Cosmology 1

2019/2020
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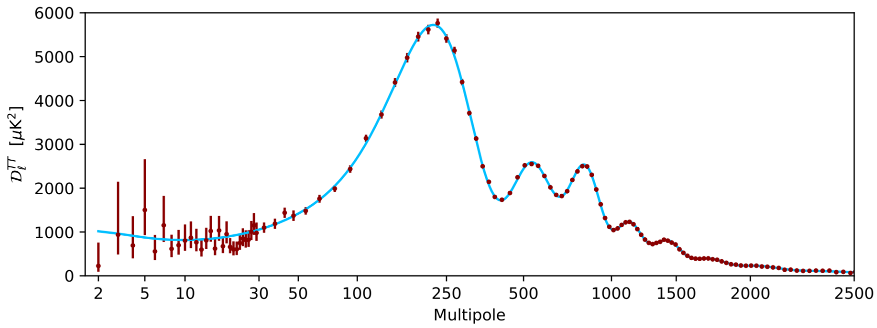
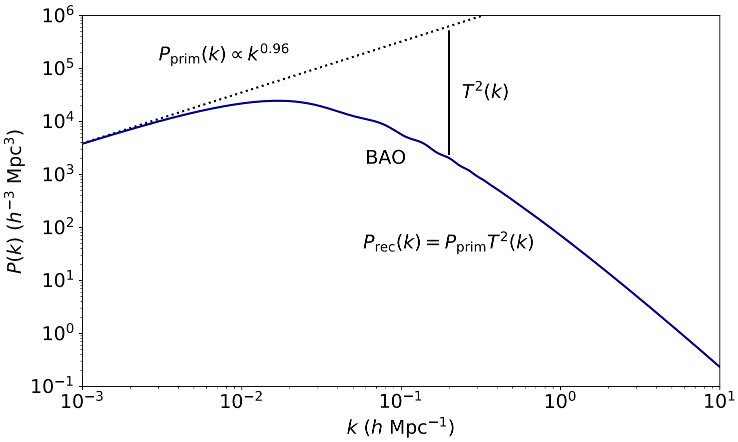
Second intermediate test
Topic: FRW models. Deadline: May 28, 13:00





A very useful standard ruler is given by the sound horizon at recombination. Before recombination, baryons and photons are tightly coupled in a highly ionised plasma, whose sound speed is $c_s = c/\sqrt{3}$; this is easily obtained from $c_s^2 = \partial p/\partial \rho$ when pressure is dominated by photons. Sound waves propagating in this plasma up to recombination imprint a spatial scale $r_{\rm sh}$, the sound horizon, that is visible as a feature both in the power spectrum of temperature fluctuations of the CMB (the acoustic peaks, see the upper figure above) and in the power spectrum of matter at later times (the Baryonic Acoustic Oscillations, BAOs, see the lower figure above). After recombination this scale expands with the scale factor.

The Planck satellite, whose cosmological parameters are given in slide 10 of Lecture 18, gives a recombination redshift of $z_{\rm rec} = 1060$ and a sound horizon comoving length of $r_{\rm sh} = 147$ Mpc.

(1) Consider a model that contains matter, radiation, cosmological constant and curvature. Compute the comoving distance and the diameter distance

as a function of redshift; numerical integration is the most obvious choice for this step. To do this, show that the comoving distance $d_c(z)$ (or $L_{\rm los}(z)$) of an object emitting light at redshift z can be written as:

$$d_c(z) = \int_{t(z)}^{t_0} \frac{cdt'}{a(t')} = \int_0^z \frac{cdz'}{H(z')}$$

Recall that the second Friedmann equation can be written as:

$$\frac{H(z)^2}{H_0^2} = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda + \Omega_K(1+z)^2$$

with $\Omega_K = 1 - \Omega_m - \Omega_r - \Omega_\Lambda$.

Hint: It is a very good idea to test the numerical result against analytic calculations for the cases in which these are available.

Describe the numerical approach adopted, and report the diameter distance (in Mpc) as a function of z, in the range $z \in [0, 10]$, for Planck cosmological parameters, assuming a flat Universe, and for the same parameters but adding ± 0.1 to Ω_m, so as to have non-flat models. *Hint: the parameters given in slide 10 of Lecture 18 assume indeed that the universe is flat, with the exception of Ω_k. We will not need parameter errorbars here. Do not forget radiation, but fix the density parameters so as to keep the universe flat.*

(2) The sound horizon at recombination can be estimated as $1/\sqrt{3}$ times the particle horizon at the recombination redshift $z_{\rm rec} = 1060$, assumed to be constant. Use the tools developed above to compute the sound horizon for the Planck cosmological parameters. Report its physical (at $z_{\rm rec}$) and comoving values in Mpc; how much does the latter differ from the 147 Mpc comoving value found by Planck? This difference is mainly due to the fact that the sound speed c_s evolves in time, details are explained in Chapter 7 of the Vittorio textbook. *Hint: you can correct for this difference by rescaling the sound horizon (i.e. multiplying it by a constant) to be* 147 *Mpc for the Planck flat cosmology.*

(3) Suppose astronomers report that, at redshift $z = 0.3$, the BAO scale $r_{\rm sh}$ subtends an angle of $\theta = 0.1166 \pm 0.0023$ rad. Keeping H_0 and Ω_r fixed to their Planck values, what region of the $\Omega_m - \Omega_\Lambda$ parameter space is consistent with this measurement at 1 and 2 σ? Look carefully at your result, and comment it, in the light of our need to get the tightest constraints on cosmological parameters. *Hint: do not bother with negative values of Ω_Λ, but do not restrict either to flat universes.*

(4) The $r_{\rm sh}$ scale is best constrained by CMB observations. Suppose that it subtends an angle of $\theta* = 0.01041 \pm 0.00031$ rad (it's actual errorbar is 100 times smaller than this!). What region of the $\Omega_m - \Omega_\Lambda$ parameter space is consistent with this measurement at 1 and 2 σ? Look carefully at your result and argue what parameter is most accurately constrained by this observation.