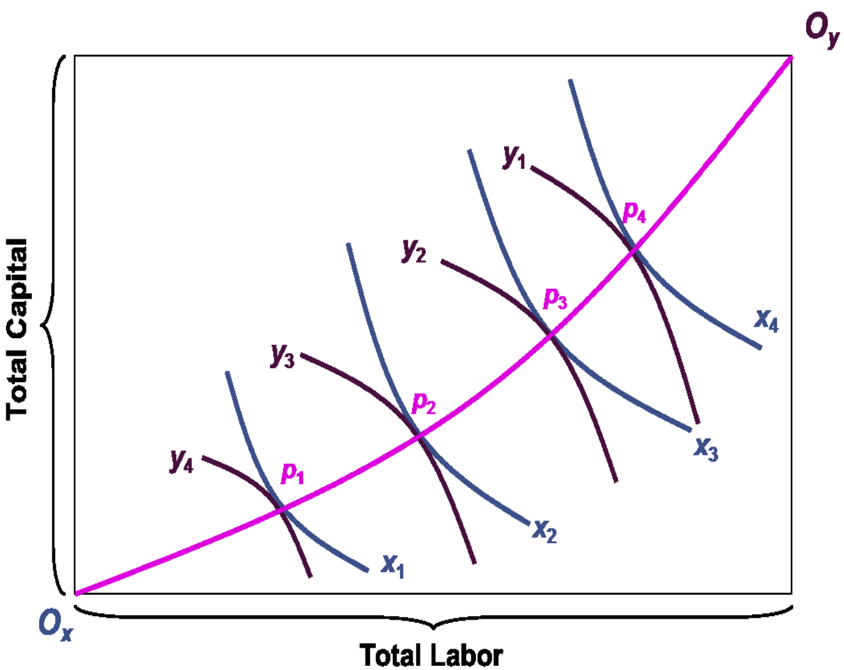
But we have assumed that inputs are homogeneous

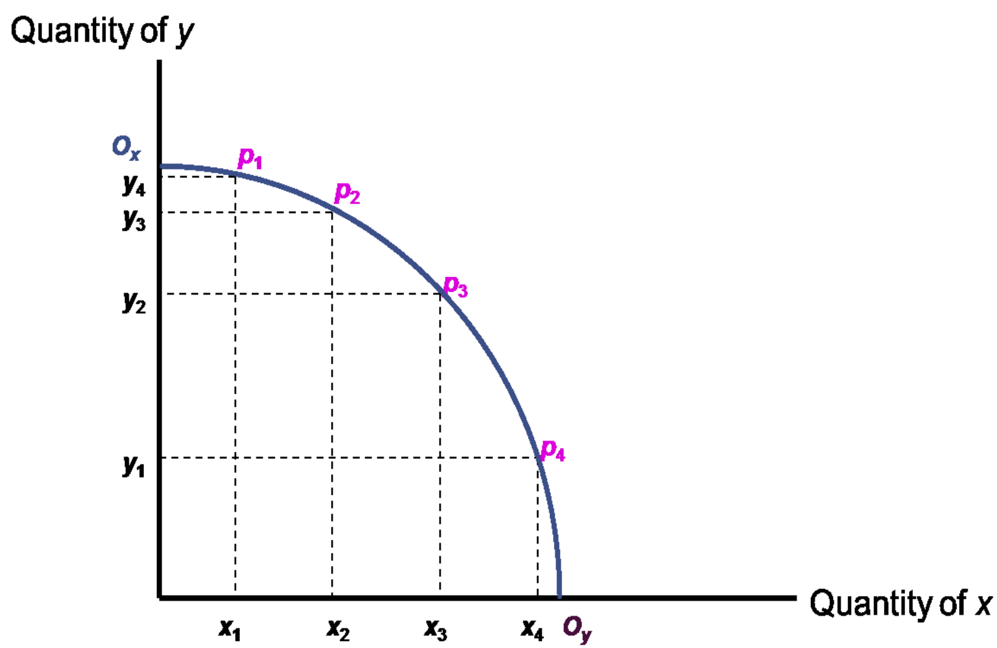
Even if we have homogeneous inputs and constant returns to scale the production possibility frontier will be concave if goods $x$ and $y$ use inputs in different intensities.

In the figure below good x is labor intensive, relative to y. Indeed the contract curve is below the diagonal. So the ratio k/l is lower in the production of good x

Given constant return of scale, if we want to produce a linear combination of two point on the frontier, say P1 and P3, we can use a proportional change of the inputs.

For example a combination that lies on the straight line between P1 and P3 in the left figure. But this point s not efficient…..



## Opportunity Cost

- The production possibility frontier demonstrates that there are many possible efficient combinations of two goods
- Producing more of one good necessitates lowering the production of the other good
  - this is what economists mean by opportunity cost
- The opportunity cost of one more unit of $x$ is the reduction in $y$ that this entails
- Thus, the opportunity cost is best measured as the *RPT* (of $x$ for $y$) at the prevailing point on the production possibility frontier
  - this opportunity cost rises as more $x$ is produced

# Example: Concavity of the Production Possibility Frontier

Suppose that the production of *x* and *y* depends only on labor and the production functions are:

$$x = f(l_x) = l_x^{0.5} \qquad y = f(l_y) = l_y^{0.5}$$

If labor supply is fixed at 100, then

$$l_x + l_y = 100$$

The production possibility frontier is

$$x^2 + y^2 = 100 \quad for \quad x, y \geq 0$$

The *RPT* can be calculated by taking the total differential:

$$2x\, dx + 2y\, dy = 0$$

$$\frac{dy}{dx} = -\frac{x}{y} = -RPT \rightarrow RPT = \frac{x}{y}$$

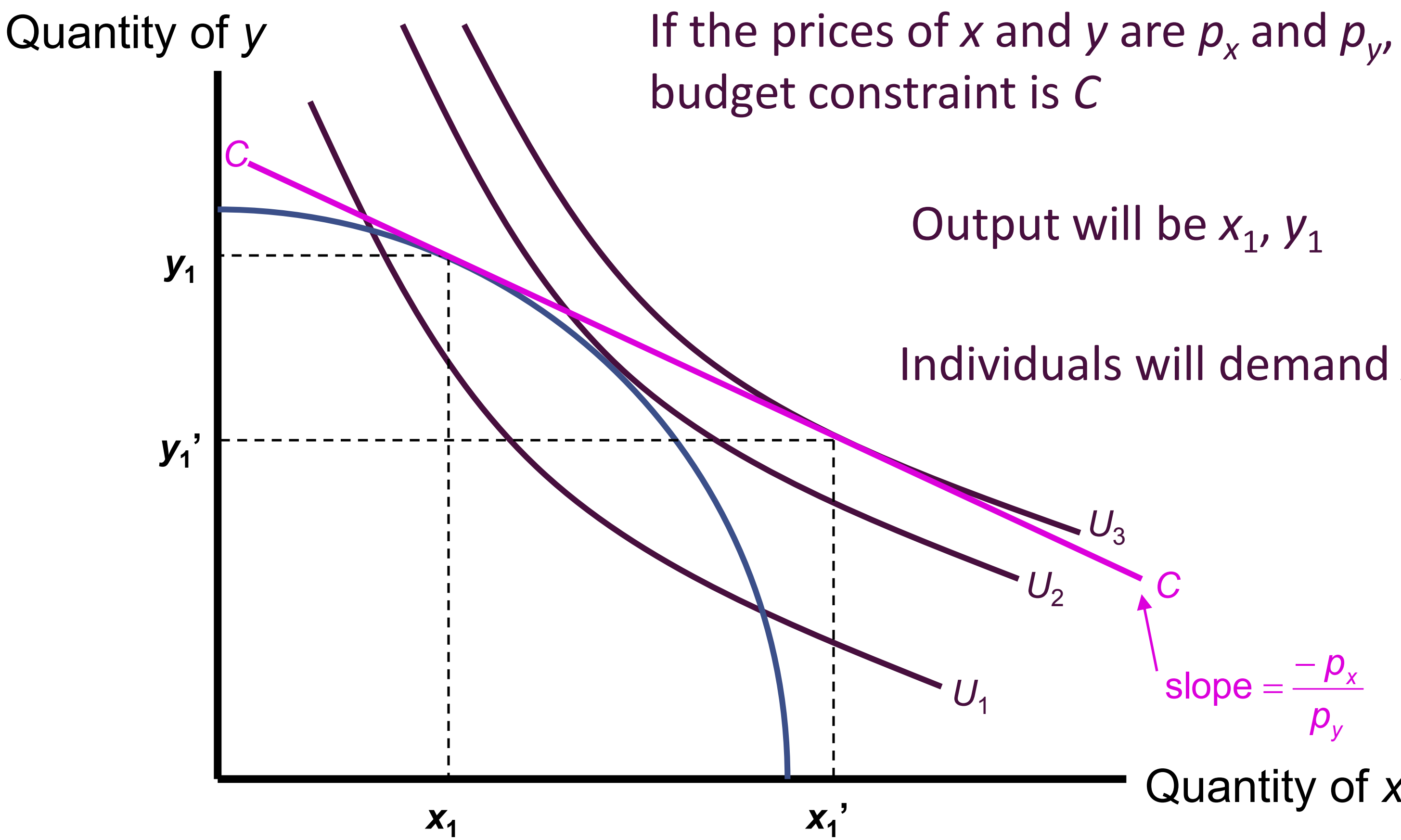
In absolute value the slope of the production possibility frontier increases as *x* output increases

the frontier is concave, indeed $\frac{d^2y}{dx^2} = -\frac{1}{y} < 0$

# Determination of Equilibrium Prices

We can use the production possibility frontier along with a set of indifference curves to show how equilibrium prices are determined

- the indifference curves represent individuals' preferences for the two goods



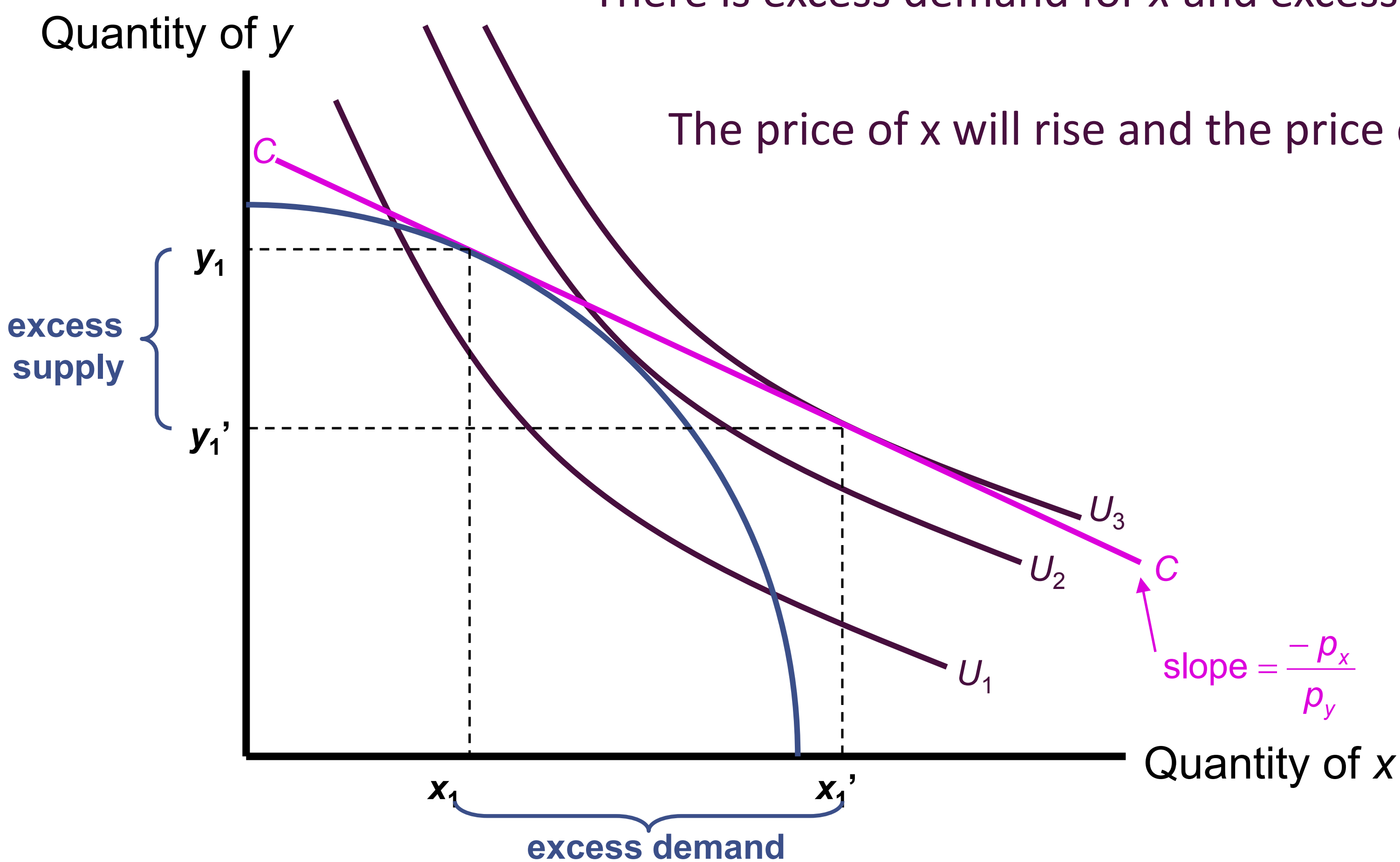

If the prices of $x$ and $y$ are $p_x$ and $p_y$, society's budget constraint is $C$

Output will be $x_1$, $y_1$

Individuals will demand $x_1'$, $y_1'$

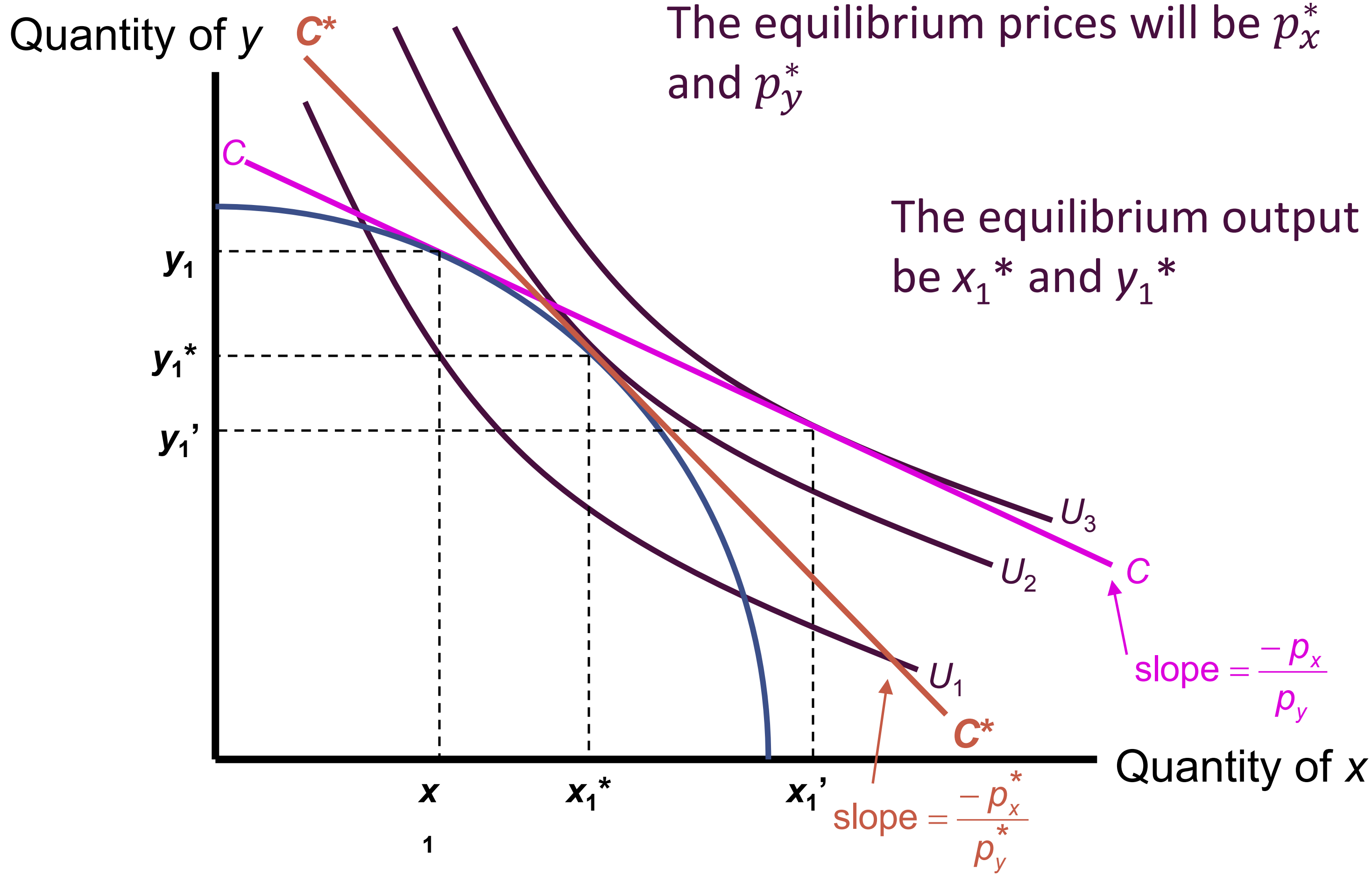
There is excess demand for *x* and excess supply of *y*

The price of x will rise and the price of y will fall



The equilibrium prices will be $p_x^*$ and $p_y^*$

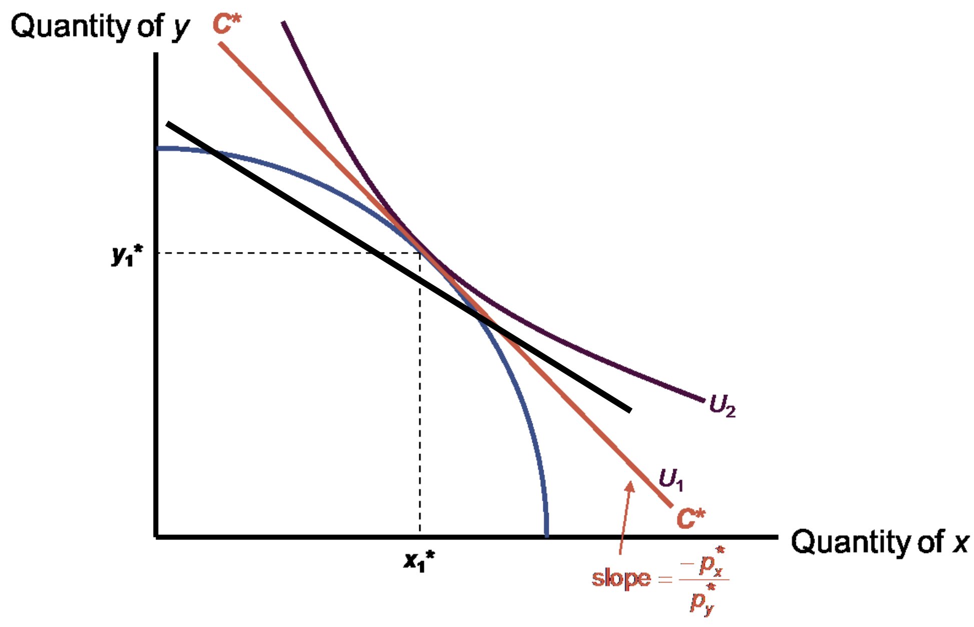
The equilibrium output will be $x_1$* and $y_1$*



The equilibrium prices will be $p_x^*$ and $p_y^*$
The equilibrium output will be $x_1$* and $y_1$*

Equilibrium conditions:

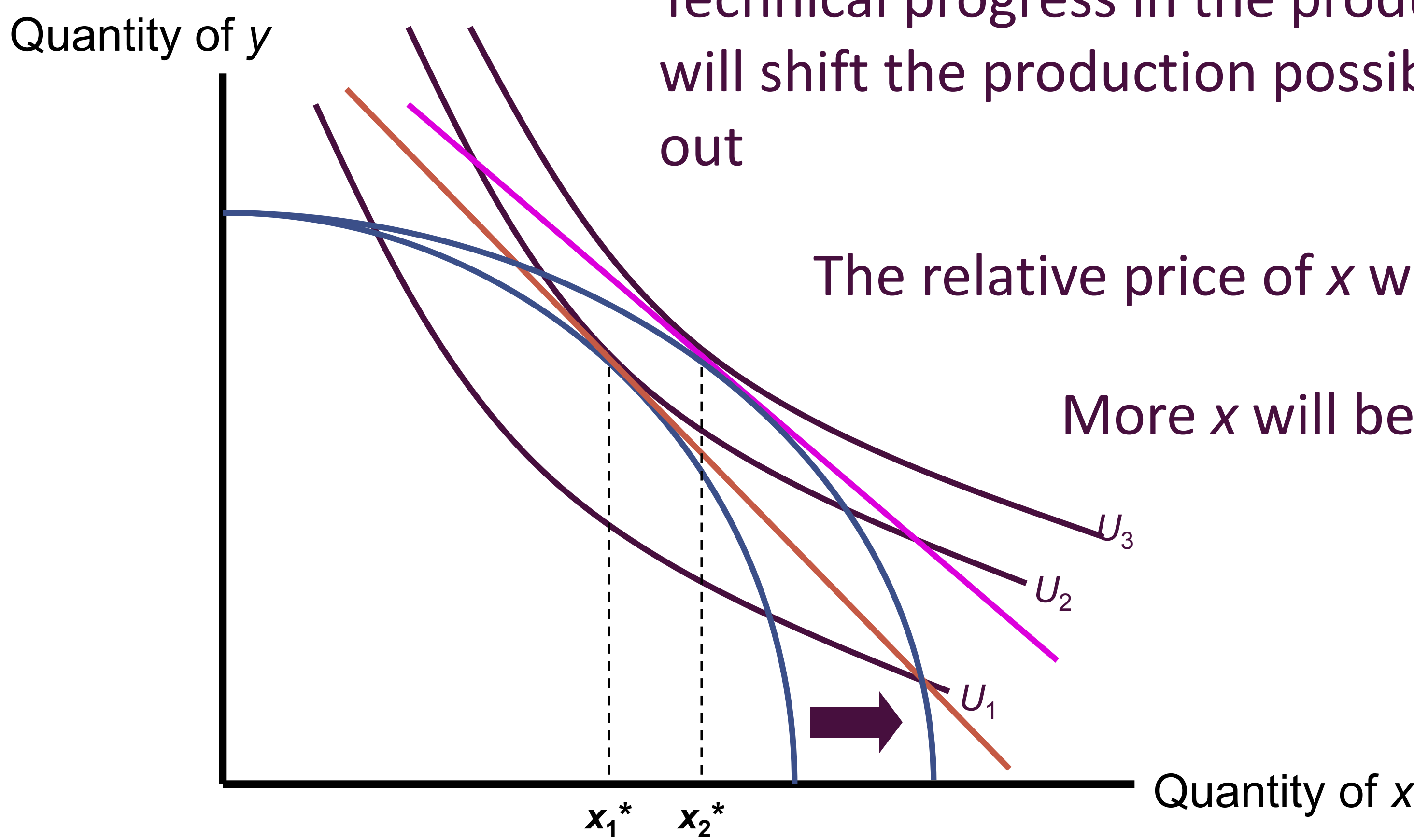
$$RPT = MRS = \frac{p_x}{p_y}$$



## Comparative Statics Analysis

- The equilibrium price ratio will tend to persist until either preferences or production technologies change
- If preferences were to shift toward good $x$, $p_x / p_y$ would rise and more $x$ and less $y$ would be produced
- we would move in a clockwise direction along the production possibility frontier
- Technical progress in the production of good $x$ will shift the production possibility curve outward
- this will lower the relative price of $x$
- more $x$ will be consumed (if $x$ is a normal good) and the effect on $y$ is ambiguous

# Example: Technical Progress in the Production of x




Technical progress in the production of *x* will shift the production possibility curve out

The relative price of *x* will fall

More *x* will be consumed

## General Equilibrium Pricing: an example

Suppose that the production possibility frontier can be represented by

$$x^2 + y^2 = 100$$

Suppose also that the community's preferences can be represented by

$$U(x,y) = x^{0.5}y^{0.5}$$

Profit-maximizing firms will equate *RPT* and the ratio of $p_x/p_y$

$$RPT = \frac{p_x}{p_y} = \frac{x}{y}$$

Utility maximization requires that

$$MRS = \frac{y}{x} = \frac{p_x}{p_y}$$

Note $MRS = \frac{\frac{dU}{dx}}{\frac{dU}{dy}} = \frac{\frac{\sqrt{y}}{2\sqrt{x}}}{\frac{\sqrt{x}}{2\sqrt{y}}} = \frac{y}{x}$

Equilibrium requires that firms and individuals face the same price ratio

$$RPT = \frac{x}{y} = \frac{p_x}{p_y} = \frac{y}{x} = MRS$$

Then

$$x^* = y^*$$

## Trade: The Corn Laws Debate

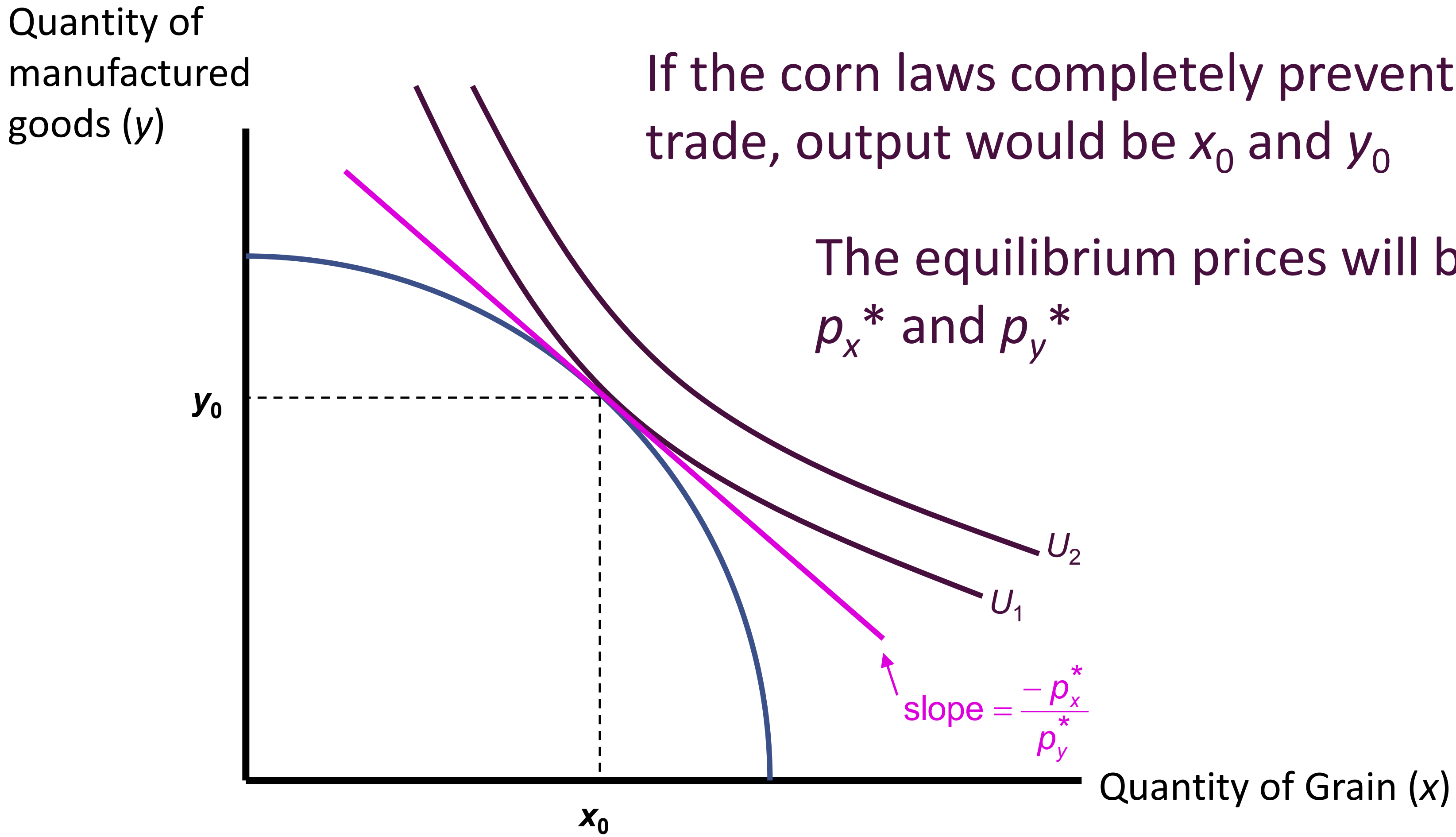
High tariffs on grain imports were imposed by the British government after the Napoleonic wars

Economists debated the effects of these “corn laws” between 1829 and 1845

- what effect would the elimination of these tariffs have on factor prices?

If the corn laws completely prevented trade, output would be $x_0$ and $y_0$

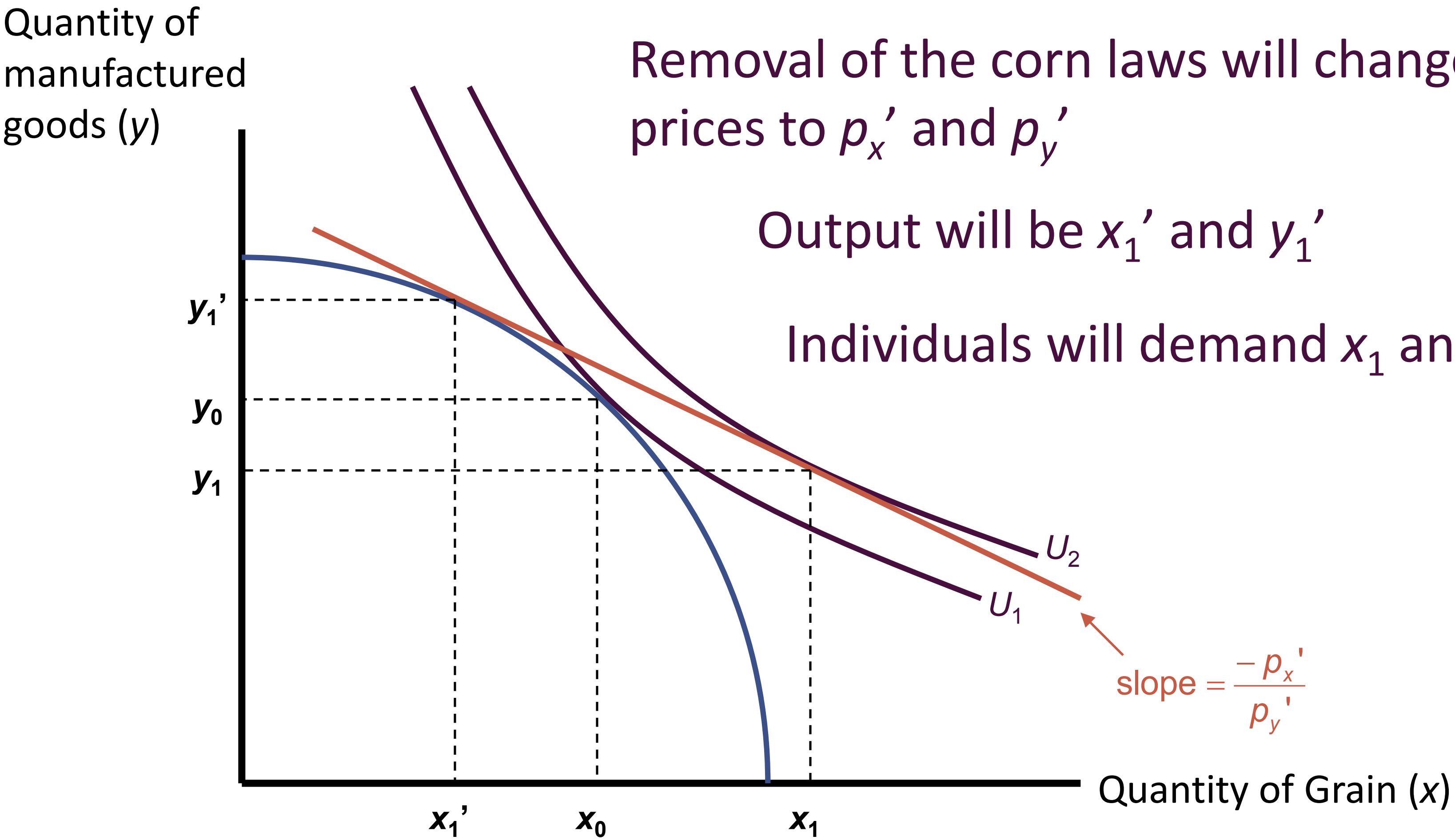
The equilibrium prices will be $p_x$* and $p_y$*



Removal of the corn laws will change the prices to $p_x'$ and $p_y'$

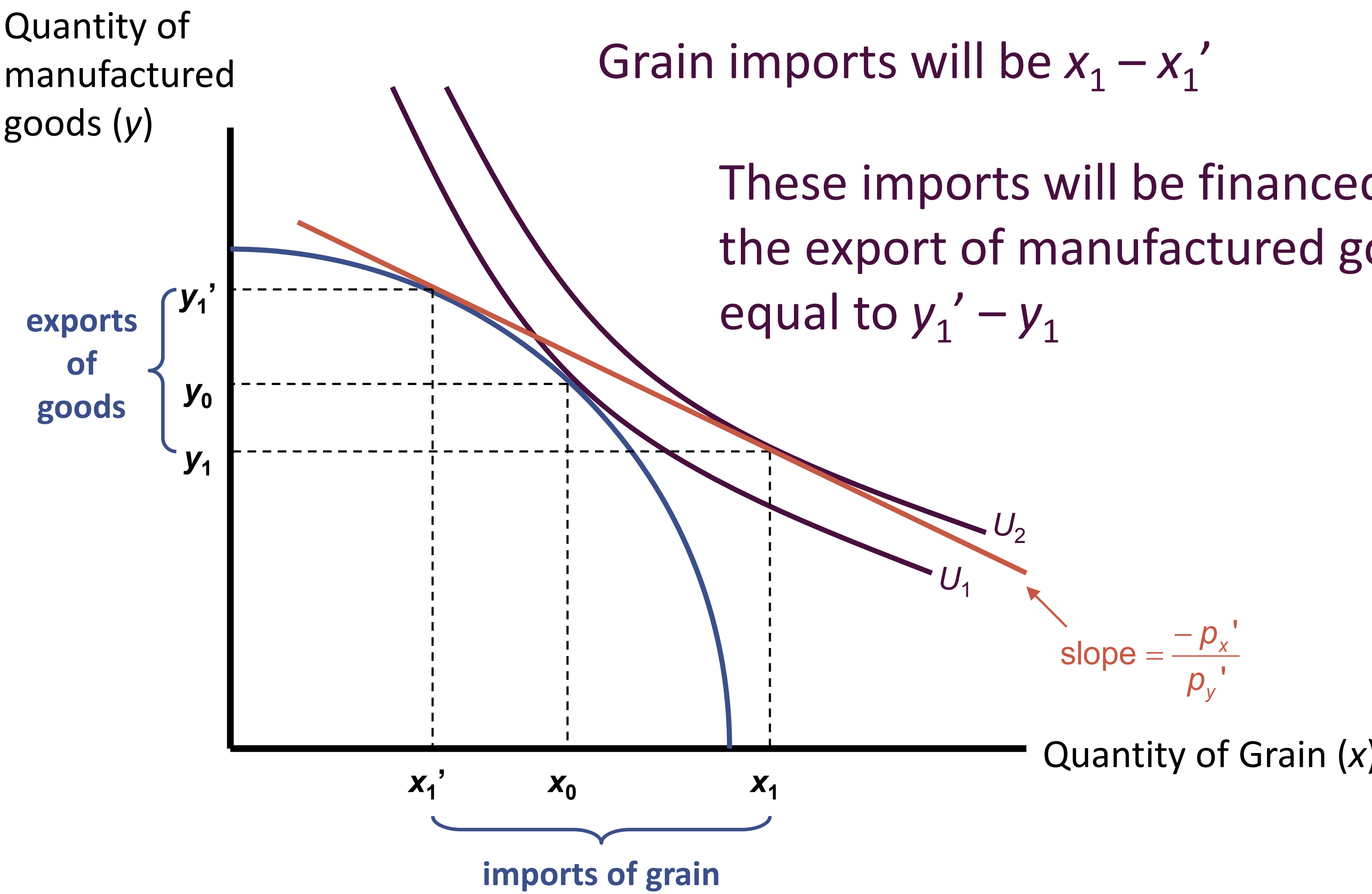
Output will be $x_1'$ and $y_1'$

Individuals will demand $x_1$ and $y_1$

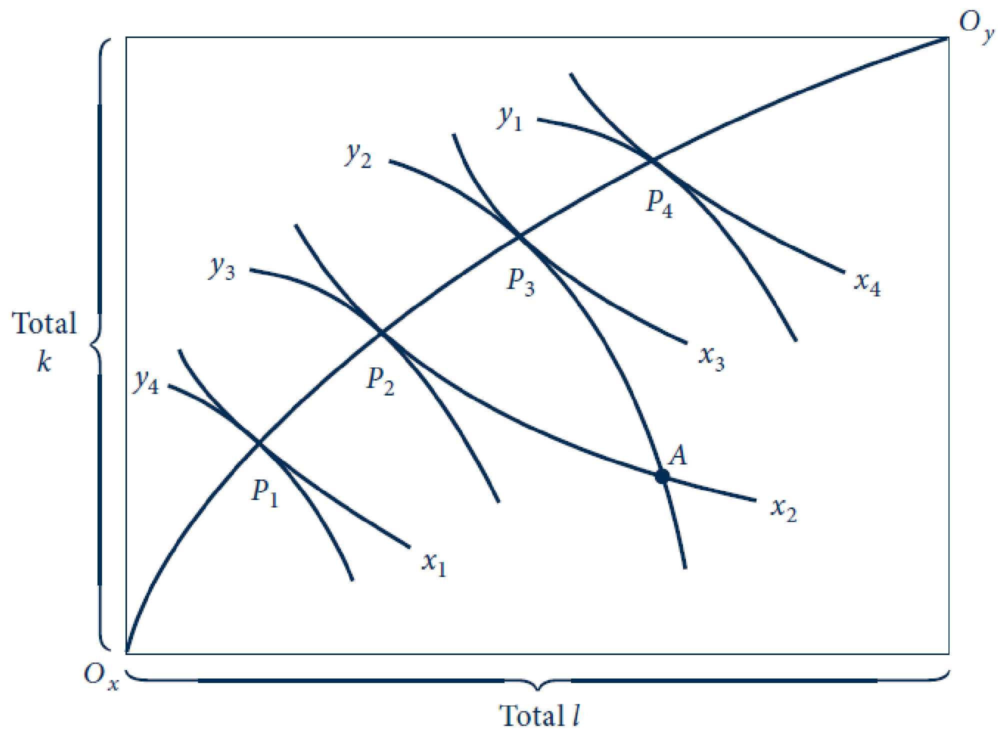


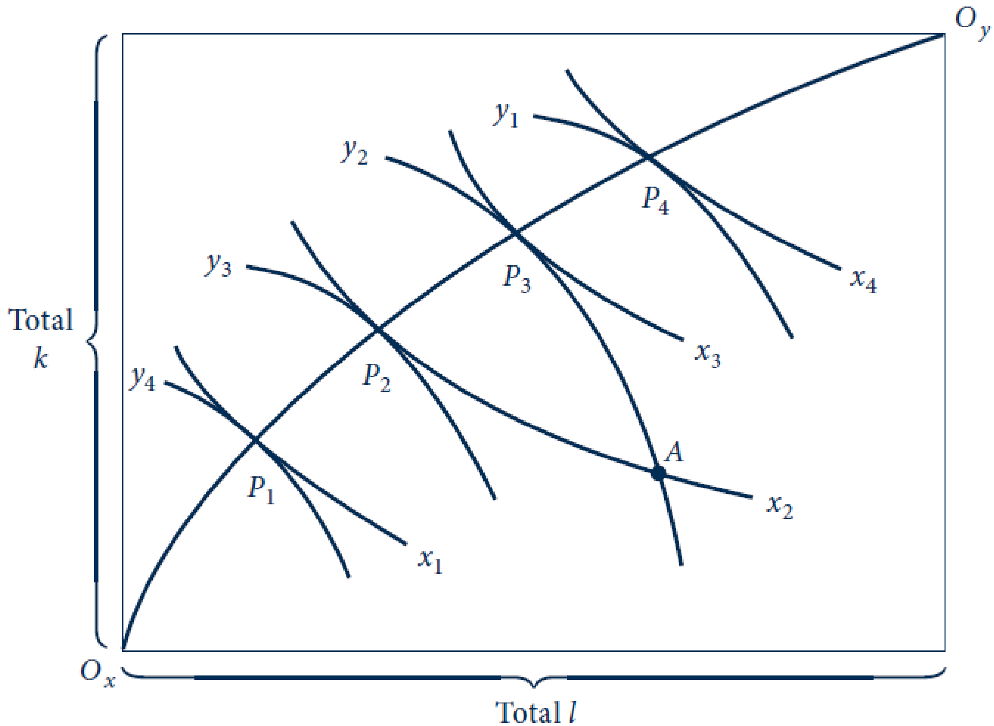
Grain imports will be $x_1 - x_1'$

These imports will be financed by the export of manufactured goods equal to $y_1' - y_1$



- We can use an Edgeworth box diagram to see the effects of tariff reduction on the use of labor and capital
- If the corn laws were repealed, there would be an increase in the production of manufactured goods and a decline in the production of grain
- A repeal of the corn laws would result in a movement from $p_3$ to $p_1$ where more *y* and less *x* is produced






- If we assume that grain production is relatively capital intensive, the movement from $p_3$ to $p_1$ causes the ratio of $k$ to $l$ to rise in both industries
    - the relative price of capital will fall
    - the relative price of labor will rise
- The repeal of the corn laws will be harmful to capital owners and helpful to laborers

# Political Support for Trade Policies

- Trade policies may affect the relative incomes of various factors of production
- In the United States, exports tend to be intensive in their use of skilled labor whereas imports tend to be intensive in their use of unskilled labor
  - free trade policies will result in rising relative wages for skilled workers and in falling relative wages for unskilled workers

# Existence of General Equilibrium Prices

- Beginning with 19th century investigations by Leon Walras, economists have examined whether there exists a set of prices that equilibrates all markets simultaneously
  - if this set of prices exists, how can it be found?

- Suppose that there are $n$ goods in fixed supply in this economy
  - let $S_i$ ($i$ =1,…,$n$) be the total supply of good $i$ available
  - let $p_i$ ($i$ =1,…$n$) be the price of good $i$
- The total demand for good $i$ depends on all prices

$$D_i (p_1,\dots,p_n) \text{ for } i = 1,\dots,n$$

- We will write this demand function as dependent on the whole set of prices ($P$)

$$D_i (P)$$

- Walras' problem: Does there exist an equilibrium set of prices such that

$$D_i (P^*) = S_i$$

for all values of $i$ ?

## Excess Demand Functions

- The excess demand function for any good *i* at any set of prices (*P*) is defined to be

$$ED_i(P) = D_i(P) - S_i$$

- This means that the equilibrium condition can be rewritten as

$$ED_i(P^*) = D_i(P^*) - S_i = 0$$

  - Demand functions are homogeneous of degree zero
    - this implies that we can only establish equilibrium relative prices in a Walrasian-type model
  - Walras also assumed that demand functions are continuous
    - small changes in price lead to small changes in quantity demanded

## Walras' Law

- A final observation that Walras made was that the *n* excess demand equations are not independent of one another
- Walras' law shows that the total value of excess demand is zero at any set of prices

$$\sum_{i=1}^{n} P_i \, ED_i(P) = 0$$

- Walras' law holds for any set of prices (not just equilibrium prices)
- There can be neither excess demand for all goods together nor excess supply

## Smith's Invisible Hand Hypothesis

- Adam Smith believed that the competitive market system provided a powerful "invisible hand" that ensured resources would find their way to where they were most valued
- Reliance on the economic self-interest of individuals and firms would result in a desirable social outcome

- Smith's insights gave rise to modern welfare economics
- The "First Theorem of Welfare Economics" suggests that there is an exact correspondence between the efficient allocation of resources and the competitive pricing of these resources

## Pareto Efficiency

- An allocation of resources is Pareto efficient if it is not possible (through further reallocations) to make one person better off without making someone else worse off
- The Pareto definition identifies allocations as being "inefficient" if unambiguous improvements are possible

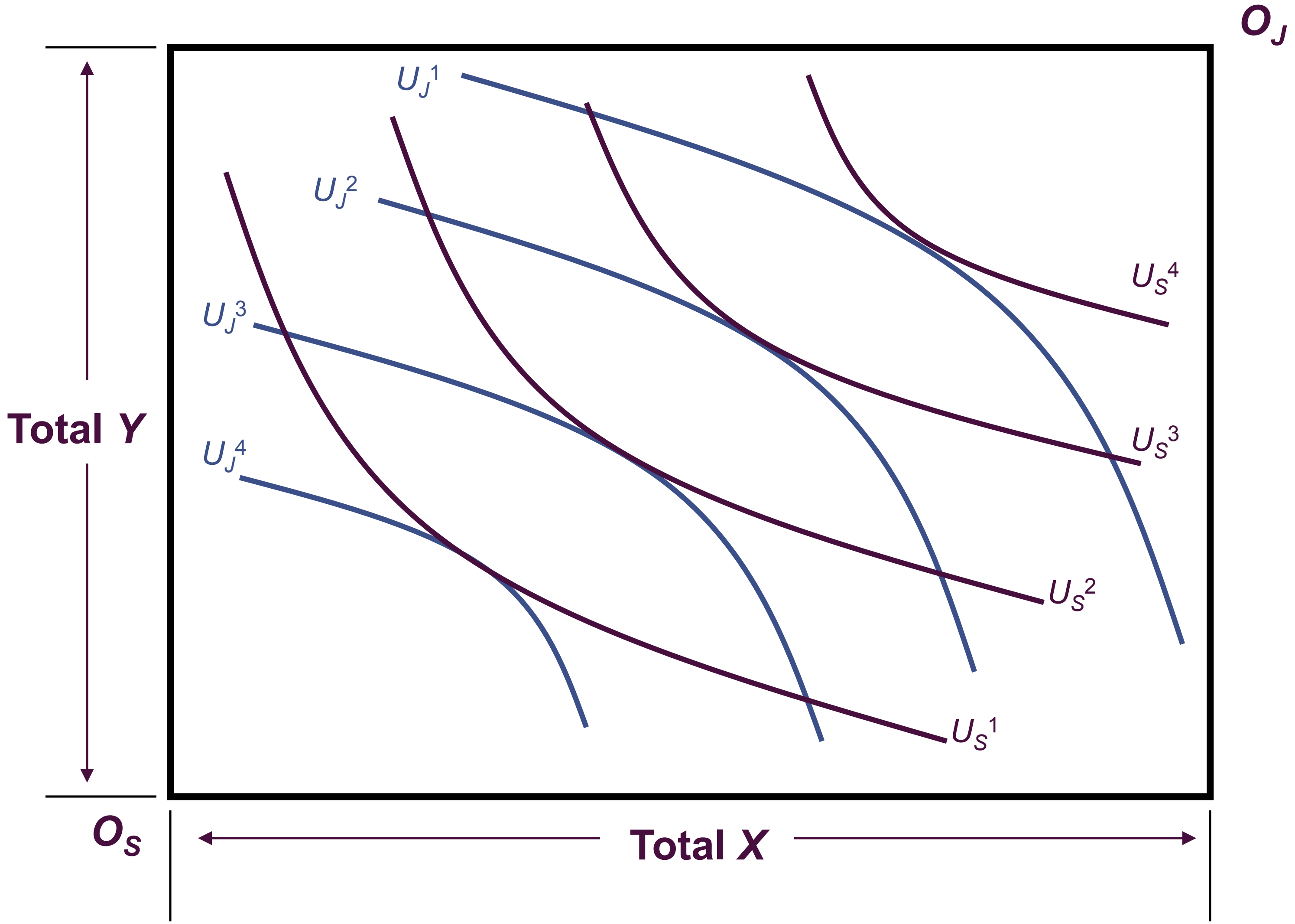
## Competitive Prices and Efficiency: The First Theorem of Welfare Economics

- Attaining a Pareto efficient allocation of resources requires that the rate of trade-off between any two goods be the same for all economic agents
- In a perfectly competitive economy, the ratio of the prices of the two goods provides the common rate of trade-off to which all agents will adjust
- Because all agents face the same prices, all trade-off rates will be equalized and an efficient allocation will be achieved
- This is the "First Theorem of Welfare Economics":

  *Every Walrasian equilibrium is efficient*

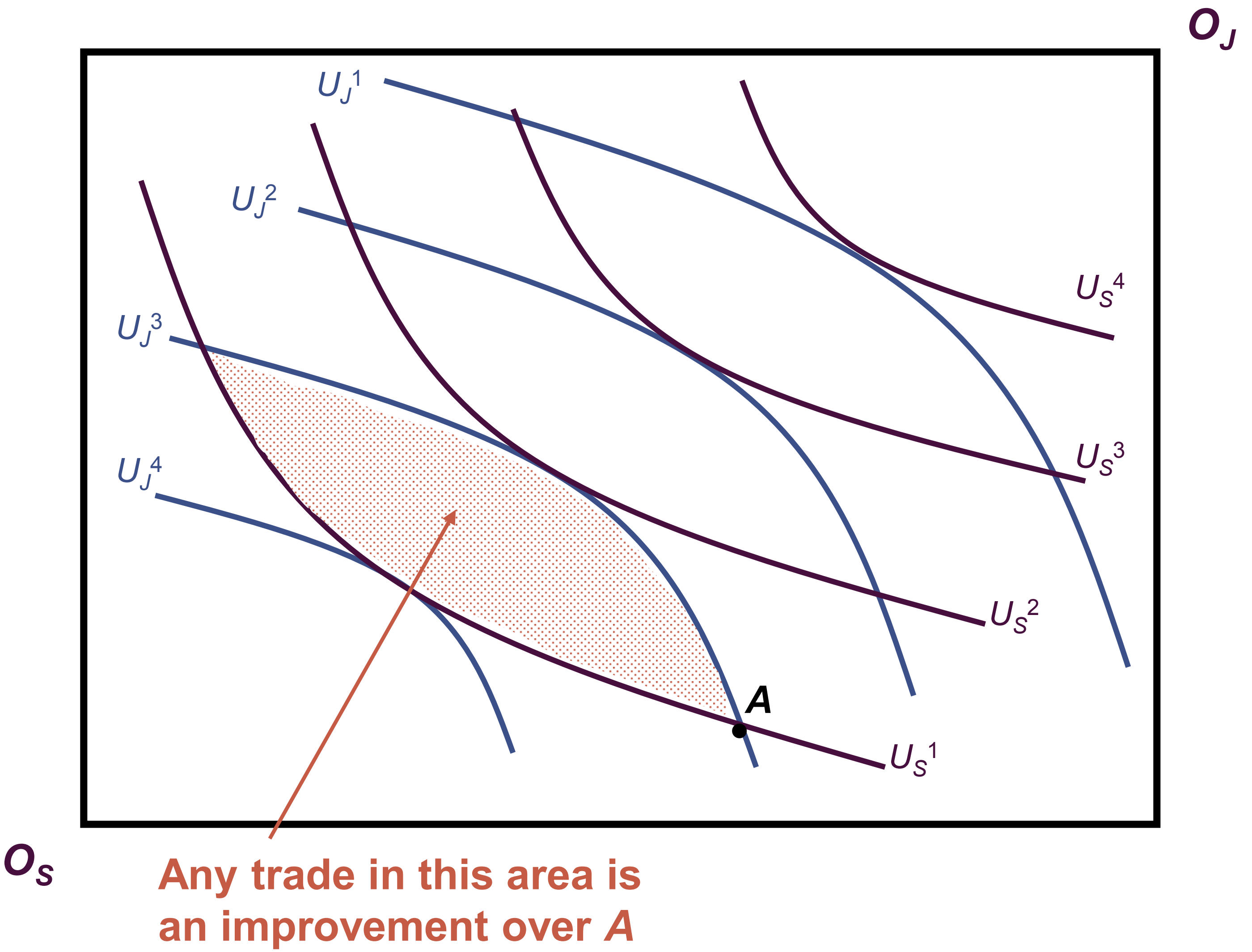
# Distribution

- Although the First Theorem of Welfare Economics ensures that competitive markets will achieve efficient allocations, there are no guarantees that these allocations will exhibit desirable distributions of welfare among individual
- Assume that there are only two people in society (Smith and Jones)
- The quantities of two goods (*x* and *y*) to be distributed among these two people are fixed in supply
- We can use an Edgeworth box diagram to show all possible allocations of these goods between Smith and Jones



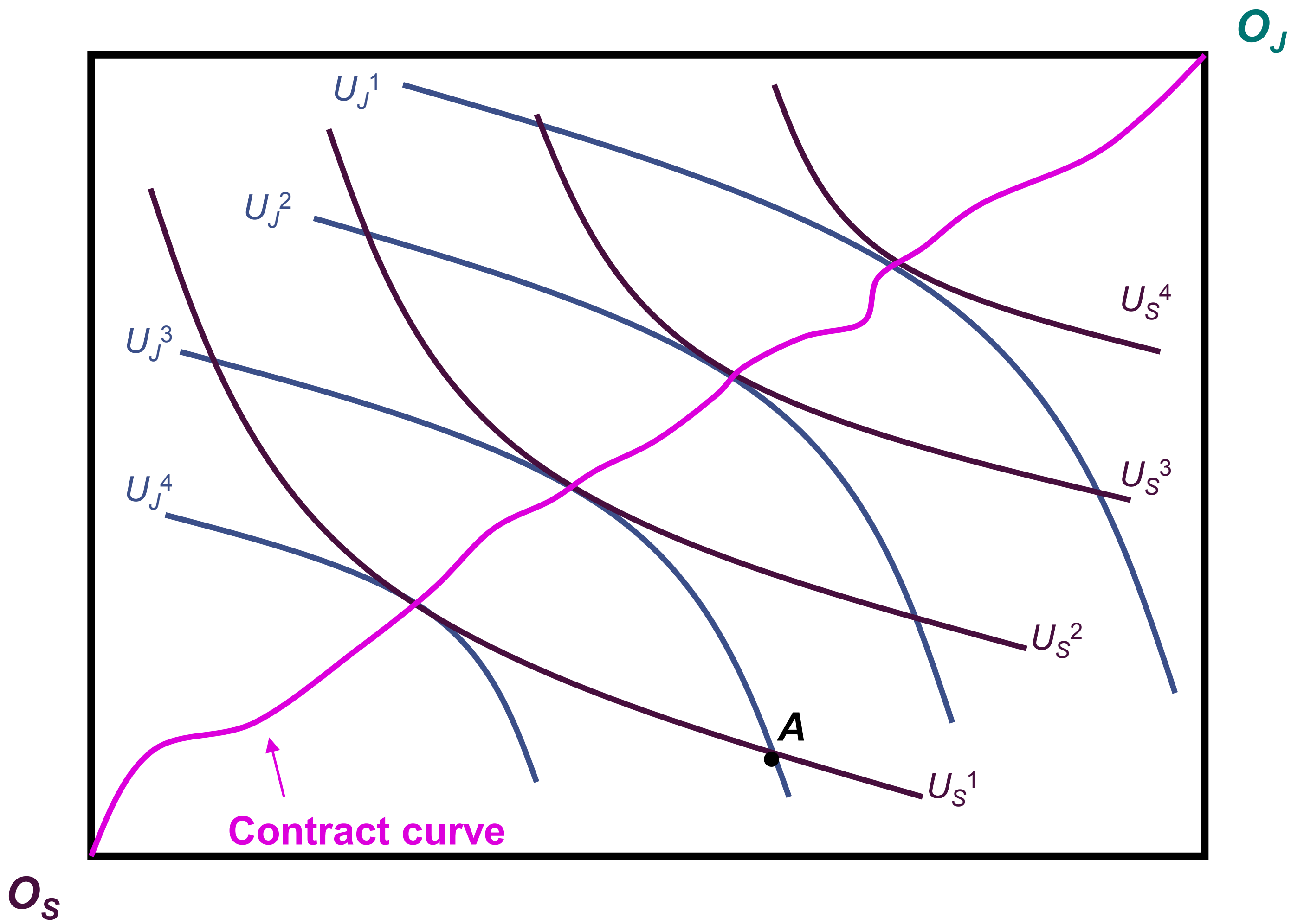
O_J
Total Y
O_S
Total X
U_J^1
U_J^2
U_J^3
U_J^4
U_S^1
U_S^2
U_S^3
U_S^4

- Any point within the Edgeworth box in which the *MRS* for Smith is unequal to that for Jones offers an opportunity for Pareto improvements
  - both can move to higher levels of utility through trade



# Contract Curve

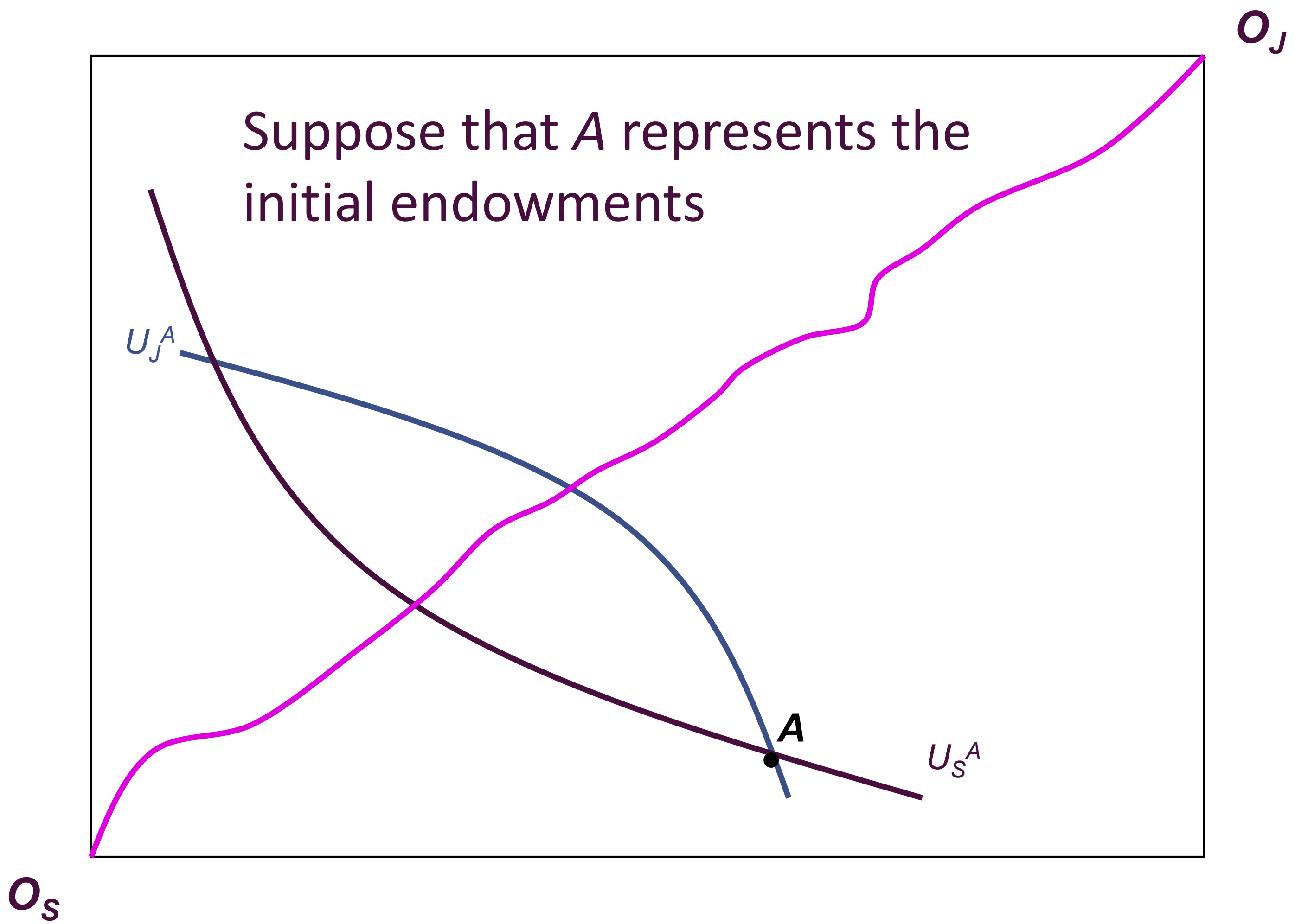
- In an exchange economy, all efficient allocations lie along a <u>contract curve</u>
  - points off the curve are necessarily inefficient
    - individuals can be made better off by moving to the curve
- Along the contract curve, individuals' preferences are rivals
  - one may be made better off only by making the other worse off



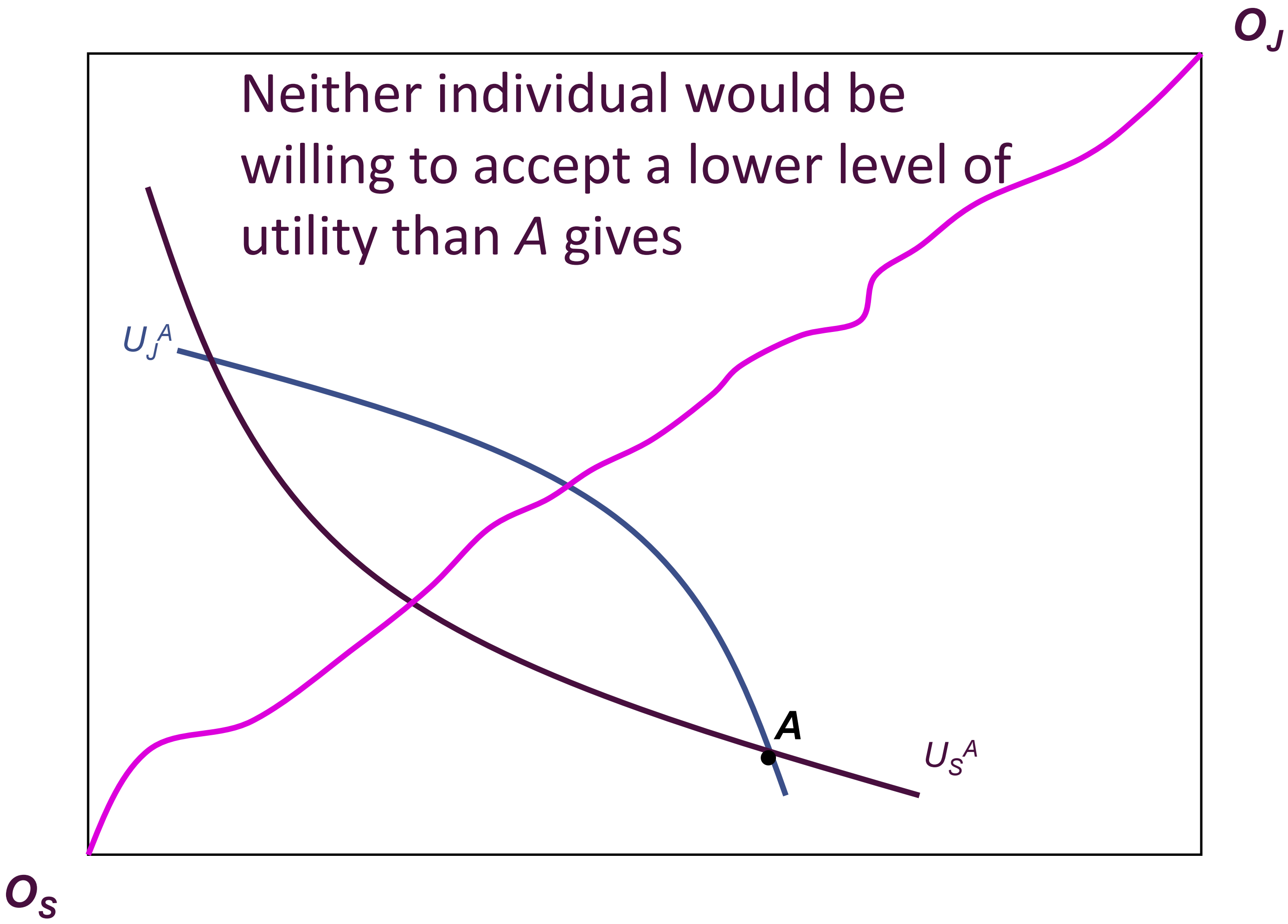
$O_J$
$U_J^1$
$U_J^2$
$U_J^3$
$U_J^4$
$U_S^4$
$U_S^3$
$U_S^2$
$U_S^1$
A
Contract curve
$O_S$

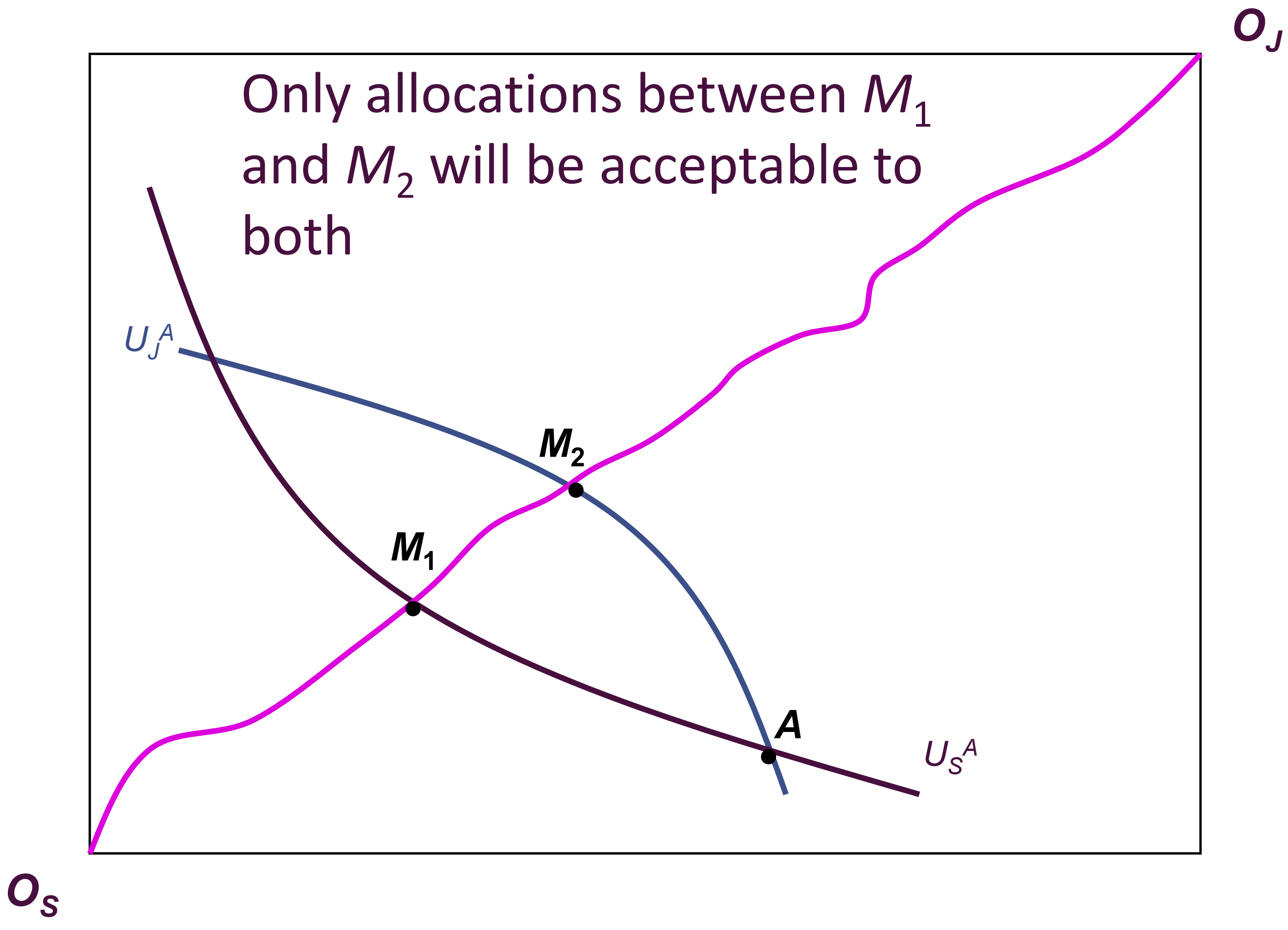
# Exchange with Initial Endowments

- Suppose that the two individuals possess different quantities of the two goods at the start
  - it is possible that the two individuals could both benefit from trade if the initial allocations were inefficient
- Neither person would engage in a trade that would leave him worse off
- Only a portion of the contract curve shows allocations that may result from voluntary exchange


Suppose that A represents the initial endowments
$O_J$
$U_J^A$
A
$U_S^A$
$O_S$

Neither individual would be willing to accept a lower level of utility than *A* gives




Only allocations between $M_1$ and $M_2$ will be acceptable to both
$O_J$
$U_J^A$
$M_2$
$M_1$
$A$
$U_S^A$
$O_S$

## The Distributional Dilemma: Second Theorem of Welfare Economics”

- If the initial endowments are skewed in favor of some economic actors, the Pareto efficient allocations promised by the competitive price system will also tend to favor those actors
  - voluntary transactions cannot overcome large differences in initial endowments
  - some sort of transfers will be needed to attain more equal results
- These thoughts lead to the “Second Theorem of Welfare Economics”:

  *any desired distribution of welfare among individuals in an economy can be achieved in an efficient manner through competitive pricing if initial endowments are adjusted appropriately*