Exercise 1.

Consider the theory of a real scalar field ϕ in $d = 4$ spacetime dimensions with Lagrangian

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi_0)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{3!}\phi^3 \ . \tag{1}$$

- What is the energy dimension of the coupling λ_0?
- Derive the expression for the superficial degree of divergence D of the theory and classify it.

Introduce the renormalized field $\phi_r = Z^{-1/2}\phi_0$ (with $Z = 1 + \delta_Z$), mass $m_0^2 Z = m^2 + \delta m$, and coupling $Z^{3/2}\lambda_0 = \lambda + \delta\lambda$.

- Compute the two-point Green function $i\Pi(p^2)$ at one-loop, regularizing the divergence in dim-reg. Renormalize the theory by imposing the renormalization conditions

$$\Pi(m^2) = 0 \qquad \text{and} \qquad \left.\frac{d\Pi(p^2)}{dp^2}\right|_{p^2=m^2} = 0 \ . \tag{2}$$

- Compute the one-point Green function $\langle 0|\phi(x)|0\rangle$ at one-loop and renormalize it in the $\overline{MS}$ scheme. Do you need to add any additional counter-term to the Lagrangian to be able to absorb this divergence?
- What is the physical interpretation of this one-point Green's function?