Problem set 8

14.2

A monopolist faces a market demand curve given by

$$Q = 70 - p.$$

a. If the monopolist can produce at constant average and marginal costs of $AC = MC = 6$, what output level will the monopolist choose to maximize profits? What is the price at this output level? What are the monopolist's profits?
b. Assume instead that the monopolist has a cost structure where total costs are described by

$$C(Q) = 0.25Q^2 - 5Q + 300.$$

With the monopolist facing the same market demand and marginal revenue, what price–quantity combination will be chosen now to maximize profits? What will profits be?
c. Assume now that a third cost structure explains the monopolist's position, with total costs given by

$$C(Q) = 0.0133Q^3 - 5Q + 250.$$

Again, calculate the monopolist's price–quantity combination that maximizes profits. What will profit be? *Hint:* Set $MC = MR$ as usual and use the quadratic formula to solve the second-order equation for Q.
d. Graph the market demand curve, the MR curve, and the three marginal cost curves from parts (a), (b), and (c). Notice that the monopolist's profit-making ability is constrained by (1) the market demand curve (along with its associated MR curve) and (2) the cost structure underlying production.

14.5

Suppose a monopoly market has a demand function in which quantity demanded depends not only on market price (P) but also on the amount of advertising the firm does (A, measured in dollars). The specific form of this function is

$$Q = (20 - P)(1 + 0.1A - 0.01A^2).$$

The monopolistic firm's cost function is given by

$$C = 10Q + 15 + A.$$

a. Suppose there is no advertising ($A = 0$). What output will the profit-maximizing firm choose? What market price will this yield? What will be the monopoly's profits?
b. Now let the firm also choose its optimal level of advertising expenditure. In this situation, what output level will be chosen? What price will this yield? What will the level of advertising be? What are the firm's profits in this case? *Hint:* This can be worked out most easily by assuming the monopoly chooses the profit-maximizing price rather than quantity.

14.6

Suppose a monopoly can produce any level of output it wishes at a constant marginal (and average) cost of $5 per unit. Assume the monopoly sells its goods in two different markets separated by some distance. The demand curve in the first market is given by

$$Q_1 = 55 - P_1,$$

and the demand curve in the second market is given by

$$Q_2 = 70 - 2P_2.$$

a. If the monopolist can maintain the separation between the two markets, what level of output should be produced in each market, and what price will prevail in each market? What are total profits in this situation?

d. Now assume the two different markets 1 and 2 are just two individual consumers. Suppose the firm could adopt a linear two-part tariff under which marginal prices charged to the two consumers must be equal but their lump-sum entry fees might vary. What pricing policy should the firm follow?

14.10 Taxation of a monopoly good

The taxation of monopoly can sometimes produce results different from those that arise in the competitive case. This problem looks at some of those cases. Most of these can be analyzed by using the inverse elasticity rule (Equation 14.1).

a. Consider first an ad valorem tax on the price of a monopoly's good. This tax reduces the net price received by the monopoly from P to $P(1 - t)$—where t is the proportional tax rate. Show that, with a linear demand curve and constant marginal cost, the imposition of such a tax causes price to increase by less than the full extent of the tax.
b. Suppose that the demand curve in part (a) were a constant elasticity curve. Show that the price would now increase by precisely the full extent of the tax. Explain the difference between these two cases.
c. Describe a case where the imposition of an ad valorem tax on a monopoly would cause the price to increase by more than the tax.
d. A specific tax is a fixed amount per unit of output. If the tax rate is τ per unit, total tax collections are τQ. Show that the imposition of a specific tax on a monopoly will reduce output more (and increase price more) than will the imposition of an ad valorem tax that collects the same tax revenue.

15.2

Suppose that firms' marginal and average costs are constant and equal to c and that inverse market demand is given by $P = a - bQ$, where $a, b > 0$.

a. Calculate the profit-maximizing price–quantity combination for a monopolist. Also calculate the monopolist's profit.
b. Calculate the Nash equilibrium quantities for Cournot duopolists, which choose quantities for their identical products simultaneously. Also compute market output, market price, and firm and industry profits.
c. Calculate the Nash equilibrium prices for Bertrand duopolists, which choose prices for their identical products simultaneously. Also compute firm and market output as well as firm and industry profits.
d. Suppose now that there are n identical firms in a Cournot model. Compute the Nash equilibrium quantities as functions of n. Also compute market output, market price, and firm and industry profits.
e. Show that the monopoly outcome from part (a) can be reproduced in part (d) by setting $n = 1$, that the Cournot duopoly outcome from part (b) can be reproduced in part (d) by setting $n = 2$ in part (d), and that letting n approach infinity yields the same market price, output, and industry profit as in part (c).

15.5

Consider the following Bertrand game involving two firms producing differentiated products. Firms have no costs of production. Firm 1's demand is

$$q_1 = 1 - p_1 + bp_2,$$

where $b > 0$. A symmetric equation holds for firm 2's demand.

a. Solve for the Nash equilibrium of the simultaneous price-choice game.
b. Compute the firms' outputs and profits.

15.7

Assume as in Problem 15.1 that two firms with no production costs, facing demand $Q = 150 - P$, choose quantities q_1 and q_2.

a. Compute the subgame-perfect equilibrium of the Stackelberg version of the game in which firm 1 chooses q_1 first and then firm 2 chooses q_2.
b. Now add an entry stage after firm 1 chooses q_1. In this stage, firm 2 decides whether to enter. If it enters, then it must sink cost K_2, after which it is allowed to choose q_2. Compute the threshold value of K_2 above which firm 1 prefers to deter firm 2's entry.
c. Represent the Cournot, Stackelberg, and entry-deterrence outcomes on a best-response function diagram.