TRADITIONAL MODELS OF IMPERFECT COMPETITION

Introduction

Here we discuss oligopoly markets, falling between the extremes of perfect competition and monopoly.

Definition of Oligopoly: A market with relatively few firms but more than one.

Oligopolies raise the possibility of strategic interaction among firms.

To analyze this strategic interaction rigorously, we will apply the concepts from game theory that were introduced in the revious lectures

- We will assume that the market is perfectly competitive on the demand side
  - there are many buyers, each of whom is a price taker
- We will assume that the good obeys the law of one price
  - this assumption will be relaxed when product differentiation is discussed
- We will assume that there is a relatively small number of identical firms (n)
  - we will initially start with n fixed, but later allow n to vary through entry and exit in response to firms' profitability
- The output of each firm is q_i (i=1,…,n)
  - symmetry in costs across firms will usually require that these outputs are equal

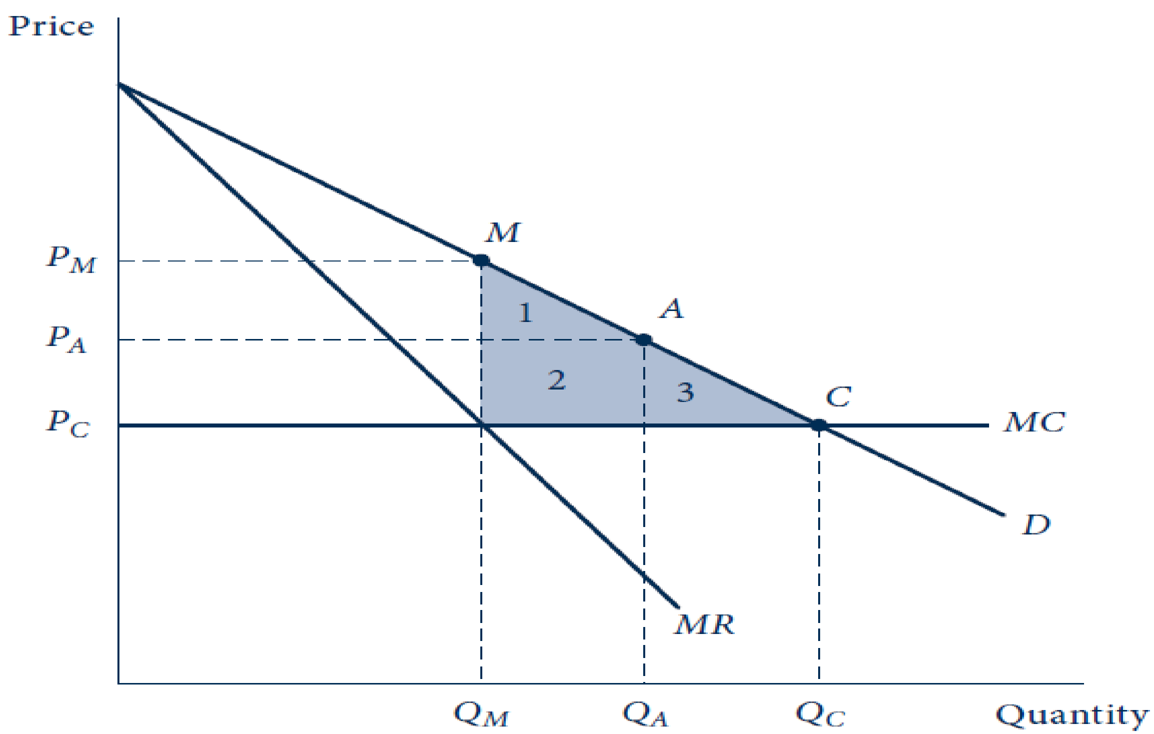
It is difficult to predict exactly the possible outcomes when there are few firms:

- prices depend on how aggressively firms compete,
- it depends on which strategic variables firms choose,
- how much information firms
- how often firms interact with each other in the market.

Bertrand game: it involves two identical firms choosing prices simultaneously for their identical products.

The Bertrand game has a Nash equilibrium at point C: the two firms behave as they were perfectly competitive, setting price equal to marginal cost and earning zero profit.

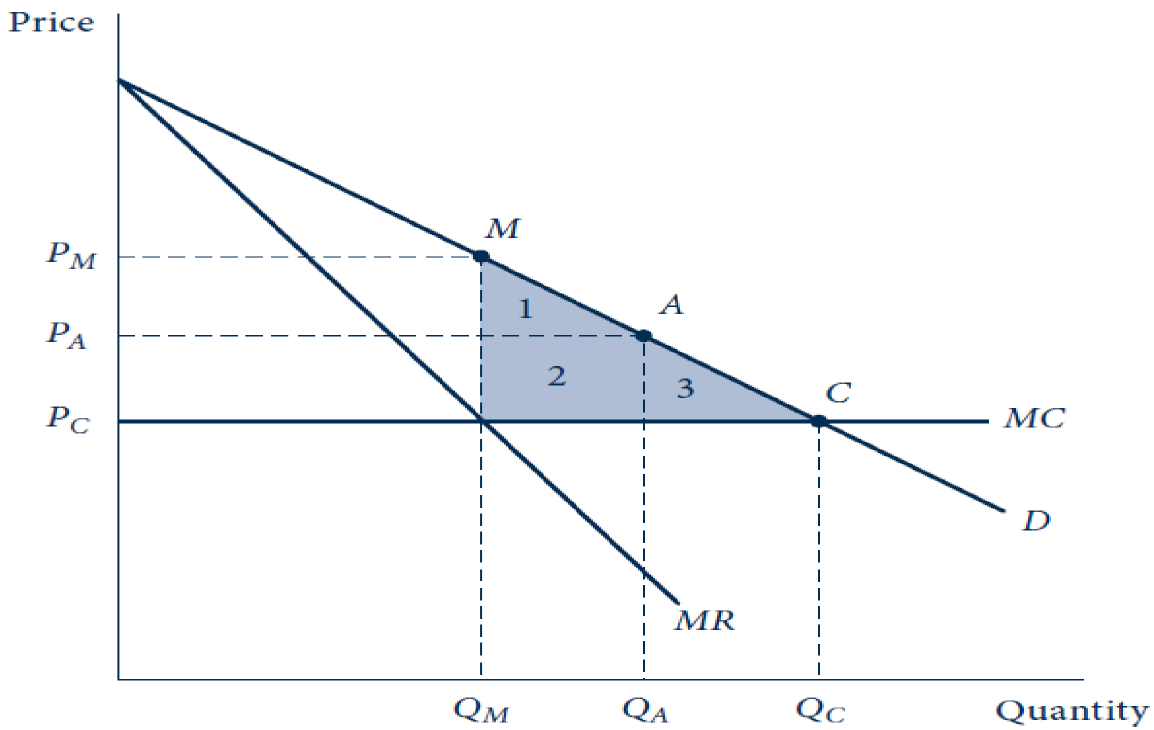
At the other extreme, point M, firms as a group may act as a cartel, recognizing that they can affect price and coordinate their decisions.



At point C, total welfare is as high as possible;

at point A, total welfare is lower by the area of the shaded triangle 3. At point M, deadweight loss is even greater and is given by the area of shaded regions 1, 2, and 3.

The closer the imperfectly competitive outcome to C and the farther from M, the higher is total welfare and the better off society will be.



Bertrand game

Two identical firms (players), labelled 1 and 2, produce identical products at a constant marginal cost (and constant average cost) c.

The firms choose prices p_1 and p_2 simultaneously in a single period of competition (prices are the players' strategies).

Because firms' products are perfect substitutes, all sales go to the firm with the lowest price.

Sales are split evenly if $p_1 = p_2$

Firms want maximize profits

Let D(p) be market demand.

We will look for the Nash equilibrium.

The only pure-strategy Nash equilibrium of the Bertrand game is $p_1 = p_2 = c$

the Nash equilibrium involves both firms charging marginal cost.

This strategy profile is a Nash equilibrium, and there is no other Nash equilibrium.

In equilibrium, firms charge a price equal to marginal cost (which is equal to average cost) and earn zero profit.

If it deviates to a higher price, then it will make no sales and therefore no profit

If it deviates to a lower price, then it will make sales but will be earning a negative profits

Because there is no possible profitable deviation for the firm, we can state that both firms' charging marginal cost is a Nash equilibrium.

Why marginal cost pricing is the only pure-strategy Nash equilibrium?

If prices exceeded marginal cost, the high-price firm would gain by undercutting the other slightly and capturing all the market demand.

Proof:

Assume firm 1 is the low-price firm—that is, $p_1 < p_2$.

There are three exhaustive cases:

1. $c > p_1$,
2. $c < p_1$
3. $c = p_1$

Case 1. $\boldsymbol{c > p_1}$

This cannot be a Nash equilibrium because firm 1 earns negative profits. Deviating to $p_1 = c$ its profits increases to 0 for sure

Case 2. $c < p_1$

Firm 2 could capture all the market demand by undercutting firm 1's price by a tiny amount ε.

The deviation would result in firm 2 moving from zero to positive profit

Then this cannot be a Nash equilibrium

Case 3. $p_1 = c$

Suppose $p_2 > p_1 = c$. This cannot be a Nash equilibrium because firm 1 can increase the price by a small amount ε (p_1 still remains below p_2) and have strictly positive profits.

Then it remains only the case $p_2 = p_1 = c$ that we previously proved to be a Nash equilibrium

Although the analysis focused on the game with two firms, the same outcome would arise for any number of firms $n \geq 2$.

The Nash equilibrium of the n-firm Bertrand game is

$$p_1 = p_2 = \cdots = p_n = c$$

Cournot game

There are n (Players) firms indexed by $i = 1, \ldots, n$ producing an identical product.

Each firm simultaneously chooses its output q_i (strategies)

The outputs are combined into a total industry output $Q = \sum_{i=1}^{n} q_i$

$P(Q)$ is the inverse demand function, that is downward sloping, i.e. $P'(Q) < 0$

Each firm i faces production costs given by $C_i(q_i)$

Profits of firm i are:

$$\pi_i = q_i P(Q) - C_i(q_i)$$

Firms want maximize profits

Nash equilibrium in the Cournot game

The problem of firm i is:

$$\max_{\{q_i\}} \ \pi_i \ \equiv \max_{\{q_i\}} q_i P(Q) - C_i(q_i)$$

The first order condition is:

$$P(Q) + q_i P'(Q) - C_i'(q_i) = 0$$

Solving it by q_i we get the best response of firm i.

In the Nash equilibrium all firms need to use a strategy that is best response to the others' strategies, then the condition above has to hold simultaneously for all firms.

Note that

$P(Q) + q_i P'(Q)$ is the marginal revenue

$C_i'(q_i)$ is the marginal cost

Then the condition above can be written as $MR_i = MC_i$

In the Nash equilibrium

$$P(Q) + q_i P'(Q) - C_i'(q_i) = 0$$

holds for all firm i.

It can be written as

$$P(Q) = C_i'(q_i) - q_i P'(Q)$$

Then the resulting price is above the competitive level ($P = MC$) but below the price of perfect *cartel* that maximizes firms' joint profits.

To see this we analyse the prediction in a market characterized by a cartel

In the cartel all n firms act as an unique firm that choses the quantity Q to produce. This is equivalent to say that, in the Cournot game, they choose the strategy profile that maximize the joint profits

The problem is:

$$\max_{\{q_1,\dots q_n\}} \sum_{i=1}^{n} \pi_i \equiv \max_{\{q_1,\dots q_n\}} P(Q) \sum_{i=1}^{n} q_i - \sum_{i=1}^{n} C_i(q_i)$$

The FOCs are

$$P(Q) + P'(Q) \sum_{i=1}^{n} q_i - C_i'(q_i) = 0 \quad \forall i$$

Solving it by q_i we get the optimal level of production for each firm

Note that in the solution

$$P(Q) = C_i'(q_i) - P'(Q) \sum_{i=1}^{n} q_i$$

Now we compare the equilibrium conditions in the Cournot model with that of the Cartel

In the cartel we have: $P(Q) = C_i'(q_i) - P'(Q)\sum_{i=1}^{n} q_i$

In Cournot model we have: $P(Q) = C_i'(q_i) - P'(Q)q_i$

The two conditions are similar except for the last term on the right

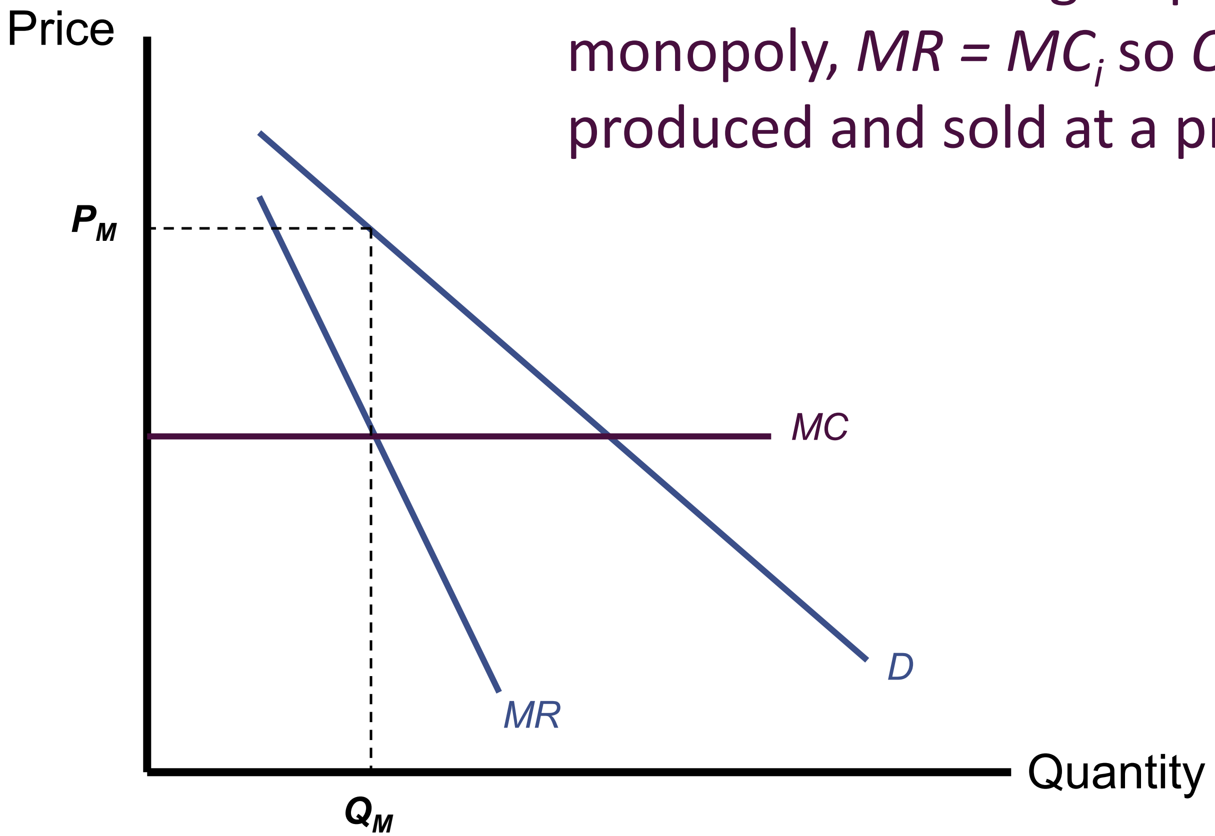
This term is larger for the cartel, then:

- the price in a cartel will be higher than in the Cournot model

- the total quantity will be greater in Cournot respect to the cartel

- In the cartel the deadweight loss is greater than in Cournot

Some comment on the Cartel Model



If the firms form a group and act as a monopoly, $MR = MC_i$ so Q_M output is produced and sold at a price of P_M

There are three problems with the cartel solution

1. these monopolistic decisions may be illegal
2. it requires that the directors of the cartel know the market demand function and each firm's marginal cost function
3. the solution may be unstable: each firm has an incentive to expand output because its best response is different from the optimal condition of the cartel.

   The optimal strategy profile it is not a Nash equilibrium

Natural Springs Duopoly

- Assume that there are two firms exploiting natural springs
- A firm's cost of pumping and bottling q_i liters is $C_i(q_i) = cq_i$
- each firm has to decide how much water to supply to the market
- The inverse demand for spring water is given by:

$$P(Q) = a - Q$$

where a is the intercept and $Q = q_1 + q_2$ is total spring water output.

We will now examine various models of how this market might operate.

Bertrand model

In the Nash equilibrium of the Bertrand game, the two firms set price equal to marginal cost c.

Hence:

- market price is $P = c$,
- total output is $Q = a - c$,
- firm profit is $\pi = 0$
- Total profits are equal to 0
- For the Bertrand quantity to be positive we must have $a > c$

Cournot model

The firms' problems are:

$$\max_{\{q_1\}} \pi_1 \equiv \max_{\{q_1\}} q_1 P(Q) - c\, q_1 \equiv \max_{\{q_1\}} q_1(a - q_1 - q_2) - c\, q_1$$

and

$$\max_{\{q_2\}} \pi_2 \equiv \max_{\{q_2\}} q_2 P(Q) - c\, q_2 \equiv \max_{\{q_2\}} q_2(a - q_1 - q_2) - c\, q_2$$

The first order conditions are:

$$a - 2q_1 - q_2 - c = 0$$
$$a - q_1 - 2q_2 - c = 0$$

Firm 1's best response is:

$$q_1 = \frac{a - q_2 - c}{2}$$

Firm 2's best response is:

$$q_2 = \frac{a - q_1 - c}{2}$$

Firms' best response are:

$$q_1 = \frac{a-q_2-c}{2}\ , \quad q_2 = \frac{a-q_1-c}{2}$$

Solving the equation system (formed by the two FOCs) we get the Nash equilibrium

$$q_1 = q_2 = \frac{a-c}{3}$$

Thus,

total output is $Q = \frac{2\,(a-c)}{3}$.

equilibrium price of $P = \frac{a+2c}{3}$

profits are $\pi_1 = \pi_2 = \frac{(a-c)^2}{9}$

Total profits are $\pi_1 + \pi_2 = \frac{2(a-c)^2}{9}$

Cartel

The cartel's problem is:

$$\max_{\{q_1,q_2\}} \pi_1 + \pi_2 \equiv \max_{\{q_1,q_2\}} (q_1+q_2)P(Q) - c(q_1+q_2) \equiv$$

and

$$\max_{\{q_1,q_2\}} (q_1+q_2)(a - q_1 - q_2) - c(q_1+q_2)$$

The first order conditions are:

$$a - 2q_1 - 2q_2 - c = 0$$
$$a - 2q_1 - 2q_2 - c = 0$$

(note that are equal to each other)

Then in the optimal solution:

$$q_1 + q_2 = \frac{a-c}{2} \text{ and } p = \frac{a+c}{2}$$

total output is $Q = \frac{a-c}{2}$.

equilibrium price of $P = \frac{a+c}{2}$

total profits are $\pi_1 + \pi_2 = \frac{(a-c)^2}{4}$

$$q_1 + q_2 = \frac{a-c}{2} \text{ and } p = \frac{a+c}{2}$$

total output is $Q = \frac{a-c}{2}$.

Suppose that firm 1 produces a share δ of Q and firm 2 produces the remaining share ($\delta \in (0,1)$).

That is $q_1 = \delta \frac{a-c}{2}$ and $q_2 = (1-\delta)\frac{a-c}{2}$

If firm 1 tries to maximize its private profits, its best response is:

$$q_1 = \frac{a - q_2 - c}{2}$$

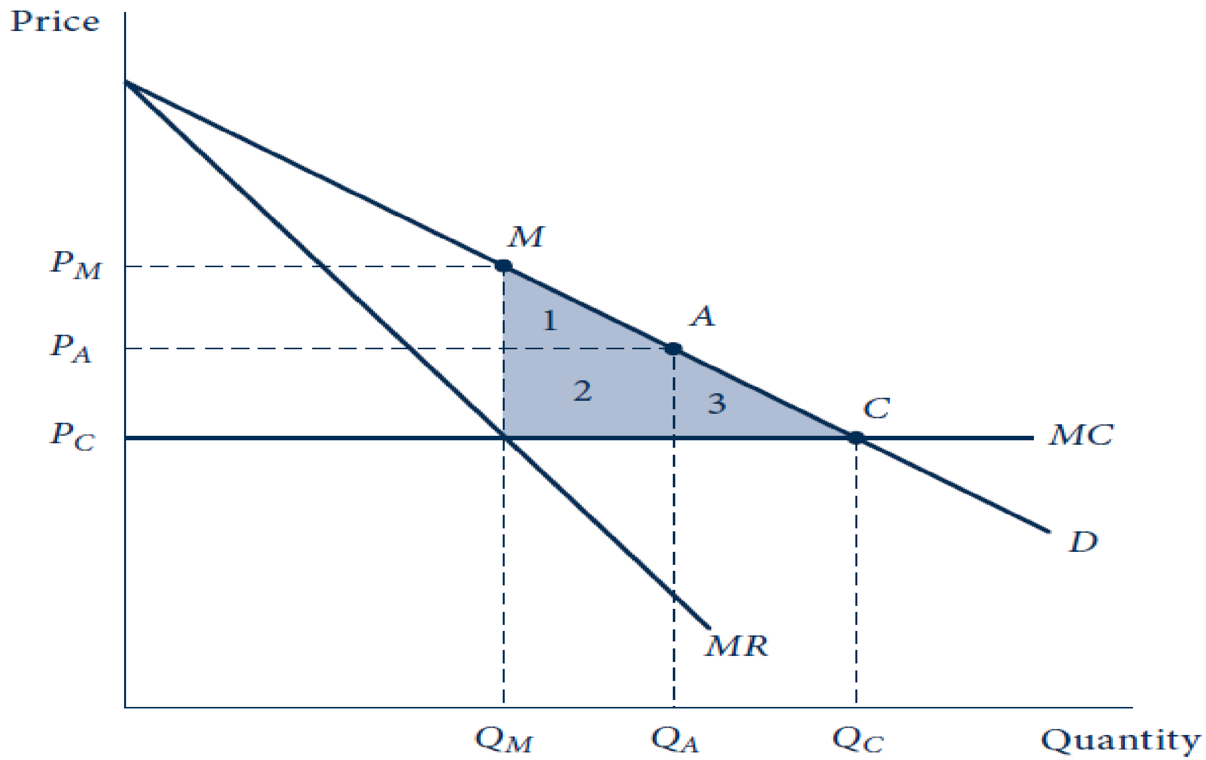
replacing $q_2 = (1-\delta)\frac{a-c}{2}$ we get

$$q_1 = \frac{a-c-(1-\delta)\frac{a-c}{2}}{2} = \frac{a-c}{4} + \frac{a-c}{4}\delta > \delta\frac{a-c}{2}$$

This prove that firm 1 has an incentive to deviate from the strategy profile of the cartel

Comparison of the three models

	Bertrand	Cournot	Cartel
Price	c	$(a+2c)/3$	$(a+c)/2$
Total quantity	$a-c$	$2\,(a-c)/3$	$(a-c)/2$
Total profits	0	$2(a-c)^2/9$	$(a-c)^2/4$
DWL	0	$(a-c)^2/18$	$(a-c)^2/8$



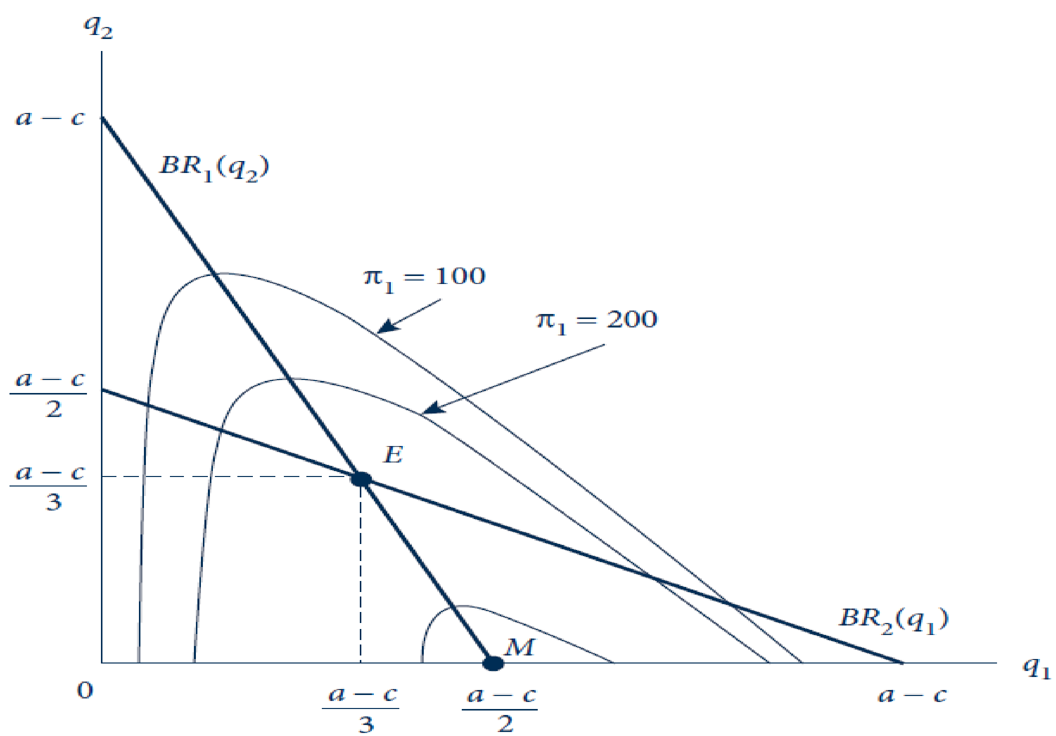
Cournot model: graphical solution

Best responses are $q_1 = \frac{a-q_2-c}{2}$, $q_2 = \frac{a-q_1-c}{2}$

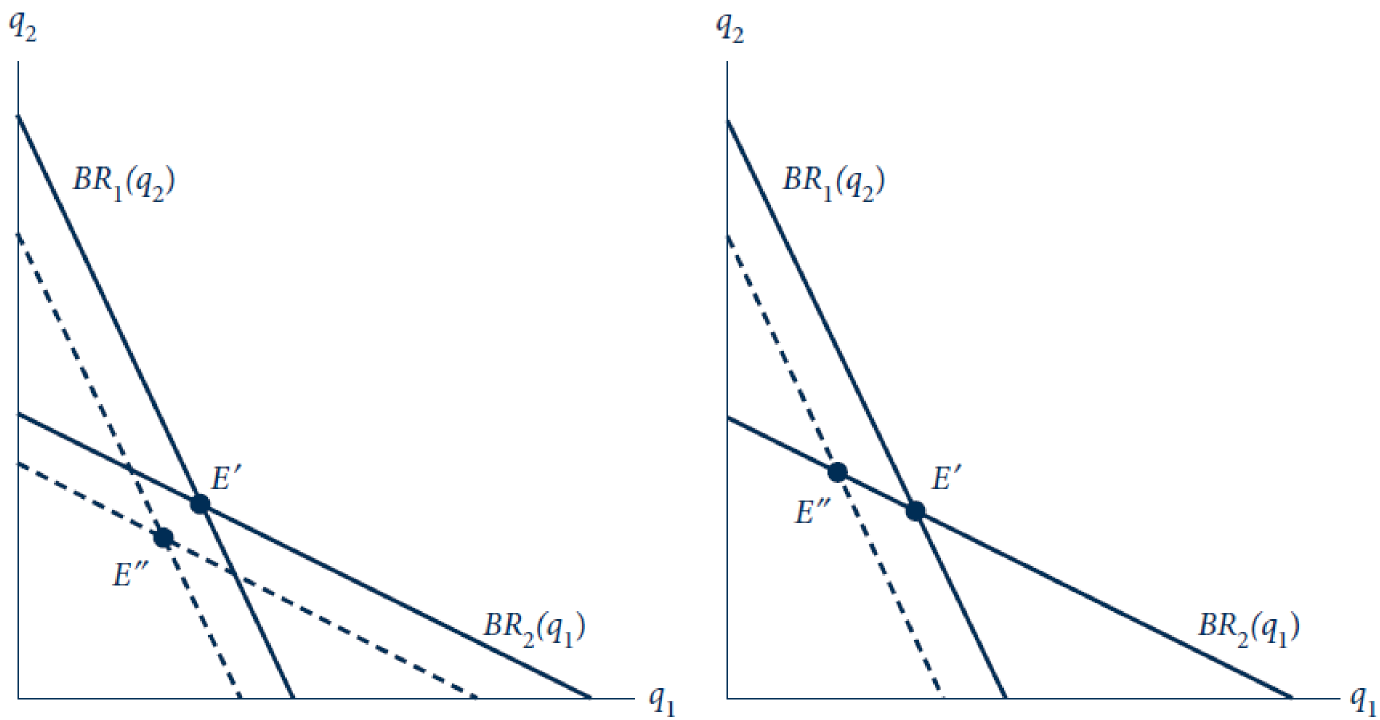
Isoprofit curves: for firm 1 is the locus of quantity pairs providing it with the same profit level.

Suppose that profits of firm 1 are equal to 100, i.e.

$q_1(a - q_1 - q_2 - c) = 100$, then $q_2 = a - q_1 - c - \frac{100}{q_1}$



Shifting best responses



(a) Increase in both firms' marginal costs (b) Increase in firm 1's marginal cost

Varying the number of Cournot firms.

The Cournot model can represent the whole range of outcomes from perfect competition to perfect cartel/monopoly.

Consider the case of identical firms. In equilibrium, firms will produce the same share of total output: $q_i = Q/n$

The FOC of the Cournot model, $P(Q) + q_i P'(Q) - C_i'(q_i) = 0,$

can be written as $P(Q) + \frac{Q}{n} P'(Q) - C_i'(q_i) = 0$

The second term disappears as n grows large;

Price approaches marginal cost and the market outcome approaches the perfectly competitive one.

As n decreases to 1, the second term increases and approaches that of a perfect cartel.

Natural Springs, version 2: Cournot Oligopoly with *n* firms

- Assume that there are n firms exploiting natural springs

- A firm's cost of pumping and bottling q_i liters is $C_i(q_i) = cq_i$

- each firm has to decide how much water to supply to the market

- The inverse demand for spring water is given by:

$$P(Q) = a - Q$$

where a is the intercept and $Q = \sum_{i=1}^{n} q_i$ is total spring water output.

The firms' problems are:

$$\max_{\{q_i\}} \pi_i \equiv \max_{\{q_1\}} q_i P(Q) - c\, q_i \equiv \max_{\{q_1\}} q_i \left(a - \sum_{i=1}^{n} q_i \right) - c\, q_i \;\; \forall i$$

The first order conditions are:

$$a - 2q_i - Q_{-i} - c = 0 \qquad \forall i$$

where Q_{-i} is the output of all firms different from firm i

Note that we can write this conditions as

$$q_i = a - Q - c \qquad \forall i$$

It implies that all firms will produce the same quantity.

Then in equilibrium

$$q_i = \frac{a - c}{n + 1} \forall i$$

Thus,

total output is $Q = \frac{n}{n+1}(a - c)$.

equilibrium price of $P = \frac{a}{n+1} + \frac{n \cdot c}{n+1}$

Total profits are $\pi_1 + \pi_2 = n\left(\frac{a-c}{n+1}\right)^2$

For $n = 1$ we have the same values of the Cartel example

For $n = 2$ we have the same value of the Cournot duopoly

For n approaching to ∞ these values converge to the competitive case (or to the Bertrand case)

Product Differentiation

Firms often devote considerable resources to differentiating their products from those of their competitors

- quality and style variations
- warranties and guarantees
- special service features
- product advertising

The law of one price may not hold, because demanders may now have preferences about which suppliers to purchase the product from

- there are now many closely related, but not identical, products to choose from

We must be careful about which products we assume are in the same market

We will assume that there are *n* firms competing in a particular product group

- each firm can choose the amount it spends on attempting to differentiate its product from its competitors (a_i)
- Each firm has to choose a price p_i

The firm's costs are now given by

$$C_i(q_i, a_i)$$

Each firm face the following demand

$$q_i(p_i, P_{-i}, a_i, A_{-i})$$

where P_{-i} is a list of all other firms' prices besides i's, and A_{-i} is a list of all other firms' attributes besides i's.

Profits are:

$$\pi_i = p_i q_i - C_i(q_i, a_i)$$

The problem of firm i is to choose the optimal level of price and attributes. Suppose the attributes is in one dimension. The firm problem is

$$\max_{\{q_i,a_i\}} p_i q_i - C_i\,(q_i, a_i)$$

FOCs are

$$q_i + p_i \frac{dq_i}{dp_i} - \frac{dC_i}{dq_i}\frac{dq_i}{dp_i} = 0$$

$$p_i \frac{dq_i}{da_i} - \frac{dC_i}{dq_i}\frac{dq_i}{da_i} - \frac{dC_i}{da_i} = 0$$

The first condition relates to the price choice and the second to the attribute choice.

The first resembles the condition MR equal to marginal costs but ……

This condition are too complex to generate general conclusions

FOCs respect to the price can be written as

$$p_i = \frac{dC_i}{dq_i} - \frac{q_i}{\frac{dq_i}{dp_i}}$$

Then the price is bigger than marginal costs (if the demand is decreasing respect the price)

Then, with differentiated products, even if firms compete as in Bertrand model, equilibrium prices are higher respect to competitive levels.

Natural Spring, version 3: Bertrand duopoly

- Assume that there are 2 firms exploiting natural springs
- A firm's cost of pumping and bottling q_i liters is $C_i(q_i) = cq_i$
- each firm has to decide the price p_i to set in the market
- The demands for spring water are:

$$q_1 = a_1 - p_1 + \frac{p_2}{2} \text{ and } q_2 = a_2 - p_2 + \frac{p_1}{2}$$

The firms' problems are:

$$\max_{\{p_1\}} (p_1 - c)\left(a_1 - p_1 + \frac{p_2}{2}\right)$$

$$\max_{\{p_2\}} (p_2 - c)\left(a_2 - p_2 + \frac{p_1}{2}\right)$$

The first order conditions are:

$$a_1 - 2p_1 + \frac{p_2}{2} + c = 0 \text{ and } a_2 - 2p_2 + \frac{p_1}{2} + c = 0$$

Firms best responses are

$$p_1 = \frac{\left(a_1 + c + \frac{p_2}{2}\right)}{2} \text{ and } p_2 = \frac{\left(a_2 + c + \frac{p_1}{2}\right)}{2}$$

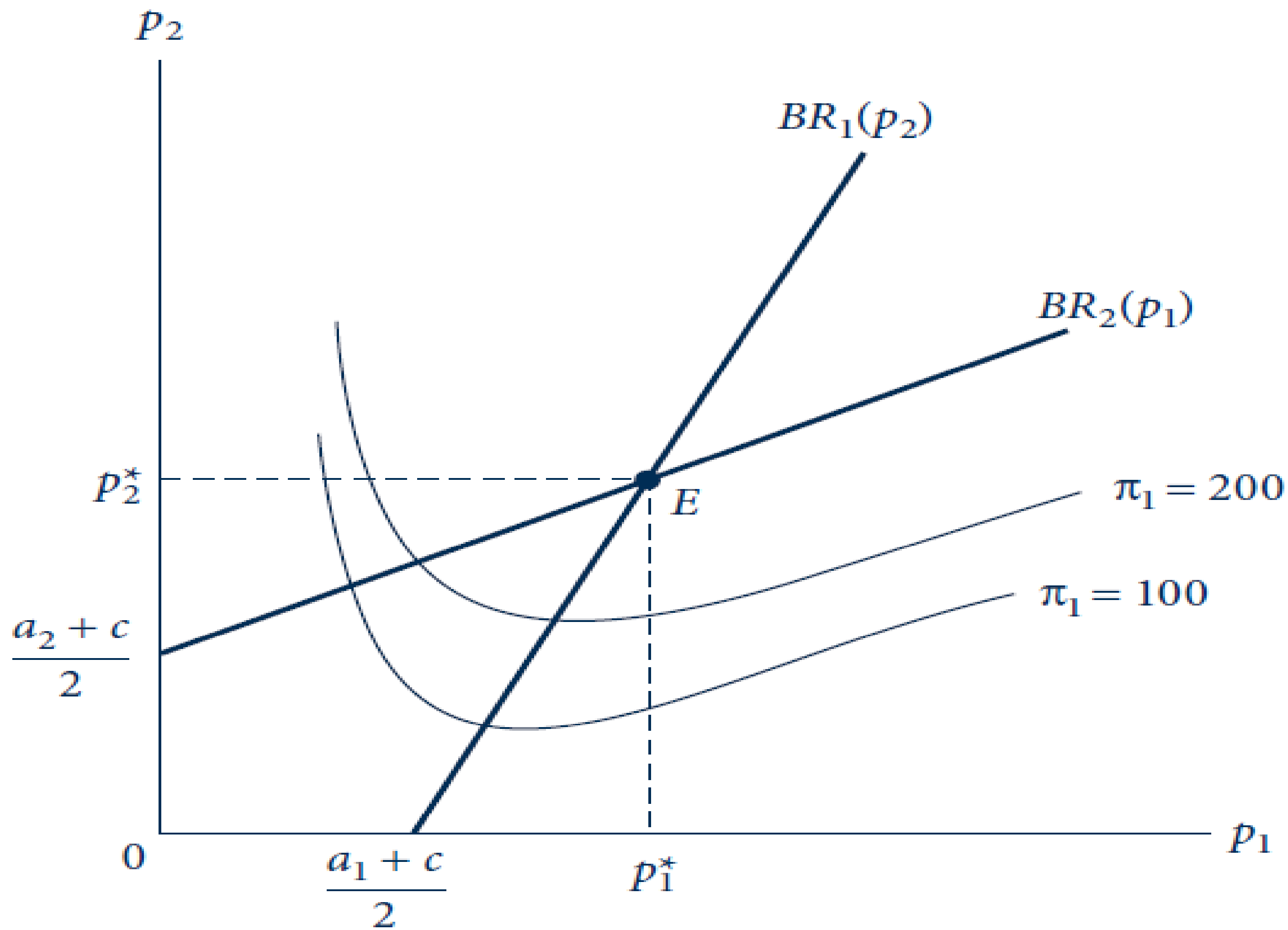
Solving the equation system given by FOCs

$$p_1 = \frac{(8a_1 + 2a_2)}{15} + \frac{2}{3}c$$

$$p_1 = \frac{(2a_1 + 8a_2)}{15} + \frac{2}{3}c$$

Then if a_1 and a_2 are not too low, price exceed marginal costs and profits are strictly positive

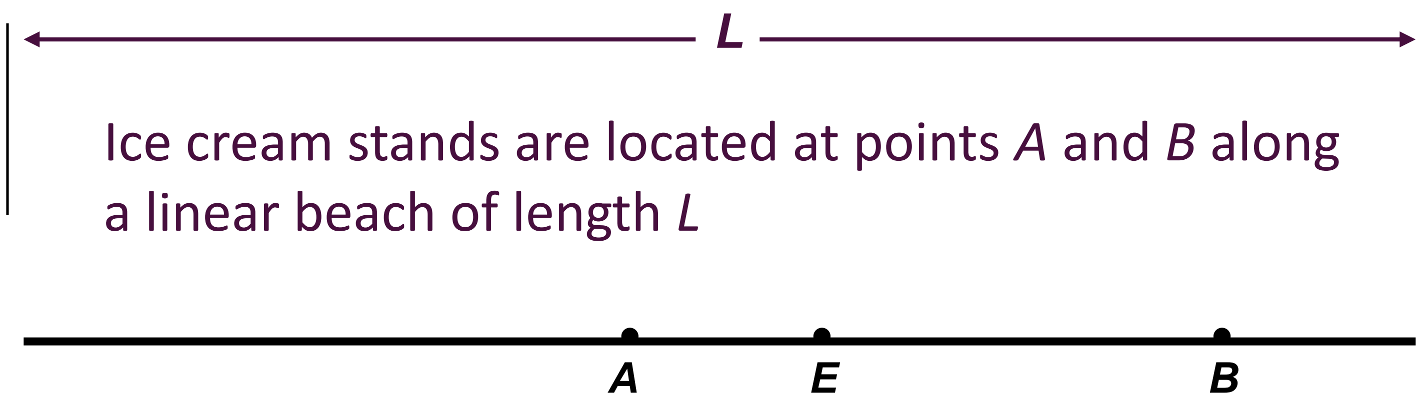
Graphical solution



Example: Spatial Differentiation

We analyze the case of ice cream stands located on a beach

- assume that demanders are located uniformly along the beach
  - one at each point of beach
  - each buyer purchases exactly one ice cream cone per period
- ice cream cones are costless to produce but carrying them back to one's place on the beach results in a **cost of *c*** per unit traveled

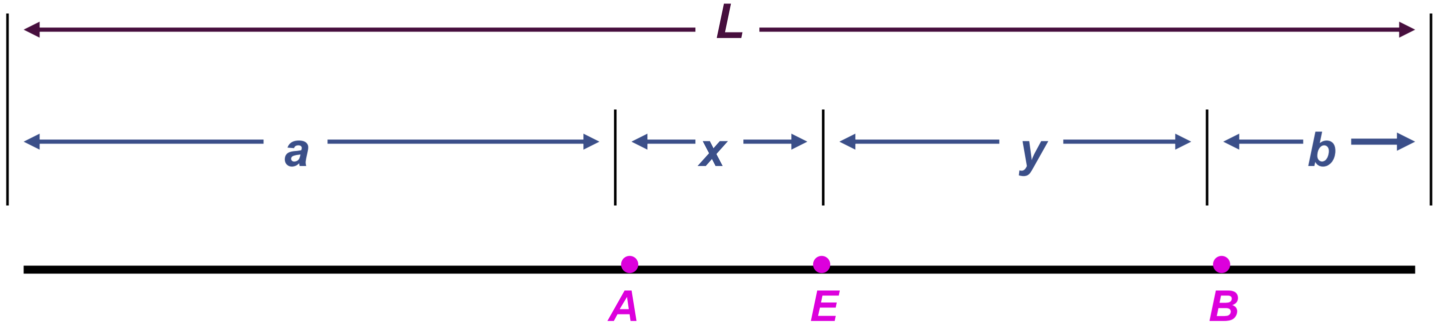


Suppose that a person is standing at point *E*

A person located at point E will be indifferent between stands A and B if

$$p_A + cx = p_B + cy$$

where p_A and p_B are the prices charged by each stand, x is the distance from E to A, and y is the distance from E to B



$$a + x + y + b = L$$

- The coordinate of point E is

$$x = \frac{p_B - p_A + cy}{c} = \frac{p_B - p_A}{c} + y$$

$$x = \frac{p_B - p_A}{c} + L - a - b - x$$

$$x = \frac{1}{2}\left(L - a - b + \frac{p_B - p_A}{c}\right)$$

$$y = \frac{1}{2}\left(L - a - b + \frac{p_A - p_B}{c}\right)$$

Profits for the two firms are:

$$\pi_A = p_A(a + x) = \frac{1}{2}(L + a - b)p_A + \frac{p_A p_B - p_A^2}{2c}$$

$$\pi_B = p_B(b + y) = \frac{1}{2}(L - a + b)p_B + \frac{p_A p_B - p_B^2}{2c}$$

Each firm will choose its price so as to maximize profits. FOCs are:

$$\frac{d\pi_A}{dp_A} = \frac{1}{2}(L + a - b) + \frac{P_B}{2c} - \frac{P_A}{c} = 0$$

$$\frac{d\pi_B}{dp_B} = \frac{1}{2}(L - a + b) + \frac{P_A}{2c} - \frac{P_B}{c} = 0$$

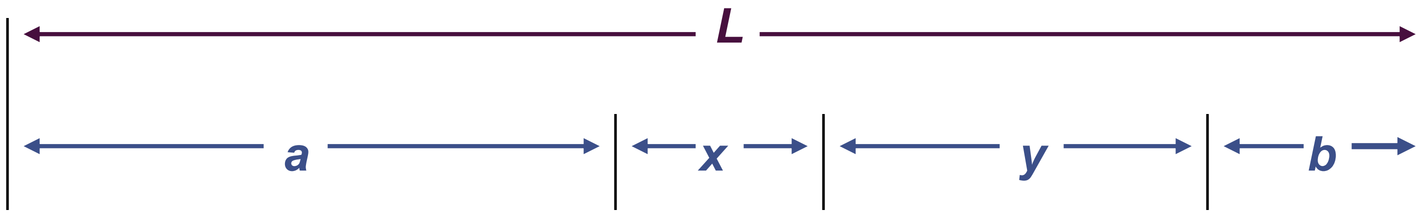
These can be solved to yield:

$$p_A = c\left(L + \frac{a-b}{3}\right)$$

$$p_B = c\left(L - \frac{a-b}{3}\right)$$

These prices depend on the precise locations of the stands and will differ from one another

Because *A* is somewhat more favorably located than *B*, p_A will exceed p_B



If we allow the ice cream stands to change their locations at zero cost, each stand has an incentive to move to the center of the beach

- any stand that opts for an off-center position is subject to its rival moving between it and the center and taking a larger share of the market
  - this encourages a similarity of products

Stackelberg Model

The simplest setting to illustrate the first-mover advantage

It is similar to a duopoly version of the Cournot model except that—rather than simultaneously choosing the quantities, firms move sequentially, with firm 1 (the leader) choosing its output first and then firm 2 (the follower) choosing after observing firm 1's output.

It is a sequential game

To solve it we use backward induction

Begin with the last firm that has to decide, i.e. firm 2.

Firm 2 chooses the output q_2 that maximizes its own profit, taking firm 1’s output as given.

In other words, firm 2 best responds to firm 1’s output.

This results in the same best-response function for firm 2 as we computed in the Cournot game from the first-order condition

Label this best response function $BR_2(q_1)$.

Consider firm 1. Firm 1 recognizes that it can influence the follower’s action because the follower best responds to 1’s observed output.

The we can substitute $BR_2(q_1)$ into the profit function for firm 1

$\pi_1 = q_1 P(Q) - C_1(q_1)$ where $Q = q_1 + q_2$

$$\pi_1 = q_1 P(q_1 + q_2) - C_1(q_1)$$

$$\pi_1 = q_1 P(q_1 + BR_2(q_1)) - C_1(q_1)$$

$$\pi_1 = q_1 P(q_1 + BR_2(q_1)) - C_1(q_1)$$

Firm 1 maximizes its profits, FOCs are

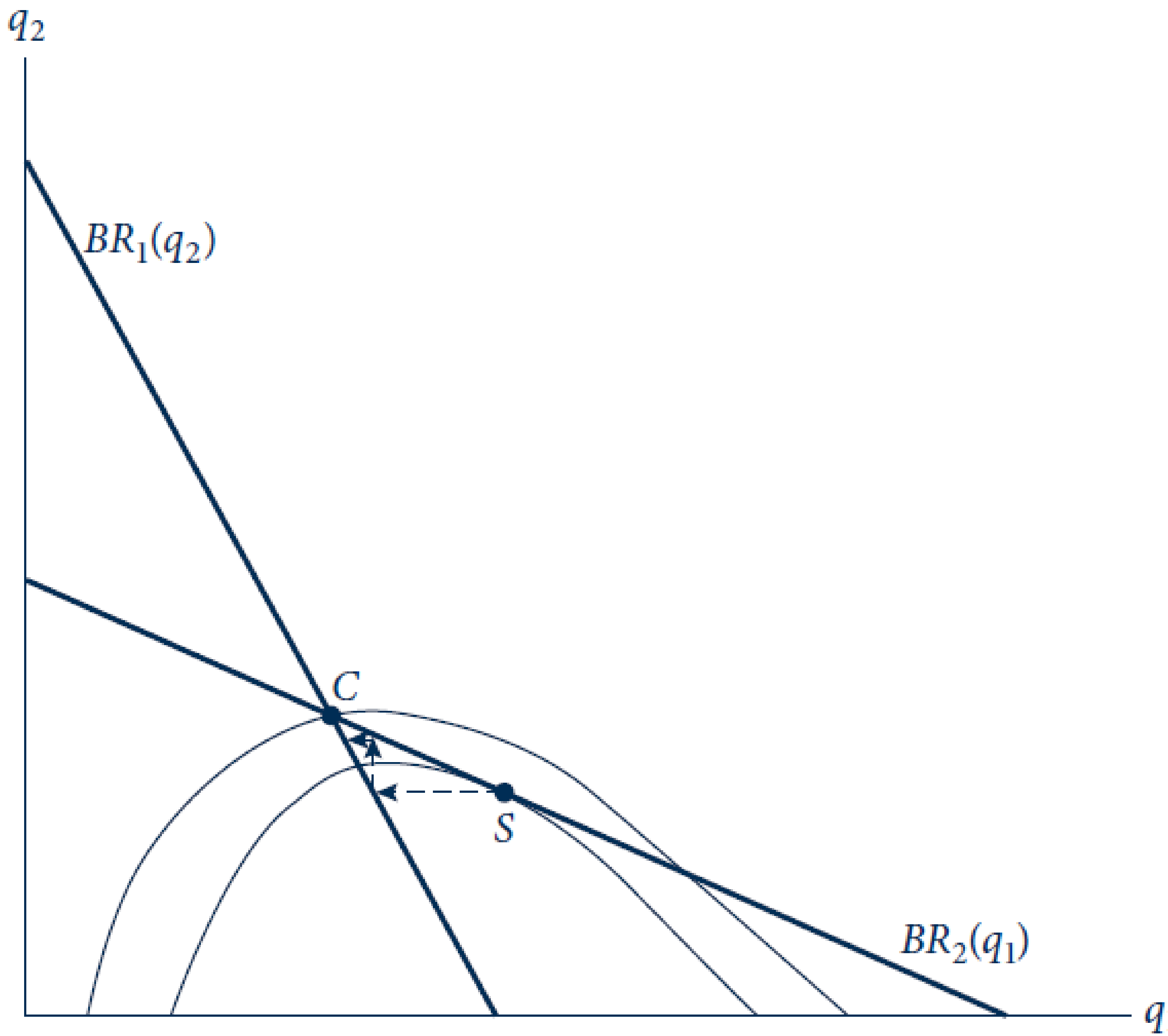
$$\frac{d\pi_1}{dq_1} = P(q_1 + BR_2(q_1)) + q_1 P'(q_1 + BR_2(q_1))(1 + BR_2'(q_1)) - C_1'(q_1) = 0$$

We can write it as

$$\frac{d\pi_1}{dq_1} = P(Q) + q_1 P'(Q) + q_1 P'(Q) BR_2'(q_1) - C_1'(q_1) = 0$$

This is the same FOCs computed in the Cournot model except for the addition of the third term, which accounts for the strategic effect of firm 1's output on firm 2's.

Then in the SPNE Firm 1 will produce more than in the NE of the Cournot game and Firm 2 will produce less


q_2
$BR_1(q_2)$
C
S
$BR_2(q_1)$
q_1

Natural Springs, version 4: Stackelberg duopoly

- Assume that there are two firms exploiting natural springs
- A firm's cost of pumping and bottling q_i liters is $C_i(q_i) = cq_i$
- each firm has to decide how much water to supply to the market
- The inverse demand for spring water is given by:

$$P(Q) = a - Q$$

where a is the intercept and $Q = q_1 + q_2$ is total spring water output.

Firm 1 (the leader) chooses its output first and then firm 2 (the follower) chooses after observing firm 1's output

We solve by backward induction

The firm 2 problem is:

$$\max_{\{q_2\}} \pi_2 \equiv \max_{\{q_2\}} q_2 P(Q) - c\, q_2 \equiv \max_{\{q_2\}} q_2(a - q_1 - q_2) - c\, q_2$$

The first order conditions are:

$$a - q_1 - 2q_2 - c = 0$$

Firm 2’s best response is:

$$BR_2(q_1) = \frac{a - q_1 - c}{2}$$

The firm 1 problem is:

$$\max_{\{q_1\}} \pi_1 \equiv \max_{\{q_1\}} q_1 P(Q) - c\, q_1 \equiv \max_{\{q_1\}} q_1(a - q_1 - q_2) - c\, q_1$$

Firm 1 recognizes that it can influence the follower's action because the follower best responds to 1's observed output.

Then its problem becomes

$$\max_{\{q_1\}} q_1\big(a - q_1 - BR_2(q_1)\big) - c\, q_1$$

$$\max_{\{q_1\}} q_1 \left(\frac{a - q_1 + c}{2}\right) - c\, q_1$$

The first order conditions are:

$$\frac{a - 2q_1 + c}{2} = 0$$

Firm 1's best strategy is:

$$q_1 = \frac{a - c}{2}$$

Firms' equilibrium strategies are:

$$q_1 = \frac{a-c}{2}\ , \quad q_2 = \frac{a-q_1-c}{2}$$

This strategy profile represents the unique SPNE

Solving the equation system (formed by the two strategies) we get the BIO

$$q_1 = \frac{a-c}{2} \quad \text{and} \quad q_2 = \frac{a-c}{4}$$

Thus,

total output is $Q = \frac{3\,(a-c)}{4}$. (respect to Cournot it is higher)

equilibrium price of $P = \frac{a+3c}{4}$ (respect to Cournot it is lower)

profits are $\pi_1 = \frac{(a-c)^2}{8}$ $\pi_2 = \frac{(a-c)^2}{16}$ (respect to Cournot, higher for firm 1, lower for firm 2)

Total profits are $\pi_1 + \pi_2 = \frac{3(a-c)^2}{16}$ (lower with respect to Cournot)

Natural Spring, version 5: price leadership game

- Assume that there are 2 firms exploiting natural springs
- A firm's cost of pumping and bottling q_i liters is $C_i(q_i) = cq_i$
- The demands for spring water are:

$$q_1 = a_1 - p_1 + \frac{p_2}{2} \text{ and } q_2 = a_2 - p_2 + \frac{p_1}{2}$$

- each firm has to decide the price p_i to set in the market
- Firm 1 (the leader) chooses its price p_1 first and then firm 2 (the follower) chooses p_2 after observing p_1
- We solve by backward induction

The firm 2 problem is:

$$\max_{\{p_2\}} (p_2 - c)\left(a_2 - p_2 + \frac{p_1}{2}\right)$$

The first order conditions are:

$$a_2 - 2p_2 + \frac{p_1}{2} + c = 0$$

Then its best response is

$$p_2 = \frac{\left(a_2 + c + \frac{p_1}{2}\right)}{2}$$

The firm 1 problem is:

$$\max_{\{p_1\}} (p_1 - c)\left(a_1 - p_1 + \frac{p_2}{2}\right)$$

Firm 1 recognizes that it can influence the follower's price because the follower best responds to 1's observed price. Then firm 1 problem is:

$$\max_{\{p_1\}} (p_1 - c)\left(a_1 - p_1 + \frac{a_2 + c + \frac{p_1}{2}}{4}\right)$$

$$\max_{\{p_1\}} (p_1 - c)\left(a_1 - \frac{7}{8}p_1 + \frac{a_2 + c}{4}\right)$$

FOCs are

$$\left(a_1 - \frac{7}{8}p_1 + \frac{a_2 + c}{4}\right) - \frac{7}{8}(p_1 - c) = 0$$

$$p_1 = \frac{4a_1 + a_2}{7} + \frac{9}{14}c$$

Replacing into the best response of firm 2 we get

$$p_2 = \frac{(a_2 + c)}{2} + \frac{4a_1 + a_2}{28} + \frac{9}{56}c =$$

$$p_2 = \frac{15a_2}{28} + \frac{a_1}{7} + \frac{37}{56}c$$

To compare with Bertrand we assume that $a_1 = a_2 = a$

$$p_1 = \frac{40a+36c}{56} \text{ and } p_2 = \frac{38\,a+37c}{56}$$

In Bertrand prices were equal

$$p_1 = p_2 = \frac{10a}{15} + \frac{2}{3}c$$

Computing profits

In Bertrand the profits of both firms are $\frac{1}{9}(c-2a)^2$

In price leadership game the profits of firms are:

Firm 1: $\frac{25}{224}(c-2a)^2$

Firm 2: $\frac{361}{3136}(c-2a)^2$

Total profits are higher than in Bertrand for both firms

Follower (firm 2) earns more than the leader (firm 1)