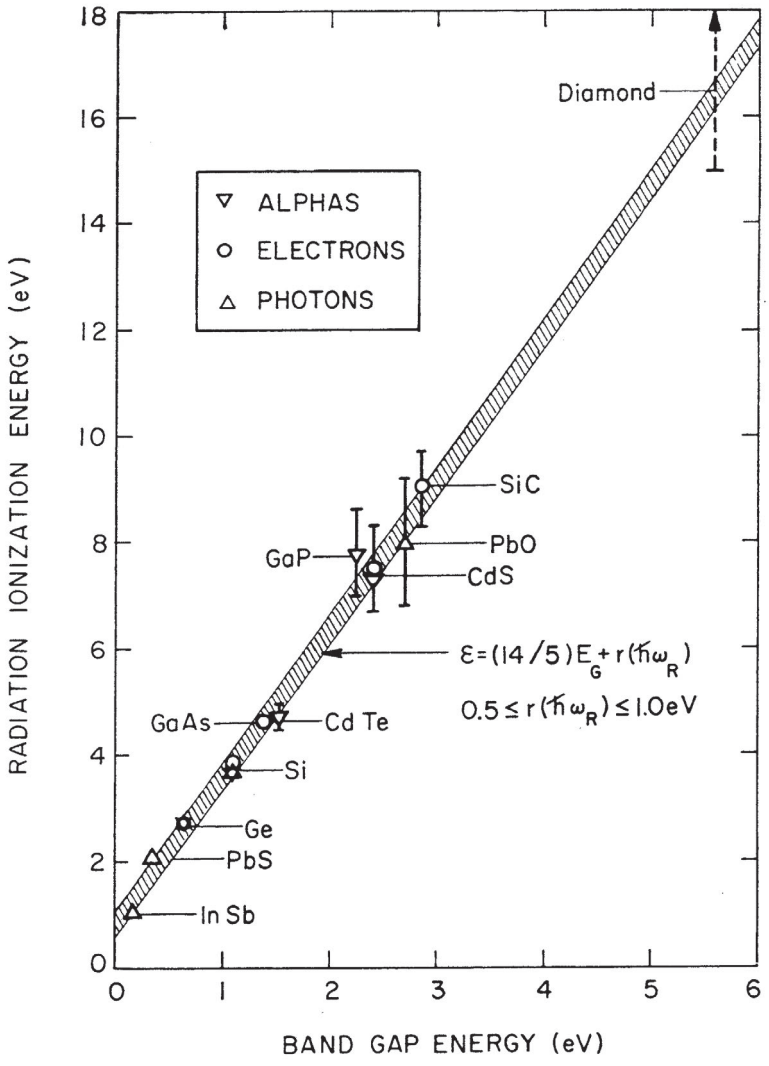
IX. Semiconductor Detectors – Part II

1. Fluctuations in the Signal Charge: the Fano Factor

It is experimentally observed that the energy required to form an electron-hole pair exceeds the bandgap.



C.A. Klein, J. Applied Physics **39** (1968) 2029

The mean ionization energy exceeds the bandgap for two reasons

1. Conservation of momentum requires excitation of lattice vibrations

2. Many modes are available for the energy transfer with an excitation energy less than the bandgap.

Two types of collisions are possible:

a) Lattice excitation, i.e. phonon production (with no formation of mobile charge).

b) Ionization, i.e. formation of a mobile charge pair.

Assume that in the course of energy deposition

N_x excitations produce N_P phonons and

N_i ionization interactions form N_Q charge pairs.

On the average, the sum of the energies going into excitation and ionization is equal to the energy deposited by the incident radiation

$$E_0 = E_i N_i + E_x N_x$$

where E_i and E_x are the energies required for a single excitation or ionization.

Assuming gaussian statistics, the variance in the number of excitations

$$\sigma_x = \sqrt{N_x}$$

and the variance in the number of ionizations

$$\sigma_i = \sqrt{N_i}$$

For a single event, the energy E_0 deposited in the detector is fixed (although this may vary from one event to the next).

If the energy required for excitation E_x is much smaller than required for ionization E_i, sufficient degrees of freedom will exist for some combination of ionization and excitation processes to dissipate precisely the total energy. Hence, for a given energy deposited in the sample a fluctuation in excitation must be balanced by an equivalent fluctuation in ionization.

$$E_x \Delta N_x + E_i \Delta N_i = 0$$

If for a given event more energy goes into charge formation, less energy will be available for excitation. Averaging over many events this means that the variances in the energy allocated to the two types of processes must be equal

$$E_i \sigma_i = E_x \sigma_x$$

$$\sigma_i = \frac{E_x}{E_i} \sqrt{N_x}$$

From the total energy $E_i N_i + E_x N_x = E_0$

$$N_x = \frac{E_0 - E_i N_i}{E_x}$$

yielding

$$\sigma_i = \frac{E_x}{E_i} \sqrt{\frac{E_0}{E_x} - \frac{E_i}{E_x} N_i}$$

Since each ionization leads to a charge pair that contributes to the signal

$$N_i = N_Q = \frac{E_0}{\varepsilon_i}$$

where ε_i is the average energy loss required to produce a charge pair,

$$\sigma_i = \frac{E_x}{E_i}\sqrt{\frac{E_0}{E_x} - \frac{E_i}{E_x}\frac{E_0}{\varepsilon_i}}$$

$$\sigma_i = \sqrt{\frac{E_0}{\varepsilon_i}} \cdot \sqrt{\frac{E_x}{E_i}\left(\frac{\varepsilon_i}{E_i} - 1\right)}$$

The second factor on the right hand side is called the Fano factor F.

Since σ_i is the variance in signal charge Q and the number of charge pairs is $N_Q = E_0/\varepsilon_i$

$$\sigma_Q = \sqrt{FN_Q}$$

In Silicon $E_x = 0.037$ eV
$E_i = E_g = 1.1$ eV
$\varepsilon_i = 3.6$ eV

for which the above expression yields $F = 0.08$, in reasonable agreement with the measured value $F = 0.1$.

$\Rightarrow$ The variance of the signal charge is smaller than naively expected

$$\sigma_Q \approx 0.3\sqrt{N_Q}$$

A similar treatment can be applied if the degrees of freedom are much more limited and Poisson statistics are necessary.

However, when applying Poisson statistics to the situation of a fixed energy deposition, which imposes an upper bound on the variance, one can not use the usual expression for the variance

$$\text{var } N = \overline{N}$$

Instead, the variance is

$$\overline{(N - \overline{N})^2} = F\overline{N}$$

as shown by Fano [1] in the original paper.

An accurate calculation of the Fano factor requires a detailed accounting of the energy dependent cross sections and the density of states of the phonon modes. This is discussed by Alkhazov [2] and van Roosbroeck [3].

References:

1. U. Fano, Phys. Rev. **72** (1947) 26

2. G.D. Alkhazov et al., NIM **48** (1967) 1

3. W. van Roosbroeck, Phys. Rev. **139** (1963) A1702