MECCANICA RAZIONALE

— LEZIONE DEL 12 MAGGIO 2020

— PRIMA PARTE

Modi normali:

Sistemi meccanici $\longrightarrow$ equazioni di Lagrange
sistema dinamico
$(\underline{p}, \underline{q})$

$\rightarrow$ equazioni differenziali complicate

$q_i = 1, \dots, l$ $\quad$ l eq 2° ordine

$2l$ eq 1° ordine

$\rightarrow$ eq. accoppiate (l'eq. per la coordinata q_1, contiene tutte le altre q_i)

$\rightarrow$ metodi di approssimazione.

Il più comune è la linearizzazione

$\rightarrow$ sol di equilibrio $\underline{q}_E$, $\underline{q} = \underline{q}_E + \underline{\eta}$

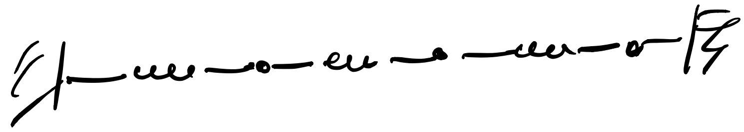
sviluppare in serie in q, tenendo l'ordine più basso

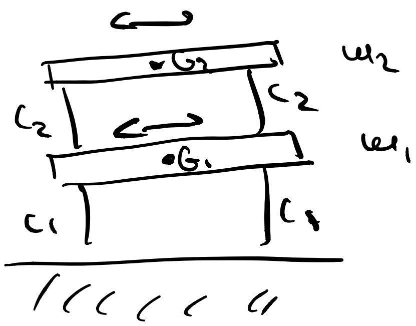
Eq. differenziali $\rightsquigarrow$ $A\ddot{\underline{x}} + C\underline{x} = 0$

$A \rightsquigarrow A(\underline{q})$ calcolato in $\underline{q}_E$

$C \rightsquigarrow \mathrm{Hess}\, V$ calcolato in $\underline{q}_E$

$K = \frac{1}{2}\dot{\underline{x}} A \dot{\underline{x}}$, $V = \frac{1}{2}\underline{x}\cdot C\cdot\underline{x}$ $\qquad \mathcal{L} = K - V$

Sistemi meccanici : 



$\to$ sistemi meccanici lineari

$A\ddot{\underline{x}} + C\underline{x} = \underline{0}$

$\to$ accoppiate.

$$\begin{pmatrix} \cdot & & \cdot \\ & \cdot & \cdot \\ \cdot & \cdot & \end{pmatrix}\begin{pmatrix}\ddot{x}_1 \\ \vdots \\ \ddot{x}_\ell\end{pmatrix} + \begin{pmatrix} - & & \cdot \\ - & & \cdot \\ - & & \cdot \end{pmatrix}\begin{pmatrix}x_1 \\ \vdots \\ x_\ell\end{pmatrix} = 0$$

diagonalizzare

$$\begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_\ell \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \vdots \\ \ddot{x}_\ell \end{pmatrix} + \dots \longrightarrow \begin{matrix} \ddot{x}_1 \lambda_1 + \dots = 0 \\ \vdots \\ \ddot{x}_\ell \lambda_\ell + \dots = 0 \end{matrix}$$

$$A = \begin{pmatrix} \vdots & \vdots \\ \vdots & \vdots \end{pmatrix} \to \det(A - \lambda I) = 0 \to \lambda$$

$$(A - \lambda I) \underline{u} = 0 \to \text{autovettori}$$

Teorema delle coordinate normali

$$K = \frac{1}{2} \dot{\underline{x}} A \dot{\underline{x}} \qquad\qquad K = \frac{1}{2} \dot{\underline{\xi}}\, \mathbb{1}\, \dot{\underline{\xi}}$$

$$\longrightarrow$$

$$V = \frac{1}{2} \underline{x} C \underline{x} \qquad\qquad V = \frac{1}{2} \underline{\xi} \tilde{C} \underline{\xi}$$

$\underline{x}$ coordinate fisiche $\longleftrightarrow$ $\underline{\xi}$ coordinate normali

K, V diagonali

$$A \ddot{\underline{x}} + C \underline{x} = \underline{0} \longleftrightarrow \ddot{\xi}_i + \gamma_i \xi_i = 0 \qquad i = 1, \dots, \ell$$

S_1		S_2		S_3
$\underline{x} = S_1 \underline{y}$	$\longrightarrow$	$y_i = \frac{1}{\sqrt{\mu_i}} z_i$	$\longrightarrow$	$\underline{z} = S_3 \underline{\xi}$
diagonalizza A		riscala		diagonalizza con trasf. ortogonale $S_3^T S_3 = 1$

$$\underline{x} = (\underline{s}_1\, \underline{s}_2\, \underline{s}_3)\,\underline{\xi} = S\,\underline{\xi}$$

$\ddot{\xi}_i + \gamma_i \xi_i = 0$ si risolve $\underline{v}^{(i)} e^{\lambda_i t}$

dove $\lambda_i^2 + \gamma_i = 0$ $\quad \underline{v}^{(i)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$ ← posizione i-esima

$$v^{(1)} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad v^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix} \quad \rightarrow$$

$$\underline{v}^{(1)} e^{\lambda_1 t} + \underline{v}^{(2)} e^{\lambda_2 t} + \ldots$$

si muove in direzione $(1, 0, \ldots, 0)$

in un'altra direzione

Casi possibili $\quad \ddot{\xi}_i + \gamma_i \xi_i = 0$

- $\gamma_i > 0 \quad \gamma_i = \omega_i^2, \ \omega_i > 0 \ \rightarrow \ \underline{v}^{(i)} \sin\omega_i t, \ \underline{v}^{(i)} \cos\omega_i t$
- $\gamma_i < 0 \quad \gamma_i = -\nu_i^2, \ \nu_i > 0 \ \rightarrow \ \underline{v}^{(i)} e^{\nu_i t}, \ \underline{v}^{(i)} e^{-\nu_i t}$
- $\gamma_i = 0 \quad \rightarrow \ \underline{v}^{(i)}, \ \underline{v}^{(i)} t$

$(\ddot{\xi} = 0)$

$\boxed{A\ddot{\underline{x}} + C\underline{x} = 0} \longrightarrow \ddot{\xi}_i + \gamma_i \xi_i = 0$ coordinate normali

$\longrightarrow$ le risolviamo $\sin\omega_i t$, $\cos\omega_i t$, exp ... $\longrightarrow$ $\boxed{\text{Torniamo indietro}}$

Torniamo indietro alle coordinate fisiche

$\underline{x} = S \underline{\xi}$

$\ddot{\xi}_i + \gamma_i \xi_i = 0$ $\qquad\qquad$ $A \ddot{\underline{x}} + C \underline{x} = 0$

La soluzione $\underline{v}^{(i)} e^{\lambda_i T}$ diventa

$$S\left(\underline{v}^{(i)} e^{\lambda_i T}\right) = e^{\lambda_i T} S\left(\underline{v}^{(i)}\right)$$

$$= e^{\lambda_i T} \underline{u}^{(i)} \qquad \underline{u}^{(i)} := S(\underline{v}^{(i)})$$

Quindi $\lambda_i^2 A \underline{u}^{(i)} + C \underline{u}^{(i)} = \underline{0}$

Abbiamo che

- $\lambda_i^2 = -\gamma_i \in \mathbb{R} \qquad i = 1, \dots, l$
- $\underline{u}^{(i)} \in \mathbb{R}^l$

$$\underline{u}^{(i)} \cdot A \underline{u}^{(j)} = (\underline{u}^{(i)})^T A \underline{u}^{(j)} =$$

$$= (\underline{v}^{(i)})^T \underbrace{S^T A S}_{\substack{\text{identità} \\ Id}} \underline{v}^{(j)} = (\underline{v}^{(i)})^T \underline{v}^{(j)}$$

$\begin{pmatrix} 1 & \end{pmatrix}$ i-esimo $\qquad \begin{pmatrix} \\ 1 \\ \end{pmatrix}$ j-esimo

$$= \delta_{ij} \quad \left(\begin{cases} 1 & \text{se } i = j \\ 0 & \text{altrimenti} \end{cases} \right)$$

D'altra parte, se partiamo da $A\ddot{\underline{x}} + C\underline{x} = \underline{0}$ e cerchiamo soluzioni del tipo $\underline{x}(t) = \underline{u}\, e^{\lambda t}$

$\Rightarrow$ troviamo $\lambda^2 A\underline{u} + C\underline{u} = 0$

$$\boxed{(\lambda^2 A + C)\,\underline{u} = 0}$$

- λ^2 è radice dell'equazione caratteristica

$$\det(\lambda^2 A + C) = 0$$

(autovalore di C relativo ad A)

- $\underline{u}$ risolve $(\lambda^2 A + C)\underline{u} = 0$

autovettore relativo

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$\underline{x}(t) = \underline{u}\, e^{\lambda t} \rightarrow \underbrace{(\lambda^2 A + C)}\,\underline{u} = 0$

$\rightarrow \det(\lambda^2 A + C) = 0$ determina λ

$$\rightarrow \quad (\lambda^2 A + C)\underline{u} = 0 \qquad \text{determino } \underline{u}$$

Consideriamo $\det(\lambda^2 A + C)$

$$\ddot{\xi}_i + \gamma_i \xi_i = 0 \quad \longrightarrow \quad (\lambda_i^2 + \gamma_i) = 0 \qquad i = 1, \dots, \ell$$

$$\prod_i (\lambda^2 + \gamma_i) = 0 \qquad \text{se } \lambda \text{ è uno degli } \lambda_i$$

$$0 = (\lambda^2 + \gamma_1)(\lambda^2 + \gamma_2) \cdots (\lambda^2 + \gamma_\ell) =$$

$$= \det(\lambda^2 I_d + F) =$$

$$F = \operatorname{diag}(\gamma_1, \dots, \gamma_\ell)$$

Ricordiamo: dal Teorema dei modi normali

$$S^T A S = I_d$$

$$S^T C S = \operatorname{diag}(\gamma_1, \dots, \gamma_\ell) = F$$

$$= \det(\lambda^2 S^T A S + S^T C S)$$

$$= \det(S^T [\lambda^2 A + C] S)$$

$$= \det S \cdot \det(\lambda^2 A + C) \cdot \det S$$

$$= \underbrace{(\det S)^2}_{\neq 0} \cdot \det(\lambda^2 A + C)$$

λ risolve $(\lambda^2+\gamma_1)(\lambda^2+\gamma_2)\cdots(\lambda^2+\gamma_\ell)=0$

(una delle sol $\lambda_i^2+\gamma_i=0$)

solo se è una radice di $\det(\lambda^2 A+C)$

Messaggio: le frequenze dei modi normali sono radici di $\det(\lambda^2 A + C) = 0$

$A\ddot{\underline{x}} + C\underline{x} = \underline{0}$

- λ^2 è radice di $\det(\lambda^2 A+C)=0$
- $\underline{u}$ nella soluzione $\underline{x}(t) = \underline{u}\, e^{\lambda t}$

soddisfa $(\lambda^2 A + C)\underline{u} = 0$

1) l' eq. algebrica $\det(\lambda^2 A + C)$ ha ℓ radici

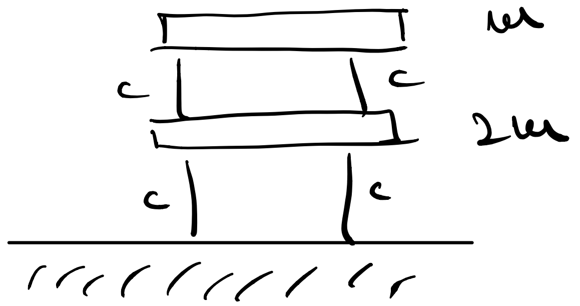
2) il sistema omogeneo $(\lambda^2 A + C)\underline{u} = \underline{0}$ ha ℓ soluzioni reali e distinte tali che

$$\underline{u}^{(i)} A\, \underline{u}^{(j)} = \delta_{ij} \qquad i,j = 1,\dots,\ell$$

condizione di ortonormalità

$A\ddot{\underline{x}} + C\underline{x} = 0 \qquad \underline{x} = \underline{u}\, e^{\lambda t}$

Esempio



$$\begin{cases} \dfrac{m}{2c}\, 2\, \dfrac{d^2}{dt^2} x_1 + 2x_1 - x_2 = 0 \\[2ex] \dfrac{m}{2c}\, \dfrac{d^2}{dt^2} x_2 - x_1 + x_2 = 0 \end{cases}$$

Cambiamo variabile : $\tau = \sqrt{\dfrac{2c}{m}}\, t$

$$\frac{m}{2c}\frac{d^2}{dt^2} = \frac{d^2}{d\left(\frac{2c}{m} t\right)^2} = \frac{d^2}{d\tau^2}$$

Adesso $A \dfrac{d^2}{d\tau^2}\underline{x} + C\underline{x} = 0$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

Cerchiamo soluzioni del tipo

$$\underline{x}(t) = \underline{u}\, e^{\lambda\tau}$$

$$\lambda^2 A + C = \begin{pmatrix} 2\lambda^2 + 2 & -1 \\ -1 & \lambda^2 + 1 \end{pmatrix}$$

$\underline{u} \longrightarrow (\lambda^2 A + C)\underline{u} = 0$

$\lambda^2 \longrightarrow \det(A\lambda^2 + C) = 0$

$$\det\left(A\lambda^2 + C\right) = 2\left(\lambda^2+1\right)^2 - 1 = 0$$

$$\left(\lambda^2+1\right) = \pm \frac{1}{\sqrt{2}} \longrightarrow \lambda^2 = -1 \pm \frac{1}{\sqrt{2}} < 0$$

Frequenze di oscillazione $(\lambda = \pm i\omega)$

$$\omega_1 = \left(1 - \frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} \qquad \omega_2 = \left(1 + \frac{1}{\sqrt{2}}\right)^{\frac{1}{2}}$$

$$\lambda_1^2 = -1 + \frac{1}{\sqrt{2}} = -\left(1 - \frac{1}{\sqrt{2}}\right) = -\omega_1^2$$

Vogliamo $\underline{u}$ tali che $(\lambda^2 A + C)\,\underline{x} = 0$

$$\begin{pmatrix} 2\lambda^2 + 2 & -1 \\ -1 & \lambda^2+1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{cases} 2(\lambda^2+1)\,u_1 - u_2 = 0 \\ -1 \cdot u_1 + (\lambda^2+1)\,u_2 = 0 \end{cases}$$

Prendiamo la prima eq.

$$\lambda_{(1)}^2 = -1 + \frac{1}{\sqrt{2}} \; : \quad 2\left(-1 + \frac{1}{\sqrt{2}} + 1\right) u_1 - u_2 =$$

$$= \frac{2}{\sqrt{2}}\,u_1 - u_2 = 0$$

Troviamo $u_2 = \sqrt{2}\,u_1$

Il vettore $\underline{u}^{(1)}$ associato a $\lambda_{(1)}^2$

è $\underline{u}^{(1)} = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} b_1 \qquad b_1 \in \mathbb{R}$

Se invece $\lambda^2_{(2)} = -1 - \frac{1}{\sqrt{2}}$

$$2\left(-1 - \frac{1}{\sqrt{2}} + 1\right) u_1 - u_2 = 0$$

$$\lambda^2_{(2)} \rightarrow \underline{u}^{(2)} = b_2 \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} \qquad b_2 \in \mathbb{R}$$

Abbiamo trovato i modi di vibrazione

$$\underline{u}^{(1)} \sin \omega_1 \tau \qquad \underline{u}^{(1)} \cos \omega_1 \tau$$

$$\underline{u}^{(2)} \sin \omega_2 \tau \qquad \underline{u}^{(2)} \cos \omega_2 \tau$$

La soluzione generale è

$$\underline{x}(T) = \underline{u}^{(1)} \left(k_1 \sin \omega_1 \tau + k_2 \cos \omega_1 \tau \right)$$

$$+ \underline{u}^{(2)} \left(k_3 \sin \omega_2 \tau + k_4 \cos \omega_2 \tau \right)$$

$$\tau = \sqrt{\frac{2c}{m}}\, t$$

Le costanti k_1, k_2, k_3, k_4 sono determinate da $\underline{x}(0)$ e $\dot{\underline{x}}(0)$

Condizioni di normalizzazione

$$\underline{u}^{(1)} A \, \underline{u}^{(1)} = b_1 (1 \;\; \sqrt{2}) \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} b_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}$$

$$\underline{u}^{(1)} = b_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix}$$

$$= b_1^2\, 4 \equiv 1 \quad \rightarrow \quad b_1 = \frac{1}{2}$$

$\uparrow$ normalizzazione

$$\underline{u}^{(2)}\, A\, \underline{u}^{(2)} = b_2^2 \begin{pmatrix} 1 & -\sqrt{2} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix}$$

$$= 4\, b_2^2 \equiv 1 \quad \Rightarrow \quad b_2 = \frac{1}{2}$$

Non è necessario perché

$$\underline{x}(t) = b_1 \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} \left(k_1 \sin \omega_1 \tau + k_2 \cos \omega_1 \tau \right)$$

$$+ b_2 \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} \left(k_3 \sin \omega_2 \tau + k_4 \cos \omega_2 \tau \right)$$

Poniamo $b_1 = b_2 = 1$

Ad es. $\underline{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad \dot{\underline{x}}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\underline{x}(0) = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} k_2 + \begin{pmatrix} 1 \\ -\sqrt{2} \end{pmatrix} k_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} k_2 + k_4 &= 1 \\ \sqrt{2} k_2 - \sqrt{2} k_4 &= 0 \end{aligned} \qquad k_2 = k_4 = \frac{1}{2}$$

$$\dot{\underline{x}}(0) = \begin{pmatrix} k_1 \omega_1 \sqrt{\frac{2c}{m}} + k_3 \omega_3 \sqrt{\frac{2c}{m}} \\ \sqrt{2} k_1 \omega_1 \sqrt{\frac{2c}{m}} - k_3 \omega_3 \sqrt{\frac{2c}{m}} \sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \quad k_1 = k_3 = 0$$

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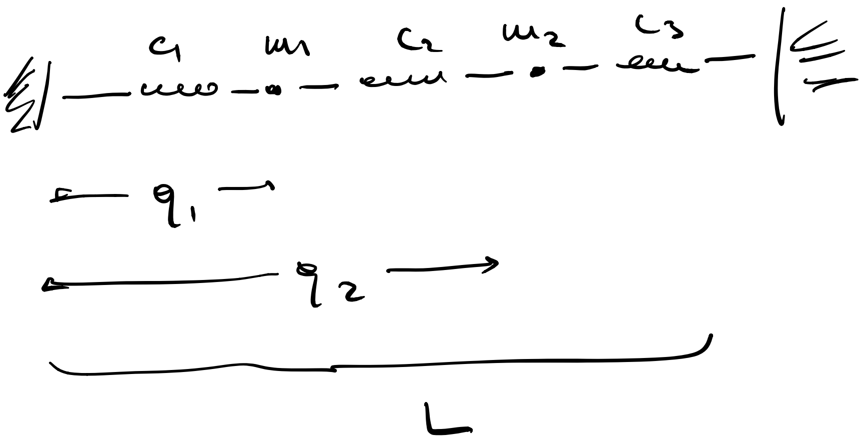
Esercizio



$m_1 = 3m$

$m_2 = 2m$

$c_1 = 2c$

$c_2 = 2c$

$c_3 = c$

1) le configurazioni di equilibrio
2) le due frequenze di oscillazione
3) i due modi normali
4) sol con dati iniziali $\begin{cases} q_1(0) = 0 \\ q_2(0) = \frac{L}{2} \end{cases}$ $\begin{cases} \dot q_1(0) = 0 \\ \dot q_2(0) = 0 \end{cases}$

1) Ricordiamo $A\underline{\ddot q} + C\underline{q} - \underline{b} = \underline{0}$

$$A = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad C = \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 0 \\ c_3 L \end{pmatrix}$$

nel nostro caso

$$A = \begin{pmatrix} 3m & 0 \\ 0 & 2m \end{pmatrix} \qquad C = \begin{pmatrix} 4c & -2c \\ -2c & 3c \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 0 \\ cL \end{pmatrix}$$

Per trovare l'eq. $C \underline{q}_{\varepsilon} = \underline{b}$

$$\left(A \ddot{\underline{q}} + C \underline{q} - \underline{b} = 0 \rightarrow \ddot{\underline{q}}_{\varepsilon} = 0 \right)$$

cioè $\begin{pmatrix} 4c & -2c \\ -2c & 3c \end{pmatrix} \begin{pmatrix} q_{1,\varepsilon} \\ q_{2,\varepsilon} \end{pmatrix} = \begin{pmatrix} 0 \\ cL \end{pmatrix}$

$$\begin{cases} 4c\, q_{1,\varepsilon} - 2c\, q_{2,\varepsilon} = 0 \\ -2c\, q_{1,\varepsilon} + 3c\, q_{2,\varepsilon} = cL \end{cases} \Rightarrow \begin{cases} q_{1,\varepsilon} = \frac{L}{4} \\ q_{2,\varepsilon} = \frac{L}{2} \end{cases}$$

Poniamo $x_1 = q_1 - \frac{L}{4}$, $x_2 = q_2 - \frac{L}{2}$

$$A \ddot{\underline{x}} + C \underline{x} = \underline{0}$$

2) Possiamo riscriverla

$$m \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \frac{d^2}{dT^2} \underline{x} + \begin{pmatrix} 4c & -2c \\ -2c & 3c \end{pmatrix} \underline{x} = \underline{0}$$

$$\frac{m}{c} \frac{d^2}{dT^2} = \frac{d^2}{d\left(\sqrt{\frac{c}{m}}T\right)^2} = \frac{d^2}{d\tau^2} \qquad \tau = \sqrt{\frac{c}{m}}\, T$$

$$\underbrace{\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}}_{\tilde{A}} \frac{d^2}{dt^2} \underline{x} + \underbrace{\begin{pmatrix} 4 & -2 \\ -2 & 3 \end{pmatrix}}_{\tilde{C}} \underline{x} = \underline{0}$$

l'eq. per le frequenze

$$\lambda^2 \tilde{A} + \tilde{C} = \begin{pmatrix} 3\lambda^2 + 4 & -2 \\ -2 & 2\lambda^2 + 3 \end{pmatrix}$$

$$\det\left(\lambda^2 \tilde{A} + \tilde{C}\right) = (3\lambda^2 + 4)(2\lambda^2 + 3) - 4$$

$$= 6\lambda^4 + 9\lambda^2 + 8\lambda^2 - 4 = 6\lambda^4 + 17\lambda^2 + 8 = 0$$

$+12$

vogliamo λ^2

$$\lambda^2 = \frac{-17 \pm \sqrt{17^2 - 4 \cdot 6 \cdot 8}}{12} = \frac{-17 \pm \sqrt{97}}{12}$$

$289 - 192$

$$\omega_1 = \sqrt{\frac{17 - \sqrt{97}}{12} \frac{c}{m}}$$

$$\omega_2 = \sqrt{\frac{17 + \sqrt{97}}{12} \frac{c}{m}}$$

3) i due modi normali, da

$$(\lambda^2 \tilde{A} + \tilde{C}) \underline{u} = 0$$

$$\begin{pmatrix} 3\lambda^2+4 & -2 \\ -2 & 2\lambda^2+3 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0$$

con i due valori di λ^2 appena trovati

$$\begin{cases} (3\lambda^2+4)\,u_1 - 2u_2 = 0 \\ \underline{-2u_1 + (2\lambda^2+3)\,u_2 = 0} \end{cases}$$

- $$\lambda^2_{(1)} = \frac{-17+\sqrt{97}}{12}$$

$$-2u_1^{(1)} + 2\left(\frac{-17+\sqrt{97}}{12} + 3\right) u_2^{(1)} = 0$$

$$\frac{-17+\sqrt{97}}{6} + \frac{18}{6}$$

$$\rightarrow \quad u_1^{(1)} = \frac{1+\sqrt{97}}{12}\, u_2^{(1)}$$

$$\underline{u}^{(1)} = \begin{pmatrix} u_1^{(1)} \\ u_2^{(1)} \end{pmatrix} = \begin{pmatrix} \frac{1+\sqrt{97}}{12} \\ 1 \end{pmatrix} \cancel{b}$$

- $$\lambda^2_{(2)} = \frac{-17-\sqrt{97}}{12} \Rightarrow -2u_1^{(2)} + \left(2\,\frac{-17-\sqrt{97}}{12} + 3\right) u_2^{(2)} = 0$$

$$u_1^{(2)} = \frac{1-\sqrt{97}}{12}\, u_2^{(2)}$$

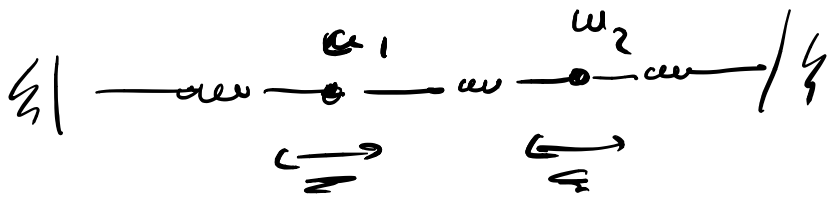
$$\Rightarrow \quad \underline{u}^{(2)} = \begin{pmatrix} \frac{1-\sqrt{97}}{12} \\ 1 \end{pmatrix} \cancel{b}$$

Abbiamo trovato i modi normali

$$\begin{pmatrix} \frac{1+\sqrt{97}}{12} \\ 1 \end{pmatrix} \left(k_1 \sin \omega_1 t + k_2 \cos \omega_2 t \right) \qquad \omega_1 = \sqrt{\frac{17-\sqrt{97}}{12} \frac{c}{m}}$$

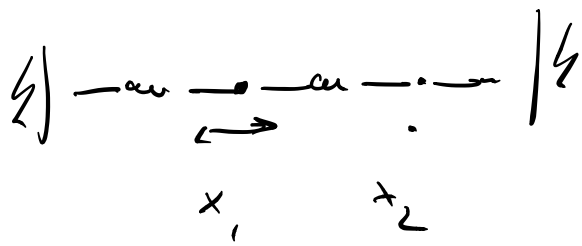
$$\begin{pmatrix} \frac{1-\sqrt{97}}{12} \\ 1 \end{pmatrix} \left(k_3 \sin \omega_2 t + k_4 \cos \omega_2 t \right) \qquad \omega_2 = \sqrt{\frac{17+\sqrt{97}}{12} \frac{c}{m}}$$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$



$$\sim \begin{pmatrix} \frac{1+\sqrt{97}}{12} \\ 1 \end{pmatrix} \sin, \cos$$

$$\omega_0 \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin, \cos$$



4) Dati iniziali $\begin{cases} q_1(0) = 0 \\ q_2(0) = \frac{1}{2} \end{cases}$ $\begin{cases} \dot{q}_1(0) = 0 \\ \dot{q}_2(0) = 0 \end{cases}$

$$\underline{x}(0) = \underline{q}(0) - \underline{q}_E = \begin{pmatrix} -\frac{L}{4} \\ 0 \end{pmatrix} \qquad \underline{\dot{x}}(0) = \underline{0}$$

$$\underline{q}_E = \begin{pmatrix} L/4 \\ L/2 \end{pmatrix}$$

$$\underline{\dot{x}}(0) = \begin{pmatrix} \cdot \ \cdot \end{pmatrix} k_1 \underbrace{\cos \omega_1 \tau}_{=1} + \begin{pmatrix} \vdots \end{pmatrix} k_3 \underbrace{\cos \omega_3 \tau}_{=1} = \underline{0}$$

$$\tau = 0$$

$$k_1 = k_3 = 0$$

$$\underline{x}(0) = \begin{pmatrix} \frac{1+\sqrt{97}}{12} k_2 + \frac{1-\sqrt{97}}{12} k_4 \\ k_2 + k_4 \end{pmatrix} = \begin{pmatrix} -\frac{L}{4} \\ 0 \end{pmatrix}$$

$$k_2 = -k_4$$

$$\frac{1+\sqrt{97}}{12} k_2 - \frac{1-\sqrt{97}}{12} k_2 = \frac{\sqrt{97}}{6} k_2 = -\frac{L}{4}$$

$$\rightsquigarrow k_2 = -\frac{6}{4} \frac{1}{\sqrt{97}} L$$