Problem set 8

14.2

A monopolist faces a market demand curve given by

$$Q = 70 - p.$$

a. If the monopolist can produce at constant average and marginal costs of $AC = MC = 6$, what output level will the monopolist choose to maximize profits? What is the price at this output level? What are the monopolist's profits?

b. Assume instead that the monopolist has a cost structure where total costs are described by

$$C(Q) = 0.25Q^2 - 5Q + 300.$$

With the monopolist facing the same market demand and marginal revenue, what price–quantity combination will be chosen now to maximize profits? What will profits be?

c. Assume now that a third cost structure explains the monopolist's position, with total costs given by

$$C(Q) = 0.0133Q^3 - 5Q + 250.$$

Again, calculate the monopolist's price–quantity combination that maximizes profits. What will profit be? *Hint:* Set $MC = MR$ as usual and use the quadratic formula to solve the second-order equation for Q.

d. Graph the market demand curve, the MR curve, and the three marginal cost curves from parts (a), (b), and (c). Notice that the monopolist's profit-making ability is constrained by (1) the market demand curve (along with its associated MR curve) and (2) the cost structure underlying production.

a) The maximizing profit condition is $MR = MC$

Revenue are: $R = Qp = (70 - Q)Q$

Marginal revenues are: $MR = 70 - 2Q$

$MR = MC \rightarrow 70 - 2Q = 6$

$Q^* = 32$

$P^* = 70 - 32 = 38$

$\pi^* = 32 \cdot 38 - 32 \cdot 6 = 1024$

b) $C = 0.25\, Q^2 - 5Q + 300$

$MR = MC \quad \rightarrow \quad 70 - 2Q = 0.5\, Q - 5 \quad \rightarrow$

$Q^* = 30$

$P^* = 70 - 30 = 40$

$\pi^* = 30 \cdot 40 - 0.25\, 30^2 + 5 \cdot 30 - 300 = 825$

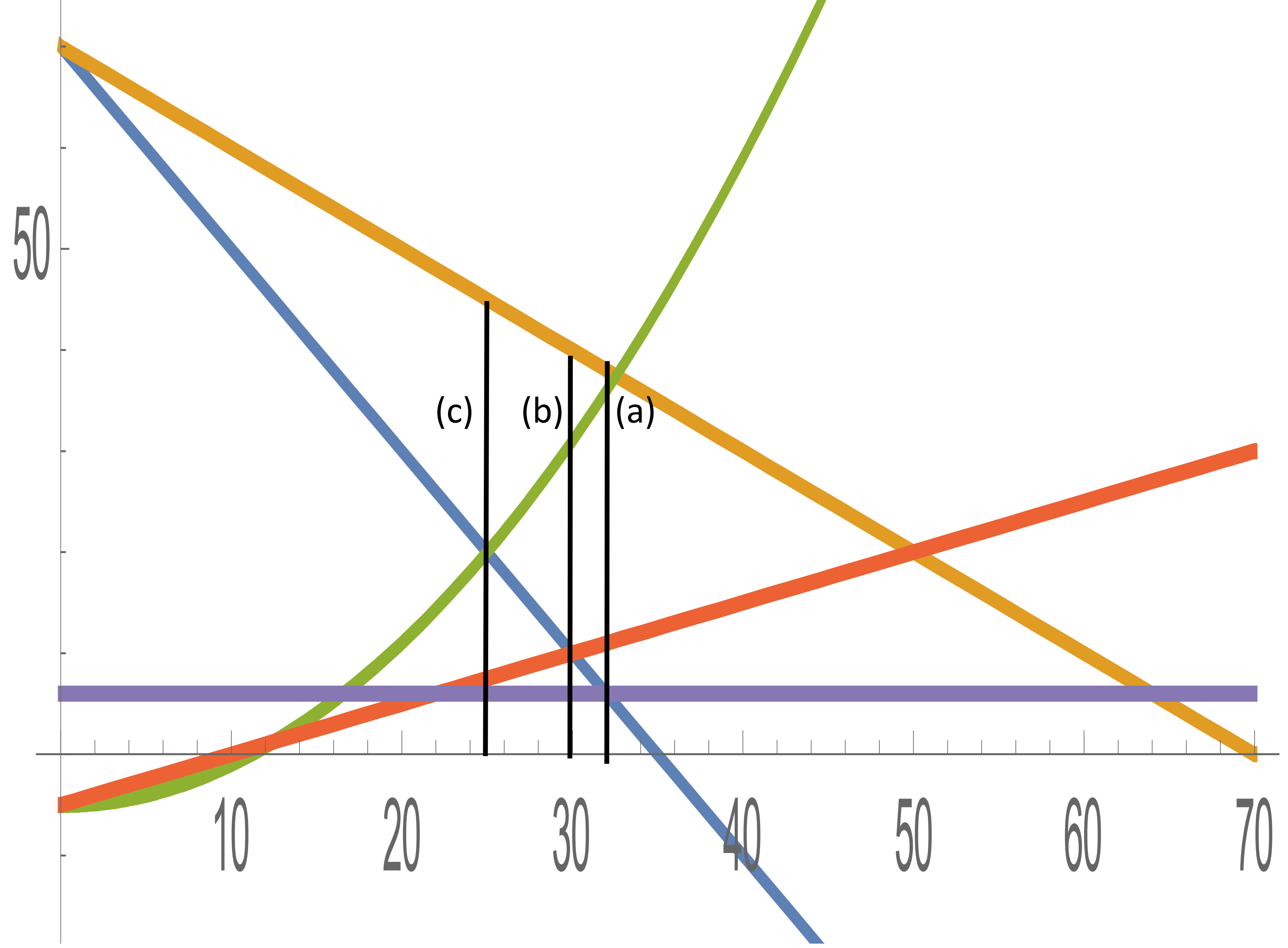
c) $C = 0.0133\, Q^3 - 5Q + 250$

$MR = MC$ → $70 - 2Q = 0.04\, Q^2 - 5$ →

$Q^* = 25$

$P^* = 70 - 25 = 45$

$\pi^* = 25 \cdot 45 - 0.0133\, 25^3 + 5 \cdot 25 - 250 = 792$


50
(c)
(b)
(a)
10
20
30
40
50
60
70

14.5

Suppose a monopoly market has a demand function in which quantity demanded depends not only on market price (P) but also on the amount of advertising the firm does (A, measured in dollars). The specific form of this function is

$$Q = (20 - P)(1 + 0.1A - 0.01A^2).$$

The monopolistic firm's cost function is given by

$$C = 10Q + 15 + A.$$

a. Suppose there is no advertising ($A = 0$). What output will the profit-maximizing firm choose? What market price will this yield? What will be the monopoly's profits?

b. Now let the firm also choose its optimal level of advertising expenditure. In this situation, what output level will be chosen? What price will this yield? What will the level of advertising be? What are the firm's profits in this case? *Hint:* This can be worked out most easily by assuming the monopoly chooses the profit-maximizing price rather than quantity.

a)

No advertising $A = 0$

Demand is $Q = 20 - P$ and $C = 10Q + 15$

Then

$R = Q(20 - Q)$ and $MR = 20 - 2Q$. $MC = 10$

Maximizing condition is: $MR = MC$ → $20 - 2Q = 10$

$Q^* = 5$ and $P^* = 15$

$\pi^* = 5 \cdot 15 - 10 \cdot 5 - 15 = 10$

b) $Q = (20 - P)(1 + 0.1A - 0.01A^2)$ and $C = 10Q + 15 + A$

$$\pi = PQ - C =$$
$$= P(20 - P)(1 + 0.1A - 0.01A^2) - 10(20 - P)(1 + 0.1A - 0.01A^2) - 15 - A$$

The Firm's problem is to maximize profits:

$$\max_{P,A} P(20 - P)(1 + 0.1A - 0.01A^2) - 10(20 - P)(1 + 0.1A - 0.01A^2) - 15 - A$$

FOCs: $$\begin{cases} (20 - 2P)(1 + 0.1A - 0.01A^2) + 10(1 + 0.1A - 0.01A^2) = 0 \\ P(20 - P)(0.1 - 0.02A) - 10(20 - P)(0.1 - 0.02A) - 1 = 0 \end{cases}$$

From the first equation can be written as

$(20 - 2P) + 10 = 0 \ \rightarrow \mathrm{P} = 15$ and $A = 16.18$

i) We replace $P = 15$ in the second equation and we get $A = 3$

ii) We replace $A = 16.18$ in the second equation and we get $P = 9.57$ and $P = 20.43$

Comparing the profit the maximizing profit solution is i) where $Q = 6.05$ and $\pi = 12.25$

14.6

Suppose a monopoly can produce any level of output it wishes at a constant marginal (and average) cost of \$5 per unit. Assume the monopoly sells its goods in two different markets separated by some distance. The demand curve in the first market is given by

$$Q_1 = 55 - P_1,$$

and the demand curve in the second market is given by

$$Q_2 = 70 - 2P_2.$$

a. If the monopolist can maintain the separation between the two markets, what level of output should be produced in each market, and what price will prevail in each market? What are total profits in this situation?

d. Now assume the two different markets 1 and 2 are just two individual consumers. Suppose the firm could adopt a linear two-part tariff under which marginal prices charged to the two consumers must be equal but their lump-sum entry fees might vary. What pricing policy should the firm follow?

a) $Q_1 = 55 - P_1 \quad Q_2 = 70 - 2P_2 \quad MC = 5 \rightarrow P_1 = 55 - Q_1$ and $P_2 = 35 - \frac{Q_2}{2}$

In each market condition $MR = MC$ has to be satisfied

$R_1 = (55 - Q_1)Q_1$ and $R_2 = (35 - \frac{Q_2}{2})Q_2$

$MR_1 = 55 - 2Q_1$ and $MR_2 = 35 - Q_2$

$55 - 2Q_1 = 5 \rightarrow Q_1 = 25 \rightarrow P_1 = 30$

$35 - Q_2 = 5 \rightarrow Q_2 = 30 \rightarrow P_1 = 20$

$$\pi = 25 \cdot 30 + 30 \cdot 20 - 5 \cdot (25 + 30) = 1075$$

b) The firm can maximize its profits setting a price equal to marginal cost, in order to maximize the consumer surplus. Then each consumers will be charged of an entry fees equal to its surplus.

With $p = 5 \rightarrow Q_1 = 50$ and $Q_2 = 60$

$CS_1 = 50 \cdot (55 - 5)/2 = 1250$ and $CS_2 = 60 \cdot (35 - 5)/2 = 900$

The entry fee of consumer 1 will be 1250 and the one for consumer 2 will be 900

The marginal price for both consumers will be 5

$Q_1 = 55 - P_1 \quad Q_2 = 70 - 2P_2 \quad MC = 5 \;\rightarrow P_1 = 55 - Q_1$ and $P_2 = 35 - \frac{Q_2}{2}$

$$\pi = CS_1 + CS_2 + (Q_1 + Q_2)(p - 5)$$

$CS_1 = \frac{(55-p)(55-p)}{2} \quad CS_2 = \frac{(35-p)(70-2p)}{2} \quad Q_1 = 55 - p \quad Q_2 = 70 - 2p$

Then the profit function is

$$\pi = \frac{(55 - p)(55 - p)}{2} + \frac{(35 - p)(70 - 2p)}{2} + (125 - 3p)(p - 5)$$

The firm problem is to maximize profits, then FOC are:

$p - 55 + 2p - 70 - 6p + 140 = 0$

$\rightarrow p = 5$

14.10 Taxation of a monopoly good

The taxation of monopoly can sometimes produce results different from those that arise in the competitive case. This problem looks at some of those cases. Most of these can be analyzed by using the inverse elasticity rule (Equation 14.1).

a. Consider first an ad valorem tax on the price of a monopoly's good. This tax reduces the net price received by the monopoly from P to $P(1 - t)$—where t is the proportional tax rate. Show that, with a linear demand curve and constant marginal cost, the imposition of such a tax causes price to increase by less than the full extent of the tax.

b. Suppose that the demand curve in part (a) were a constant elasticity curve. Show that the price would now increase by precisely the full extent of the tax. Explain the difference between these two cases.

c. Describe a case where the imposition of an ad valorem tax on a monopoly would cause the price to increase by more than the tax.

d. A specific tax is a fixed amount per unit of output. If the tax rate is τ per unit, total tax collections are τQ. Show that the imposition of a specific tax on a monopoly will reduce output more (and increase price more) than will the imposition of an ad valorem tax that collects the same tax revenue.

Inverse elasticity rule

$$\frac{P - MC}{P} = -\frac{1}{e}$$

$$p = MC\frac{e}{1+e} \qquad \mathrm{e} < -1$$

Note that $\frac{e}{1+e}$ is increasing by e

a) linear demand

$e = \frac{dQ}{dP}\frac{P}{Q}$ → decreases by P (more elastic)

$$p' = \frac{MC}{1-t}\frac{e'}{1+e'} \qquad e' < e < -1$$

$$\frac{e'}{1+e'} < \frac{e}{1+e}$$

Monopolist receives a lower price

b) Constant elasticity demand

$$p' = \frac{MC}{1-t}\frac{e}{1+e}$$

$$p'(1-t) = MC\frac{e}{1+e} = p$$

Monopolist receives the same price

c) If the monopoly operates on a negatively sloped portion of its marginal cost curve we have (in the constant elasticity case)

$$p' = \frac{MC'}{1-t}\frac{e}{1+e} > \frac{MC}{1-t}\frac{e}{1+e} = \frac{p}{1-t}$$

Monopolist receives an higher price

d) requirement of equal tax revenues. That is,

$$tP_aQ_a = tQ_s$$

where the subscripts refer to the monopoly's choices under the two tax regimes. Assuming constant MC, profit maximization requires

$$P_a(1-t)\frac{e}{1+e} = MC$$

$$P_s\frac{e}{1+e} - \tau = MC$$

Using the condition of equal tax revenues you can prove that $P_s > P_a$

15.2

Suppose that firms' marginal and average costs are constant and equal to c and that inverse market demand is given by $P = a - bQ$, where $a, b > 0$.

a. Calculate the profit-maximizing price–quantity combination for a monopolist. Also calculate the monopolist's profit.
b. Calculate the Nash equilibrium quantities for Cournot duopolists, which choose quantities for their identical products simultaneously. Also compute market output, market price, and firm and industry profits.
c. Calculate the Nash equilibrium prices for Bertrand duopolists, which choose prices for their identical products simultaneously. Also compute firm and market output as well as firm and industry profits.
d. Suppose now that there are n identical firms in a Cournot model. Compute the Nash equilibrium quantities as functions of n. Also compute market output, market price, and firm and industry profits.
e. Show that the monopoly outcome from part (a) can be reproduced in part (d) by setting $n = 1$, that the Cournot duopoly outcome from part (b) can be reproduced in part (d) by setting $n = 2$ in part (d), and that letting n approach infinity yields the same market price, output, and industry profit as in part (c).

a) monopolist

$R = (a - bQ)Q \rightarrow MR = a - 2bQ$

The profit maximizing condition is $MR = MC \rightarrow a - 2bQ = c \rightarrow Q = \frac{a-c}{2b}$

$$P = a - bQ = a - b\frac{a-c}{2b} = \frac{a+c}{2}$$

$$\pi = (P - c)Q = \frac{(a-c)^2}{4b}$$

b) Cournot duopolists → let be q_1 and q_2 the quantities chosen by the two firms

The profit function of firm 1 is

$$\pi_1 = (a - b(q_1 + q_2) - c)q_1$$

Its problem is

$$\max_{q_1}(a - b(q_1 + q_2) - c)q_1$$

FOC is: $a - 2bq_1 - bq_2 - c = 0$

The FOC of the problem of firm 2 is: $a - bq_1 + 2bq_2 - c = 0$

Then we have to solve an equation system formed by the two conditions $\begin{cases} a - 2bq_1 - bq_2 - c = 0 \\ a - bq_1 - 2bq_2 - c = 0 \end{cases}$

$$q_1 = q_2 = \frac{a - c}{3b}$$

$$P = a - b(q_1 + q_2) = a - b2\frac{a - c}{3b} = a - 2\frac{a - c}{3} = \frac{a + 2c}{3}$$

$$\pi_2 = \pi_1 = (P - c)q_1 = \frac{(a - c)^2}{9b}$$

c) Bertand duopolists → let be p_1 and p_2 the prices chosen by the two firms

In the Nash equilibrium $p_1 = p_2 = c$

$$q_1 + q_2 = \frac{a - P}{b} = \frac{a - c}{b}$$

and

$$\pi_2 = \pi_1 = 0$$

d) Cournot with n firms→ let be q_i the quantities chosen by firm i, $i \in \{1, 2, \dots, n\}$

The profit function of firm j is

$$\pi_j = \left(a - b\sum_{i=1}^{n} q_i - c\right) q_j$$

And its problem is

$$\max_{q_j} \left(a - b\sum_{i=1}^{n} q_i - c\right) q_j$$

FOC is: $a - 2bq_j - b\sum_{\substack{i=1,\\ i\neq j}}^{n} q_i - c = 0$

This can be written as $bq_j = a - b\sum_{i=1}^{n} q_i - c = 0$ so that we can conclude that firms produce equal quantities q.

Then equation can be written as $a - (n+1)bq_i - c = 0$

$$q_i = \frac{a-c}{(n+1)b} \qquad P = a - b\left(n\frac{a-c}{(n+1)b}\right) = a - n\frac{a-c}{(n+1)} = \frac{a+nc}{n+1}$$

$$\pi_i = (P-c)q_1 = \frac{(a-c)^2}{(n+1)^2 b}$$

e)

$$q_i = \frac{a-c}{(n+1)b} \qquad P = \frac{a+nc}{n+1} \qquad \pi_i = \frac{(a-c)^2}{(n+1)^2 b}$$

$$q_i = \frac{a-c}{2b} \qquad P = \frac{a+c}{2} \qquad \pi_i = (P-c)q_1 = \frac{(a-c)^2}{4b}$$

ii) $n = 2$

$$q_i = \frac{a-c}{3b} \qquad P = \frac{a+2c}{3} \qquad \pi_i = \frac{(a-c)^2}{9b}$$

iii) $n = \infty$

$$q_i = 0 \qquad P = c \qquad \pi_i = 0$$

Note that the total quantity does not approach to 0

$$Q = n\frac{a-c}{(n+1)b} \qquad \lim_{n\to\infty} n\frac{a-c}{(n+1)b} = \frac{a-c}{b}$$

15.5

Consider the following Bertrand game involving two firms producing differentiated products. Firms have no costs of production. Firm 1's demand is

$$q_1 = 1 - p_1 + bp_2,$$

where $b > 0$. A symmetric equation holds for firm 2's demand.

a. Solve for the Nash equilibrium of the simultaneous price-choice game.
b. Compute the firms' outputs and profits.

a) The firm's strategy is to choose a price

$q_1 = 1 - p_1 + \mathrm{b}p_2$ and $q_2 = 1 - p_2 + \mathrm{b}p_1$

The profit of firm 1 is written as a function of the prices

The firm 1's problem is

$$\max_{p_1}(1 - p_1 + \mathrm{b}p_2)p_1$$

The FOC is

$$1 - 2p_1 + \mathrm{b}p_2 = 0$$

Then the best response of firm 1 is

$$p_1 = \frac{1 + \mathrm{b}p_2}{2}$$

Using the same steps we find the firm 2's best response

$$p_2 = \frac{1 + \mathrm{b}p_1}{2}$$

Solving the equation system formed by the two best response functions, we get

$$p_1 = p_2 = \frac{1}{2 - b}$$

b) $q_1 = 1 - p_1 + \mathrm{b}p_2$ and $q_2 = 1 - p_2 + \mathrm{b}p_1$

$p_1 = p_2 = \frac{1}{2-b}$

$$q_1 = 1 - p_1 + \mathrm{b}p_2 = 1 - \frac{1}{2-b} + \mathrm{b}\frac{1}{2-b} = \frac{2-b-1+b}{2-b} = \frac{1}{2-b}$$

$$\pi_1 = p_1 q_1 = \frac{1}{2-b}\frac{1}{2-b} = \frac{1}{(2-b)^2}$$

15.7

Assume as in Problem 15.1 that two firms with no production costs, facing demand $Q = 150 - P$, choose quantities q_1 and q_2.

a. Compute the subgame-perfect equilibrium of the Stackelberg version of the game in which firm 1 chooses q_1 first and then firm 2 chooses q_2.

b. Now add an entry stage after firm 1 chooses q_1. In this stage, firm 2 decides whether to enter. If it enters, then it must sink cost K_2, after which it is allowed to choose q_2. Compute the threshold value of K_2 above which firm 1 prefers to deter firm 2's entry.

c. Represent the Cournot, Stackelberg, and entry-deterrence outcomes on a best-response function diagram.

$Q = 150 - P$ and $c = 0$

In Cournot the best responses are $q_1 = \frac{150 - q_2}{2}$ and $q_2 = \frac{150 - q_1}{2}$

The Nash equilibrium is $q_1 = q_2 = 50$

a) Stackelberg model: firm 1 moves first, firm 2 observes and then move.

We use backward induction to find the subgame perfect Nash equilibrium

Start with the problem of firm 2 (the last to move)

$$\max_{q_2} \big(150 - (q_1 + q_2)\big) q_2$$

Its problem is perfectly equivalent to the one it has in the Cournot model. Then its best response is $q_2 = \frac{150 - q_1}{2}$

Now consider the problem of firm 1. Its problem is

$$\max_{q_1}(150 - q_1 - q_2)q_1$$

Firm 1 can anticipate the best response of firm 2. In a sense can “drive” the decision of firm 2.

So its problem becomes:

$$\max_{q_1}\left(150 - q_1 - \frac{150 - q_1}{2}\right)q_1$$

Where we have replaced q_2 by the best response of firm 2

FOC is $150 - 2q_1 - \frac{150-2q_1}{2} = 0$ → $q_1 = 75$

Replacing in the best response of firm 2 we get $q_2 = \frac{150-q_1}{2} = 37.5$

b) Again we use backward induction to solve the game

If firm 2 enters (and pay k_2) its problem is the same than in the previous point.

Then its best response is $q_2 = \frac{150-q_1}{2}$

Its profits are $\pi_2 = \left(150 - q_1 - \frac{150-q_1}{2}\right)\frac{150-q_1}{2} - k_2 = \left(\frac{150-q_1}{2}\right)^2 - k_2$

Then for firm 2 the decision to enter is a best response if profits are nonnegative, i.e.

$$\left(\frac{150 - q_1}{2}\right)^2 - k_2 \geq 0$$

Solving the inequality we get $q_1 \leq -2(-75 + \sqrt{k_2})$ or $q_1 \geq 2(75 + \sqrt{k_2})$

Only the last condition is relevant for this problem

Then the best response of Firm 2 is to enter if $q_1 \leq -2(-75 + \sqrt{k_2})$ and then produce $q_2 = \frac{150-q_1}{2}$

Then the optimal quantity of firm 2 will be $q_2 = \begin{cases} \frac{150-q_1}{2} & if \;\; q_1 \leq -2(-75+\sqrt{k_2}) \\ 0 & otherwise \end{cases}$

Now we consider firm 1 that anticipate the best response of firm 2

So its problem becomes is to maximize

$$\pi_1 = \begin{cases} (150-q_1-q_2)q_1 & if \;\; q_1 \leq -2(-75+\sqrt{k_2}) \\ (150-q_1)q_1 & otherwise \end{cases}$$

To deter firm 2's entry firm 1 has to produce at least $q_1 = -2\left(-75+\sqrt{k_2}\right) = 150 - 2\sqrt{k_2}$

Then its profits are $(150-q_1)q_1 = 2\sqrt{k_2}\left(150-2\sqrt{k_2}\right) = 300\sqrt{k_2} - 4k_2$

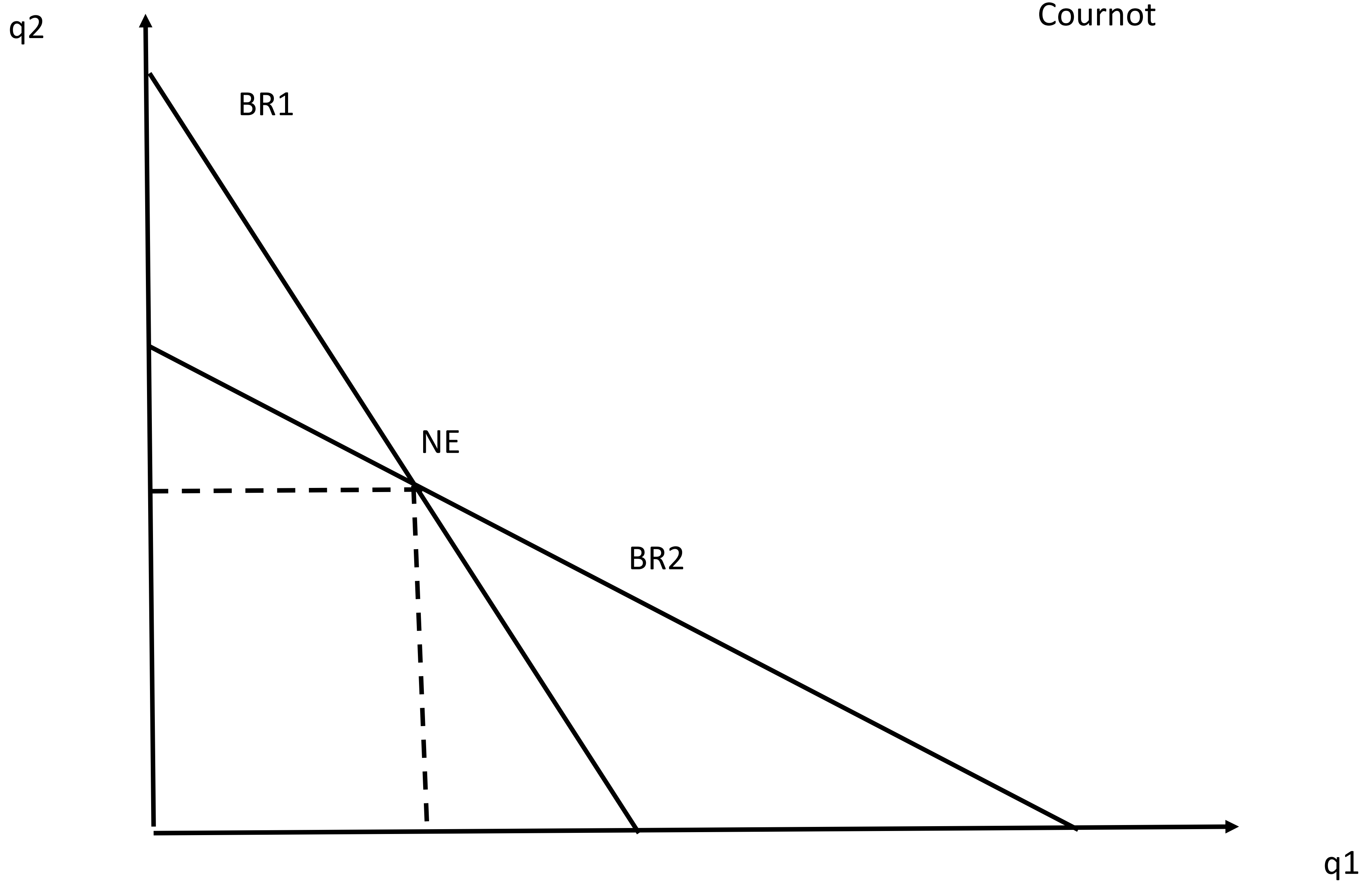
If Firm 1 does not deter firm 2's entry, its profits are as in the previous point of the problem

$$\pi_1 = (150-q_1-q_2)q_1 = (150-75-37.5)75 = 2812$$

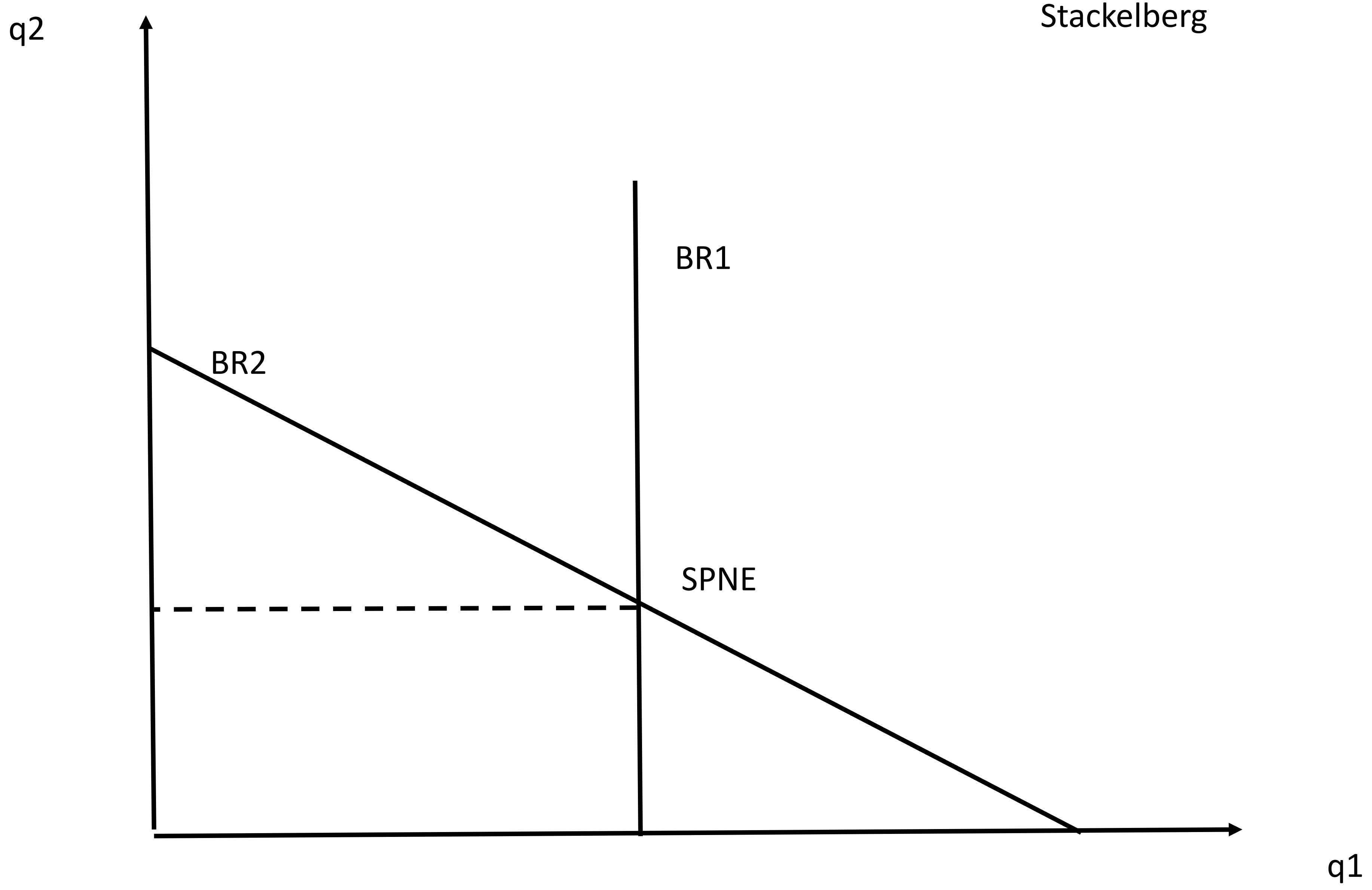
Firm 1 prefers to deter firm 2's entry if

$$300\sqrt{k_2} - 4k_2 \geq 2812$$
$$k_2 \geq 120.6$$

Cournot



Stackelberg



Entry game