LABOR MARKETS

Allocation of Time

Individuals must decide how to allocate the fixed amount of time they have

We will initially assume that there are only two uses of an individual's time

- engaging in market work at a real wage rate of w

- leisure (nonwork)

Assume that an individual's utility depends on consumption (c) and hours of leisure (h)

$$utility = U(c, h)$$

In seeking to maximize utility, the individual is bound by two constraints

$$l + h = 24$$

$$c = wl$$

Combining the two constraints, we get

$$c = w(24 - h)$$

$$c + wh = 24w$$

An individual has a "full income" of $24w$ *(or potential income).*

He may spend the full income either by working (for real income and consumption) or by not working (enjoying leisure)

The opportunity cost of leisure is w

The individual problem is

$$\max_{c,h} U(c, h)$$

subject to

$$c + wh = 24w$$

Utility Maximization

The individual's problem is to maximize utility subject to the full income constraint

Setting up the Lagrangian

$$\boldsymbol{L} = U(c,h) + \lambda(24w - c - wh)$$

The first-order conditions are

$$\partial \boldsymbol{L}/\partial c = \partial U/\partial c - \lambda = 0$$

$$\partial \boldsymbol{L}/\partial h = \partial U/\partial h - \lambda w = 0$$

Combining the two equations, we get

$$\frac{dU/dh}{dU/dc} = w = MRS\left(= \frac{dc}{dh}\right)$$

To maximize utility, the individual should choose to work that number of hours for which the *MRS* (of h for c) is equal to w (Necessary condition)

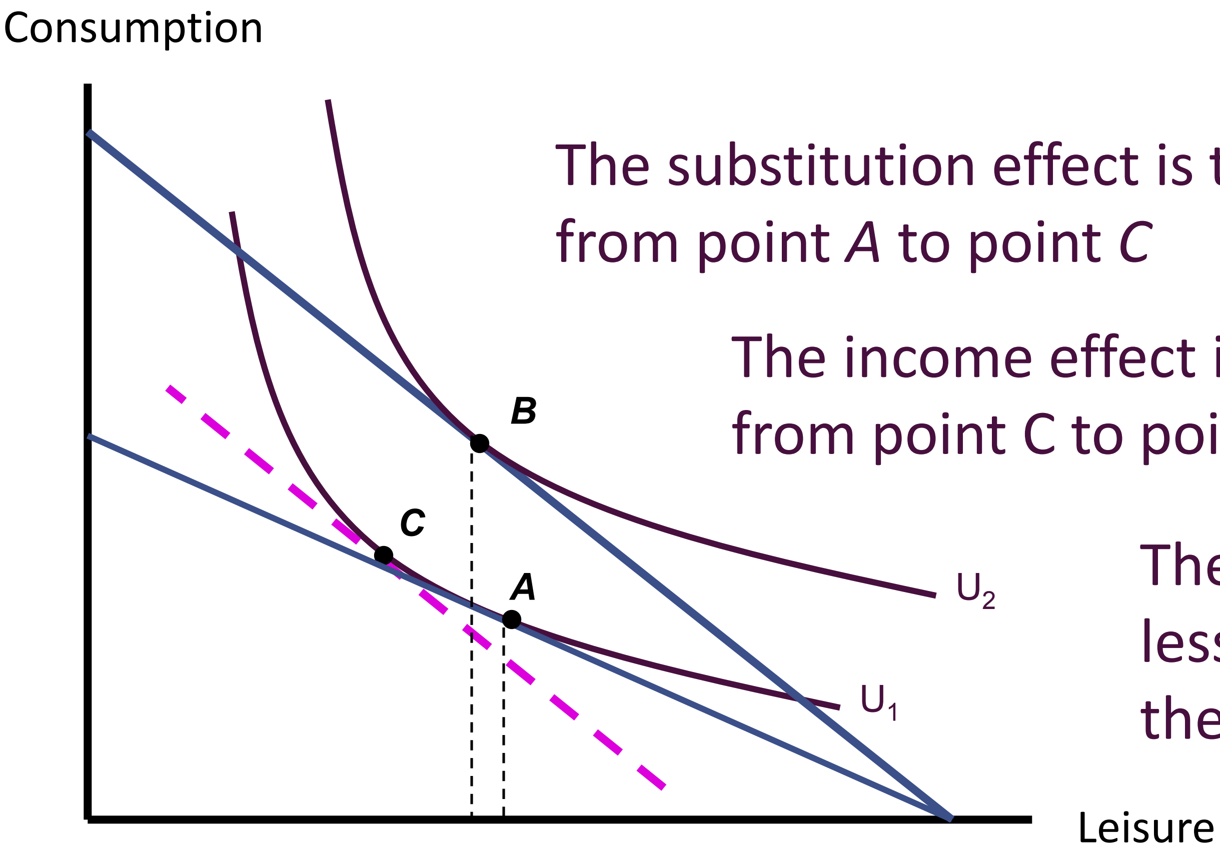
Sufficient condition for a maximum is that the *MRS* (of h for c) must be diminishing

Income and Substitution Effects of a change in w

Both a substitution effect and an income effect occur when w changes

- when w rises, the price of leisure becomes higher and the individual will choose less leisure (substitution effect)
- because leisure is a normal good, an increase in w leads to an increase in leisure (income effect)

- The income and substitution effects move in opposite directions

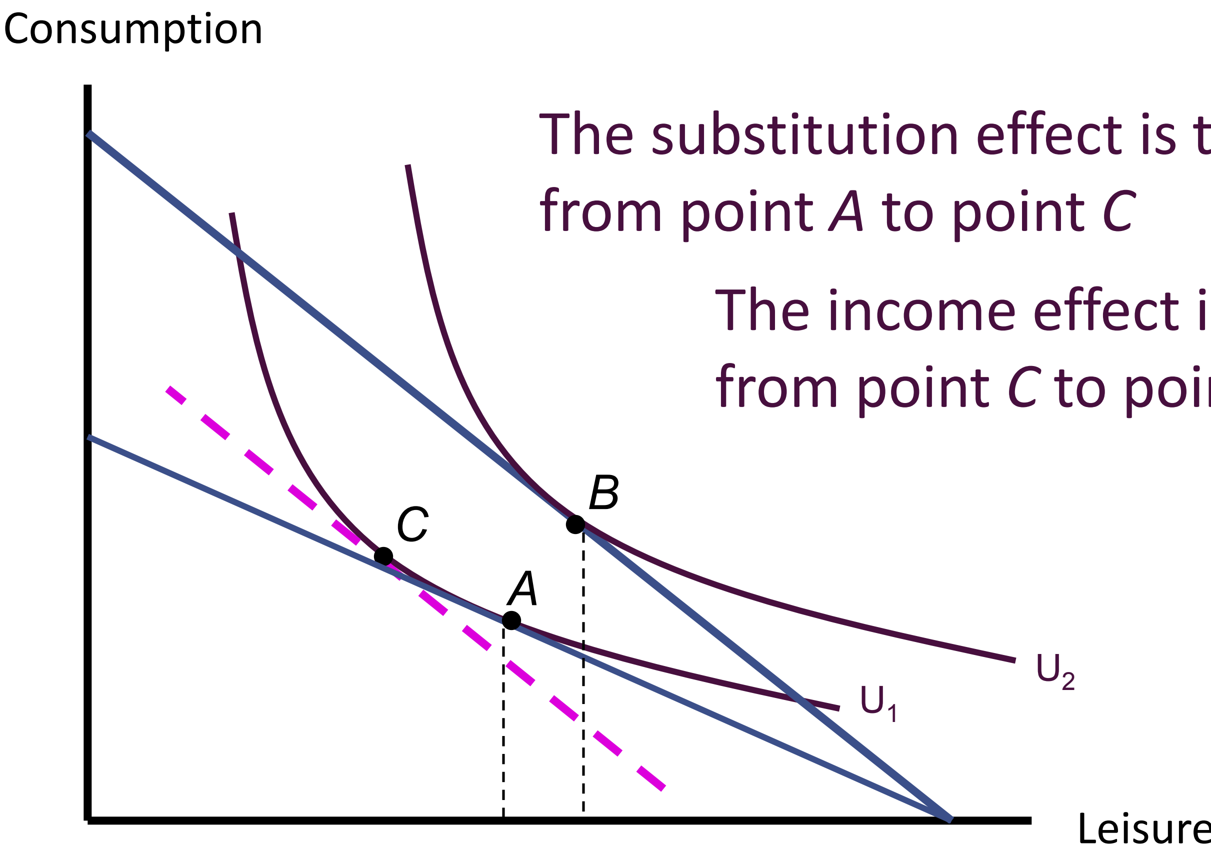


The substitution effect is the movement from point *A* to point *C*

The income effect is the movement from point C to point B

The individual chooses less leisure as a result of the increase in w

substitution effect > income effect



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The individual chooses more leisure as a result of the increase in *w*

substitution effect < income effect

A Mathematical Analysis of Labor Supply

We will start by amending the budget constraint to allow for the possibility of nonlabor income

$$c = wl + n$$

Now the individual problem is

$$\max_{c,h} U(c,h)$$

subject to

$$c + wh = 24w$$

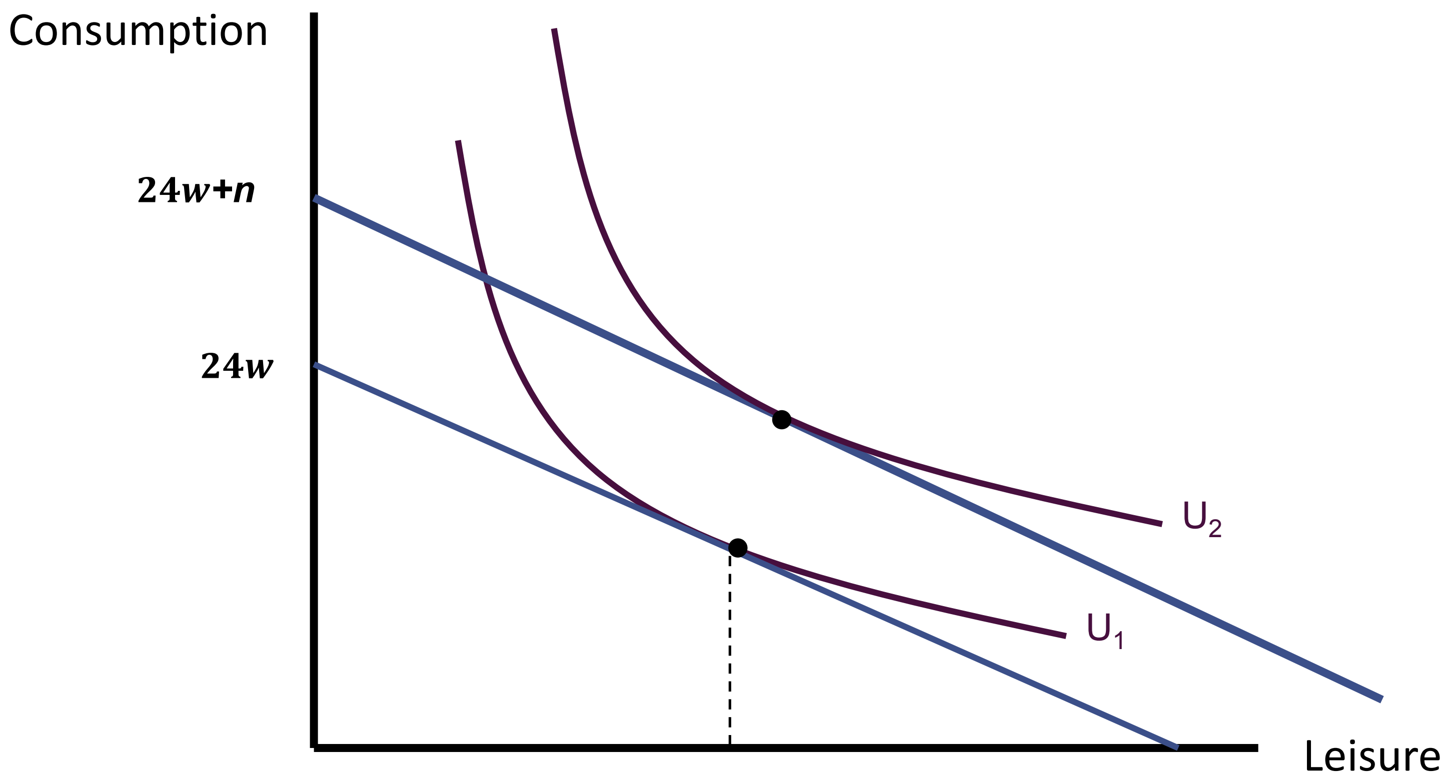
Maximization of utility subject to this constraint yields identical results as long as n is unaffected by the labor-leisure choice

The only effect of introducing nonlabor income is that the budget constraint shifts out (or in) in a parallel fashion

We can now write the individual's labor supply function as:

$$\boldsymbol{l(w, n)}$$

- hours worked will depend on both the wage and the amount of nonlabor income
- since leisure is a normal good, $\partial l / \partial n < 0$



Dual Statement of the Problem

The dual problem can be phrased as choosing levels of c and h so that the amount of expenditure ($E = c + wh$) required to obtain a given utility level ($U_0 = U(c,h)$) is as small as possible

$$\min_{c,h} c + wh \quad s.t.\ U_0 = U(c,h)$$

Solving this minimization problem will yield exactly the same solution as the utility maximization problem

$$\boldsymbol{L} = c + wh + \lambda(U_0 - U(c,h))$$

FOCs are

$$1 - \lambda \frac{dU}{dc} = 0$$

$$w - \lambda \frac{dU}{dh} = 0$$

Combining the two equations, we get

$$\frac{dU/dh}{dU/dc} = w = MRS\left(= \frac{dc}{dh}\right)$$

In the solution we have that the total expenditure is a function:

$$E(w, U_0)$$

Using the envelope theorem (see chapter 2) we can make the following conclusions.

A small change in w will change the minimum expenditures required by

$$\partial E/\partial w \;=\; -l$$

this is the extent to which labor earnings are increased by the wage change

This means that a labor supply function can be calculated by partially differentiating the expenditure function because utility is held constant, this function should be interpreted as a "compensated" (constant utility) labor supply function:

$$\boldsymbol{l^c(w, U)}$$

The excess expenditure function

$$E^*(w, U_0) = E(w, U_0) - 24w = c - wl$$

i.e. it represents the additional cash that individual needs on the top of what he earns by labour. Note that $\partial E^*/\partial w \;=\; -l$

Slutsky Equation of Labor Supply

The excess expenditure function play the role of nonlabor income in the primary utility-maximization problem, i.e. $n = E^*(w, U_0)$

$l^c(w, U) = l(w, n) = l[w, E^*(w, U_0)]$

In other words, we can adjust the unearned income (n) so to maintain the same level of utility while changing wage w.

Partial differentiation of both sides with respect to *w* gives us

$$\frac{\partial l^c}{\partial w} = \frac{\partial l}{\partial w} + \frac{\partial l}{\partial n} \cdot \frac{\partial E^*}{\partial w}$$

Replacing $\partial E/\partial w = -l$ we get the :

$$\frac{\partial l^c}{\partial w} = \frac{\partial l}{\partial w} - \frac{\partial l}{\partial n} \cdot l$$

Introducing a different notation for l^c , and rearranging terms gives us the Slutsky equation for labor supply:

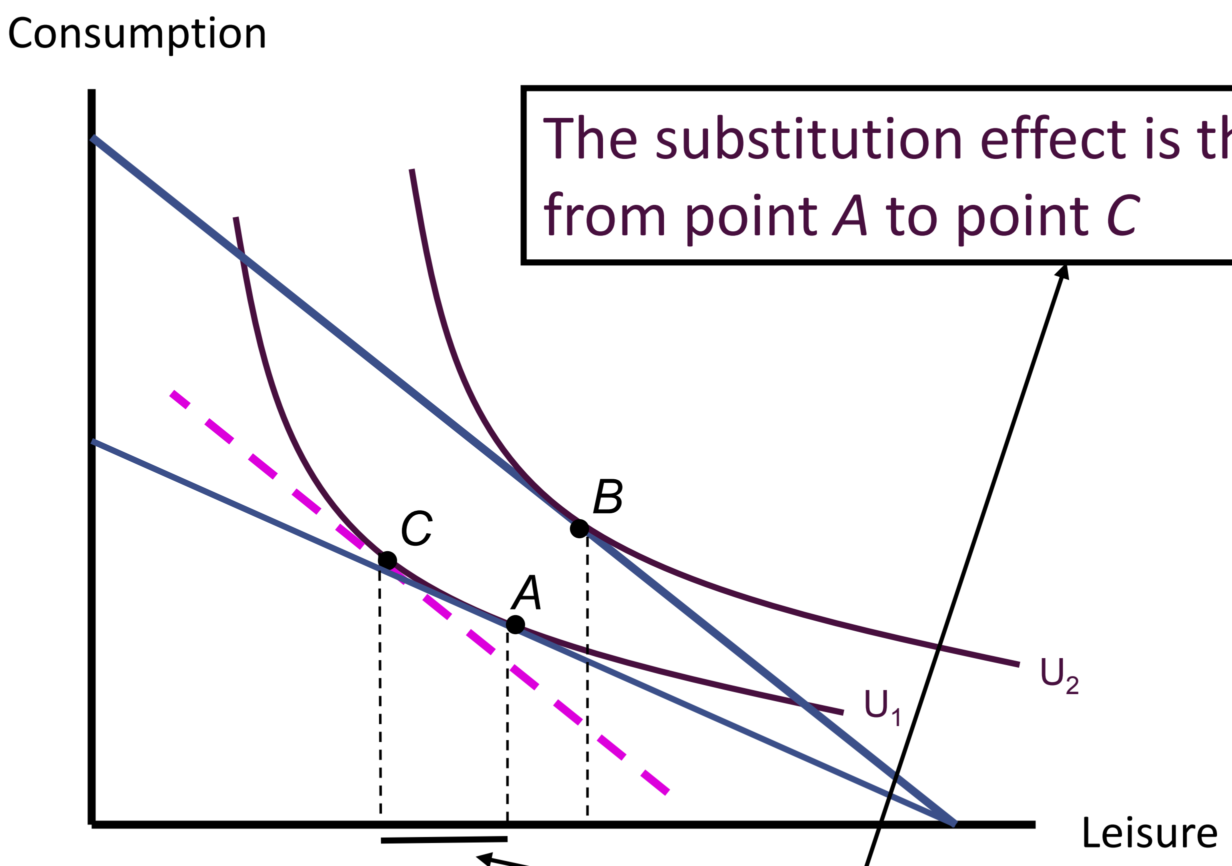
$$\frac{\partial l^c}{\partial w} = \left.\frac{\partial l}{\partial w}\right|_{U=U_0}$$

And replacing into the previous equation

$$\frac{\partial l}{\partial w} = \left.\frac{\partial l}{\partial w}\right|_{U=U_0} + \frac{\partial l}{\partial n} \cdot l$$

That is the **Slutsky equation**

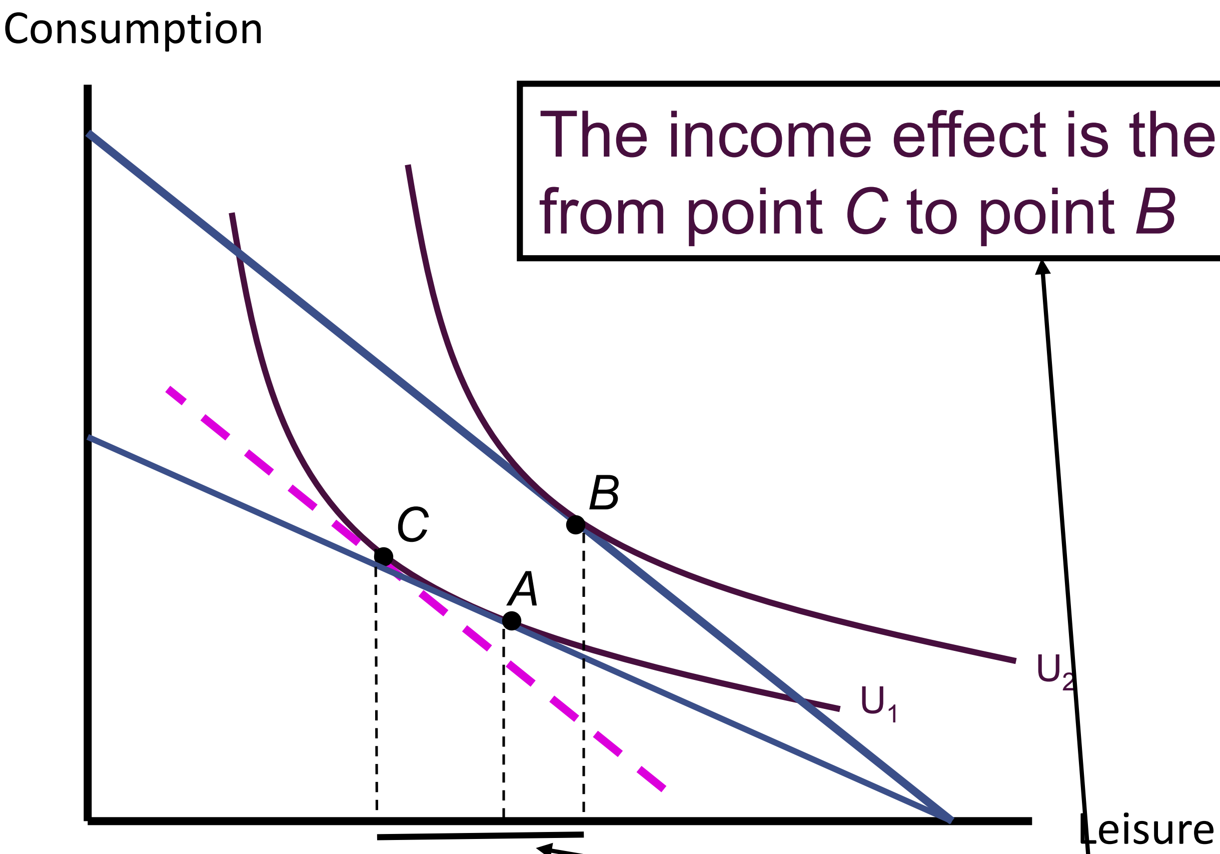
The first term on the LHS represents the substitution effect and the second term represents the income effect



The first term on the LHS represents the substitution effect

$$\frac{\partial l}{\partial w} = \left.\frac{\partial l}{\partial w}\right|_{U=U_0} + \frac{\partial l}{\partial n} \cdot l$$

The income effect is the movement from point *C* to point *B*



the second term represents the income effect

$$\frac{\partial l}{\partial w} = \left.\frac{\partial l}{\partial w}\right|_{U=U_0} + \frac{\partial l}{\partial n} \cdot l$$

Examples of labour supply functions

Suppose a Cobb-Douglas utility function

$$U = c^{\alpha} h^{\beta}$$

And assume $\alpha + \beta = 1$

The budget constraints is

$$c = wl + n$$

and the time constraint is

$$l + h = 1$$

note that we have set maximum work time to 1 hour for convenience

The individual problem is

$$\max_{c,h} c^{\alpha} h^{\beta}$$

subject to $c + wh = w + n$

The Lagrangian expression for utility maximization is

$$\boldsymbol{L} = c^{\alpha}h^{\beta} + \lambda(w + n - wh - c)$$

First-order conditions are (using $\alpha + \beta = 1$)

$$\partial\boldsymbol{L}/\partial c = \alpha c^{-\beta}h^{\beta} - \lambda = 0$$

$$\partial\boldsymbol{L}/\partial h = \beta c^{\alpha}h^{-\alpha} - \lambda w = 0$$

$$\partial\boldsymbol{L}/\partial\lambda = w + n - wh - c = 0$$

From the first two equations we get

$$wh = \frac{(1-\alpha)c}{\alpha}$$

Replacing in the third one we get

$c = \alpha(w + n)$ and $h = \beta(w + n)/w$

the person spends α of his income on consumption and $\beta = 1 - \alpha$ on leisure

the labor supply function is: $l(w, n) = 1 - h = 1 - \beta - \frac{\beta n}{w}$

$$l(w,n) = 1 - \beta - \frac{\beta n}{w} \qquad h = \beta(w+n)/w$$

- Note that if $n = 0$, the person will work $(1-\beta)$ of each hour no matter what the wage is
  - the substitution and income effects of a change in w offset each other and leave l unaffected
- If $n > 0$, $\partial l/\partial w \ > \ 0$
  - the individual will always choose to spend βn on leisure
  - Since leisure costs w per hour, an increase in w means that less leisure can be bought with n
- Note that $\partial l/\partial n \ < \ 0$
  - an increase in nonlabor income allows this person to buy more leisure
    - income transfer programs are likely to reduce labor supply
    - lump-sum taxes will increase labor supply

Suppose a CES utility function

$$U(c,h) = \frac{c^\delta}{\delta} + \frac{h^\delta}{\delta}$$

The individual problem is

$$\max_{c,h} \frac{c^\delta}{\delta} + \frac{h^\delta}{\delta}$$

subject to $c + \ wh \ = \ w + n$

The Lagrangian expression for utility maximization is

$$\boldsymbol{L} \ = \frac{c^\delta}{\delta} + \frac{h^\delta}{\delta} + \lambda(w + n - wh - c)$$

First-order conditions are

$$\partial \boldsymbol{L}/\partial c \ = c^{\delta-1} - \ \lambda \ = \ 0$$

$$\partial \boldsymbol{L}/\partial h \ = h^{\delta-1} - \ \lambda w \ = \ 0$$

$$\partial \boldsymbol{L}/\partial \lambda \ = \ w + n - wh - c \ = \ 0$$

From the first two we get

$$h = cw^{\frac{1}{\delta-1}}$$

Replacing in the third we get the solution for c and h.

Dividing by the budget $(n + w)$ we get the budget share equations are given by

$$s_c = \frac{c}{w+n} = \frac{1}{1+w^k}$$

$$s_h = \frac{wh}{w+n} = \frac{1}{1+w^{-k}}$$

where $k = \delta/(\delta - 1)$

Solving for leisure demand we have:

$$h = \frac{w + n}{w + w^{1-k}}$$

and labor supply is

$$l(w, n) = 1 - h = \frac{w^{1-k} - n}{w + w^{1-k}}$$

Assume that

$\delta = 0.5$ then $k = -1$

$$l(w, n) = \frac{w^2 - n}{w + w^2}$$

If $n = 0$ then $\frac{dl}{dw} > 0$

The high grade of substitutability between consumption and leisure makes that the substitution effect dominates the income effect labor supply is

$$l(w,n) = 1 - h = \frac{w^{1-k} - n}{w + w^{1-k}}$$

Assume that

$\delta = -1$ then $k = 0.5$

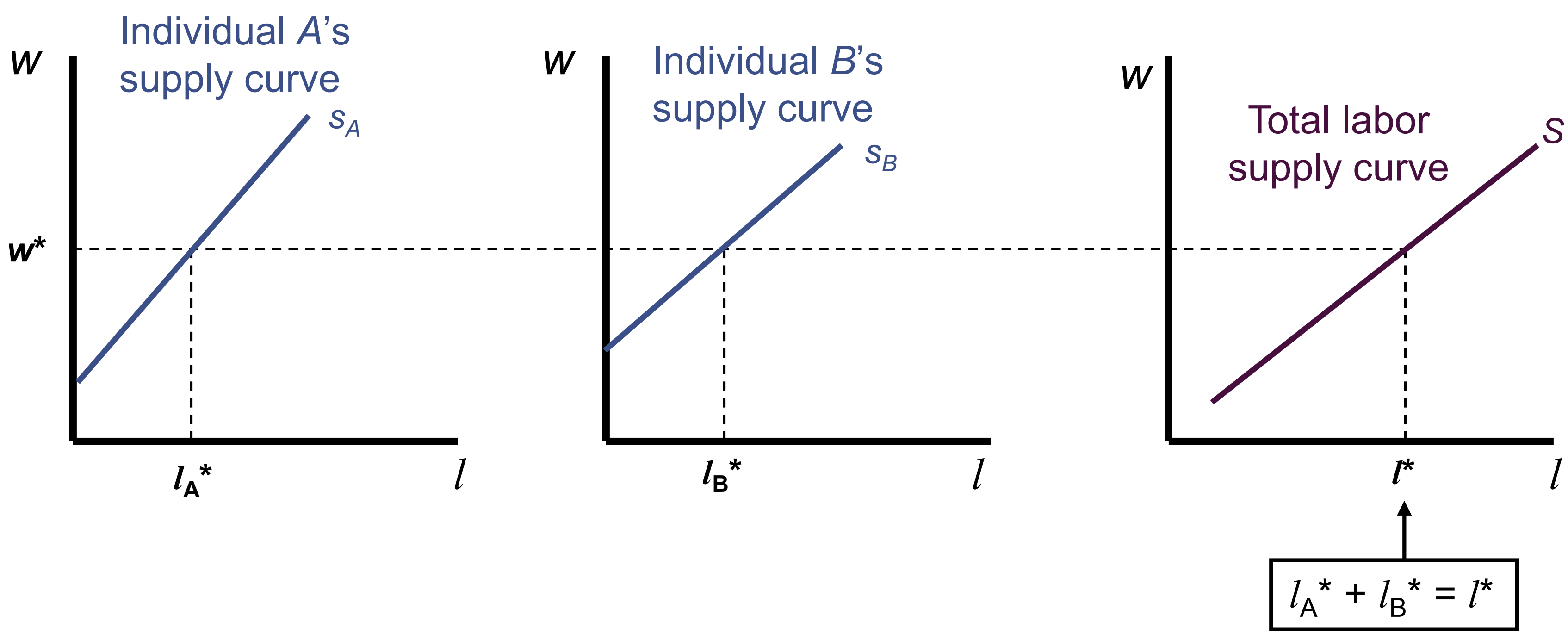
$$l(w,n) = \frac{w^{0.5} - n}{w + w^{0.5}}$$

If $n = 0$ then $\frac{dl}{dw} < 0$

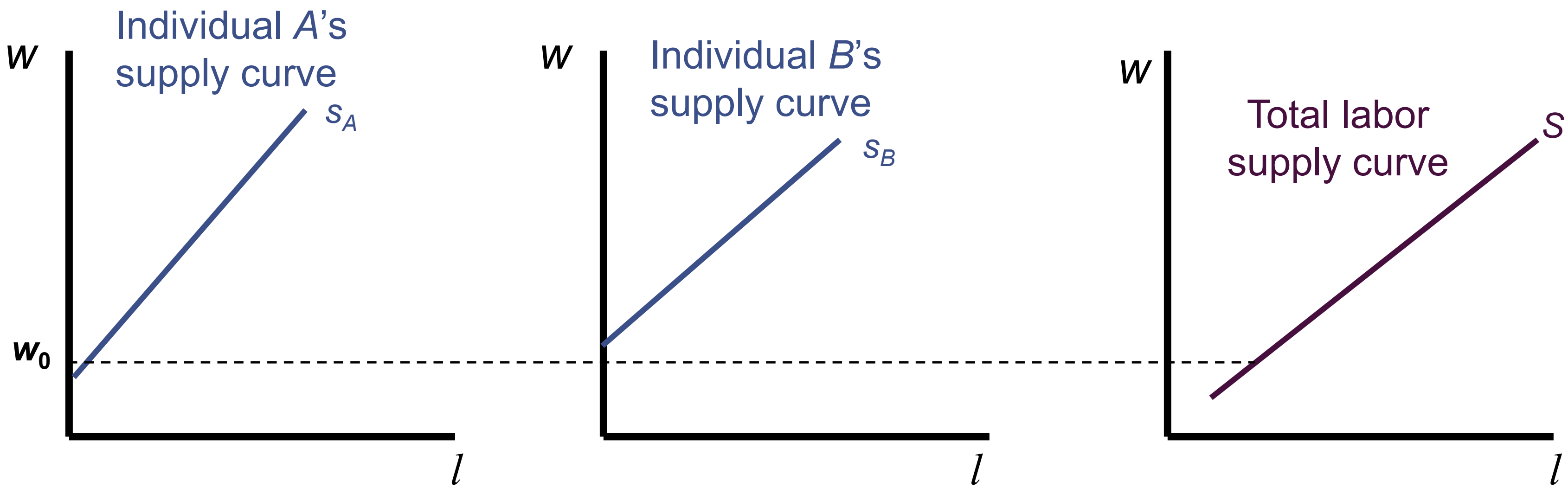
The smaller grade of substitutability between consumption and leisure makes that the income effect dominates the substitution effect

Market Supply Curve for Labor

To derive the market supply curve for labor, we sum the quantities of labor offered at every wage



Note that at w0, individual B would choose to remain out of the labor force

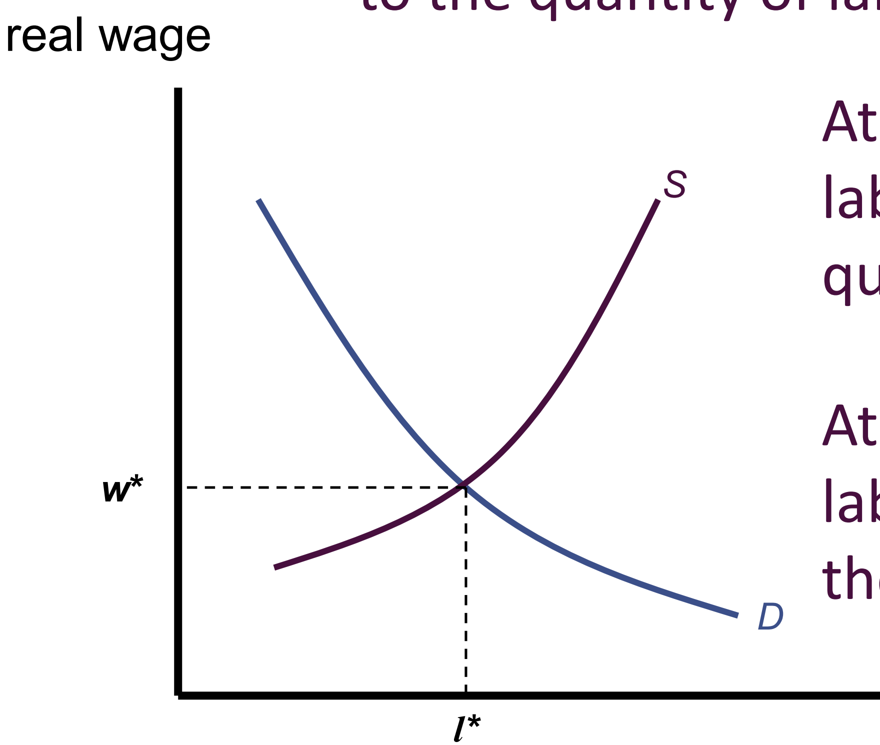


As w rises, l rises for two reasons: increased hours of work and increased labor force participation

Labor Market Equilibrium

Equilibrium in the labor market is established through the interactions of individuals’ labor supply decisions with firms’ decisions about how much labor to hire

At w*, the quantity of labor demanded is equal to the quantity of labor supplied



At any wage above w*, the quantity of labor demanded will be less than the quantity of labor supplied

At any wage below *w**, the quantity of labor demanded will be greater than the quantity of labor supplied

Example: Mandated Benefits

A number of new laws have mandated that employers provide special benefits to their workers

- health insurance
- paid time off
- minimum severance packages

The effects of these mandates depend on how much the employee values the benefit

Suppose that, prior to the mandate, the supply and demand for labor are

$$l_S = a + bw$$

$$l_D = c - dw$$

Setting $l_S = l_D$ yields an equilibrium wage of

$$w^* = \frac{c-a}{b+d}$$

Suppose that the government mandates that all firms provide a benefit to their workers that costs t per unit of labor hired

- unit labor costs become $w + t$

Suppose also that the benefit has a value of k per unit supplied

- the net return from employment rises to $w + k$

- Equilibrium in the labor market then requires that

$$a + b(w + k) = c - d(w + t)$$

- This means that the net wage is

$$w^{**} = \frac{c - a}{b + d} - \frac{bk - dt}{b + d} = w^{*} - \frac{bk - dt}{b + d}$$

$$w^{**} = \frac{c-a}{b+d} - \frac{bk+dt}{b+d} = w^{*} - \frac{bk+dt}{b+d}$$

If workers derive no value from the mandated benefits (*k* = 0), the mandate is just like a tax on employment. They pay a share oif the tax equal to $\frac{d}{b+d}$

Similar results will occur as long as $k < t$

If *k = t*, the new wage falls precisely by the amount of the cost, i.e. $w^{**} = w^{*} - t$, and the equilibrium level of employment does not change

If *k > t (*i.e. workers value the benefit more than it cost to the firm), the new wage falls by more than the cost of the benefit and the equilibrium level of employment rises

Wage Variation

It is impossible to explain the variation in wages across workers with the tools developed so far

we must consider the heterogeneity that exists across workers and the types of jobs they take

Human Capital

- differences in human capital translate into differences in worker productivities
- workers with greater productivities would be expected to earn higher wages
- while the investment in human capital is similar to that in physical capital, there are two differences
  - investments are sunk costs
  - opportunity costs are related to past investments

Compensating Differentials

- individuals prefer some jobs to others
- desirable job characteristics may make a person willing to take a job that pays less than others
- jobs that are unpleasant or dangerous will require higher wages to attract workers
- these differences in wages are termed <u>compensating differentials</u>

Job search

- Wage differences can also arise from differences in the success that workers have in finding good job matches.
- The primary difficulty is that the job search process involves uncertainty. Workers new to the labor force may have little idea of how to find work.
- Workers who have been laid off from jobs face special problems, in part because they lose the returns to the specific human capital they have accumulated unless they are able to find another job that uses these skills
- In general workers could differ for the cost of the job search. This reflects on the intensity if the job search and on the reservation wage.
- Who search with more intensity or who has an higher reservation wage probably will get a job with higher salary

Monopsony in the Labor Market

In many situations, the supply curve for an input (l) is not perfectly elastic

We will examine the polar case of monopsony, where the firm is the single buyer of the input in question

- the firm faces the entire market supply curve
- to increase its hiring of labor, the firm must pay a higher wage to all workers

The marginal expense (*ME*) associated with any input is the increase in total costs of that input that results from hiring one more unit

- if the firm faces an upward-sloping supply curve for that input, the marginal expense will exceed the wage rate

If the total cost of labor is wl, then

$$ME_l = \frac{dwl}{dl} = w + l\frac{dw}{l}$$

In the competitive case, $\partial w/\partial l = 0$ and $ME_l = w$

If the firm face a positively sloped labor supply curve, then

$\partial w/\partial l > 0$ and $ME_l > w$

Marginal input expense: The marginal expense (ME) associated with any input is the increase in total costs of the input that results from hiring one more unit.

If the firm faces an upward-sloping supply curve for the input, the marginal expense will exceed the market price of the input.

Suppose a good x is produced by production function $q = f(l)$ where l is quantity of labor and q the quantity of x produced. Good x is sold on the market at fixed price p. Also assume that w is the cost of l

The firm problem is to maximize its profits

$$\max_l pf(l) - wl$$

FOCs are: $pf'(l) = w$

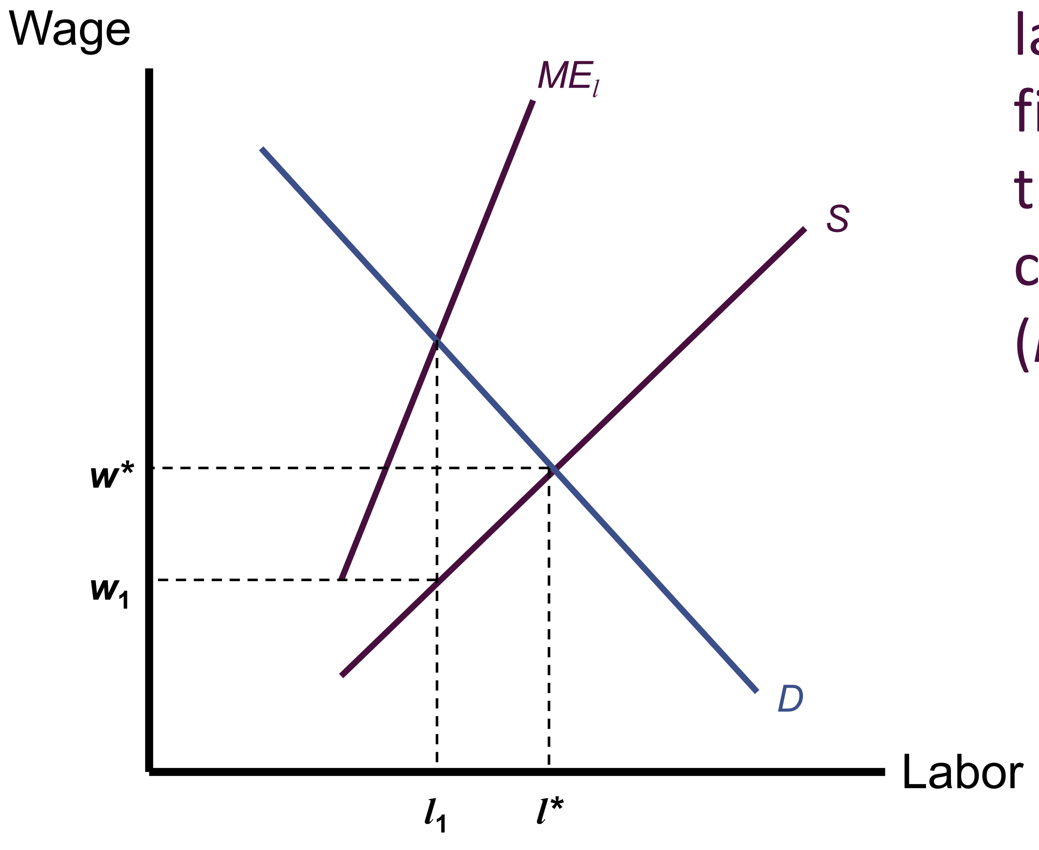
This is the inverse demand of labor..

Now assume the inverse labor supply is $w = h(l)$, monopsony case. The firm's problem is

$$\max_l pf(l) - h(l)l$$

FOCs are: $pf'(l) = h'(l)l + h(l)$

Labor demand (on the left) is equal to marginal expenditure (on the right)



Note that the quantity of labor demanded by this firm falls short of the level that would be hired in a competitive labor market (l*)

The wage paid by the firm will also be lower than the competitive level (w*)

Example: Monopsonistic Hiring

Suppose that a coal mine's workers can dig 2 tons per hour and coal sells for $10 per ton

- this implies that MRP_l = $20 per hour

If the coal mine is the only hirer of miners in the local area, it faces a labor supply curve of the form

$$l = 50w \quad (or\ w = \frac{l}{50})$$

The firm's wage expenditure is

$$wl = l^2/50$$

The marginal expense associated with hiring miners is

$$ME_l = \partial wl/\partial l = l/25$$

Setting $ME_l = MRP_l$, we find that the optimal quantity of labor is 500 and the optimal wage is $10

Labor Unions

If association with a union was wholly voluntary, we can assume that every member derives a positive benefit

With compulsory membership, we cannot make the same claim

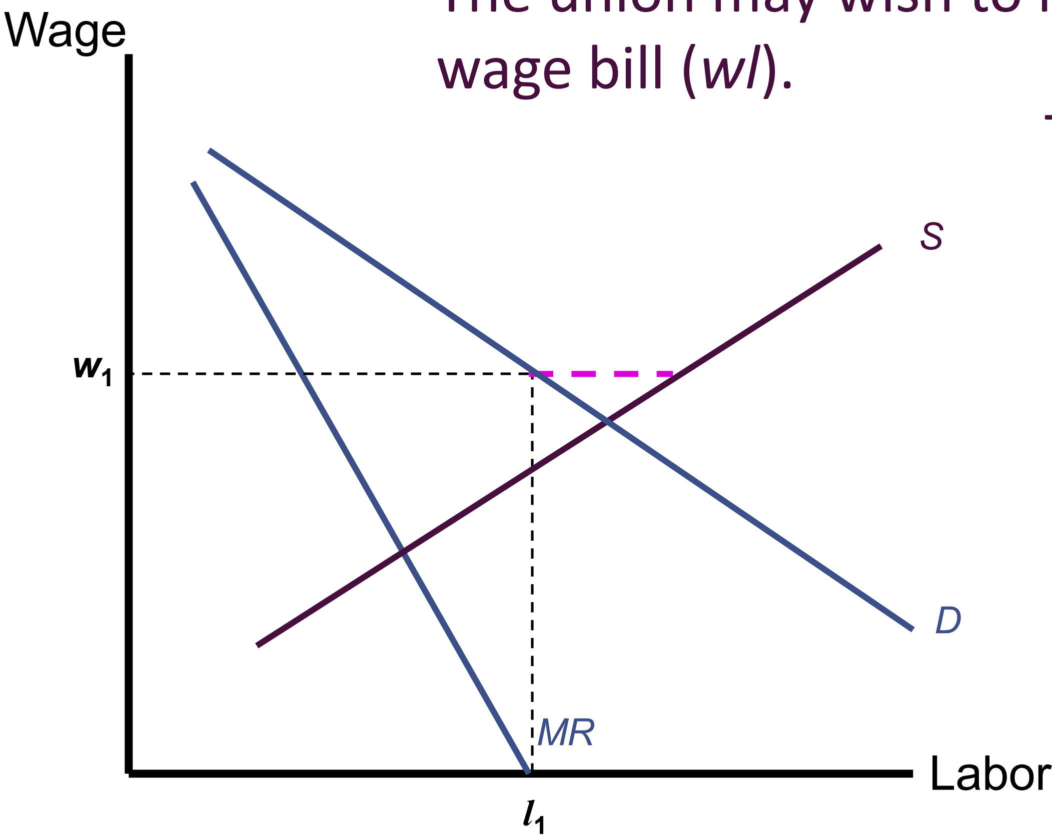
- even if workers would benefit from the union, they may choose to be “free riders”

We will assume that the goals of the union are representative of the goals of its members

In some ways, we can use a monopoly model to examine unions

- the union faces a demand curve for labor
- as the sole supplier, it can choose at which point it will operate
  - this point depends on the union’s goals

The union may wish to maximize the total wage bill (*wl*).

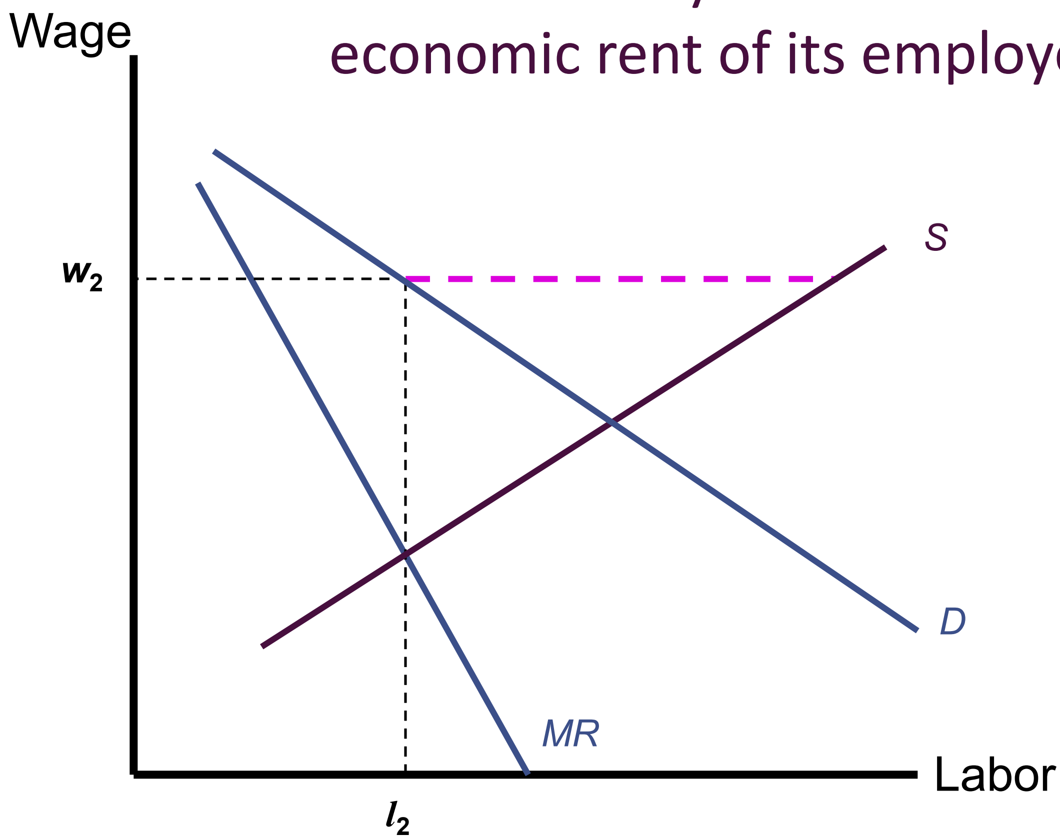


This occurs where MR=0

l_1 workers will be hired and paid a wage of w_1

This choice will create an excess supply of labor

The union may wish to maximize the total economic rent of its employed members

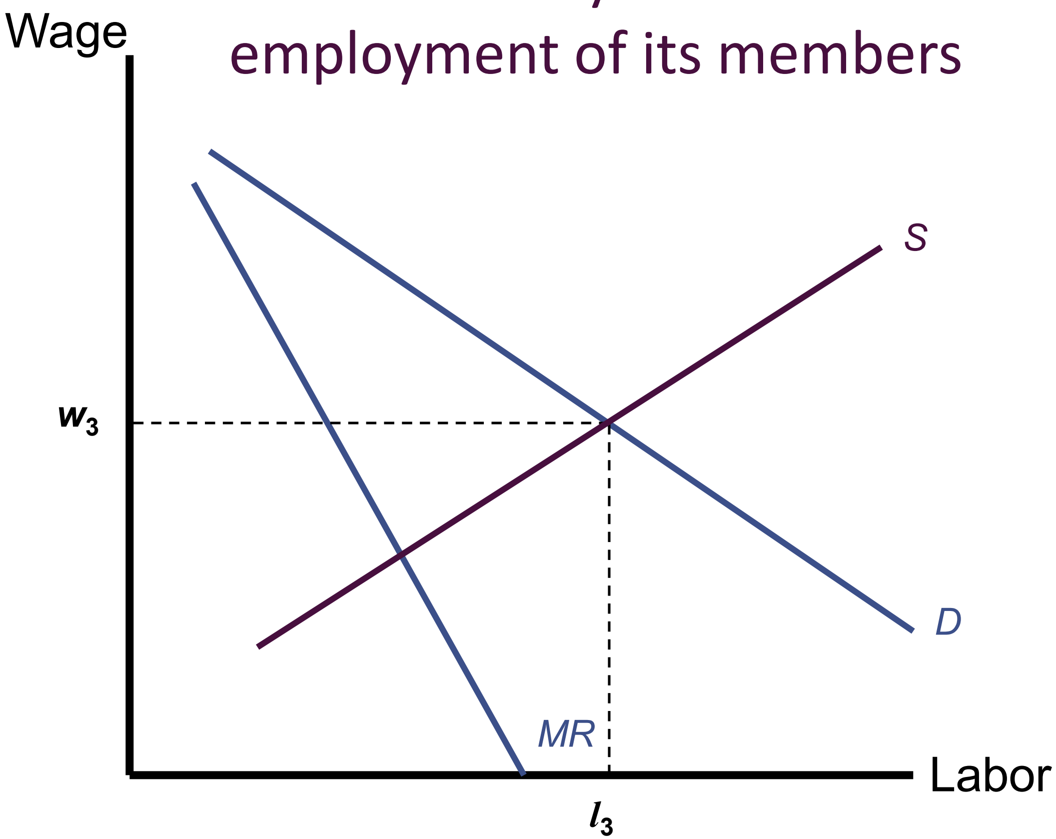


This occurs where *MR* = *S*

l_2 workers will be hired and paid a wage of w_2

Again, this will cause an excess supply of labor

The union may wish to maximize the total employment of its members



This occurs where *D* = *S*

l_3 workers will be hired and paid a wage of w_3

Example: Modeling a Union

A monopsonistic hirer of coal miners faces a supply curve of

$$l = 50w$$

Assume that the monopsony has a MRP_L curve of the form

$$MRP_l = 70 - 0.1l$$

The monopsonist will choose to hire 500 workers at a wage of $10

If a union can establish control over labor supply, other options become possible

- competitive solution where l = 583 and w = $11.66
- monopoly solution where l = 318 and w = $38.20

Example: A Union Bargaining Model

Suppose a firm and a union engage in a two-stage game

- first stage: union sets the wage rate its workers will accept
- second stage: firm chooses its employment level

This two-stage game can be solved by backward induction

The firm's second-stage problem is to maximize its profits:

$$\pi = R(l) - wl$$

The first-order condition for a maximum is

$$R'(l) = w$$

Assuming that $l^*(w)$ solves the firm's problem, the union's goal is to choose w to maximize utility

$$U(w, l) = U[w, l^*(w)]$$

and the first-order condition for a maximum is

$$U_1 + U_2 l' = 0$$

$$U_1/U_2 = l'$$

This implies that the union should choose w so that its *MRS* is equal to the slope of the firm's labor demand function

The result from this game is a Subgame Perfect Nash equilibrium

(see lecture 6, from slide 51 example "wage and employment")