

# Tecniche di programmazione in chimica computazionale

## Examples

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# Matrix transpose

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- Transpose conjugated of a matrix: example `tconjug.f90`

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# Franck-Condon factors

Born-Oppenheimer approximation

$$\Psi_{e\nu}(q_e, q_N) = \psi_e(q_e; q_N) \chi_\nu^e(q_N)$$

$$\mu_{e,\nu; e',\nu'} = \int dq_N \chi_\nu^{e,*}(q_N) M_{ee'} \chi_{\nu'}^{e'}(q_N)$$

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Electronic contribution to transition dipole moment **not varying** with  $q_N$

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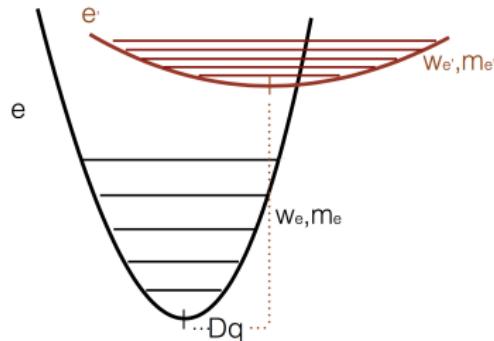
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$$\begin{aligned}S_{\nu,\nu'}^{e,e'} &= \int dq_N \chi_\nu^{e,*}(q_N) \chi_{\nu'}^{e'}(q_N) \\ \text{FC}_{\nu,\nu'}^{e,e'} &= |S_{\nu,\nu'}^{e,e'}|^2\end{aligned}$$

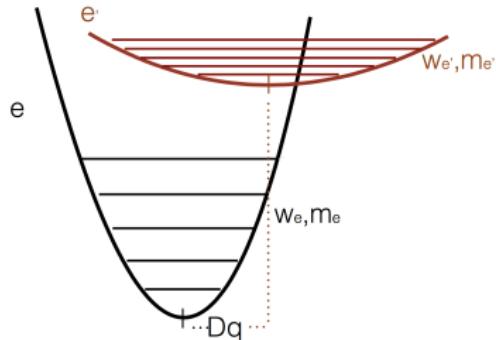
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- Harmonic eigenfunctions

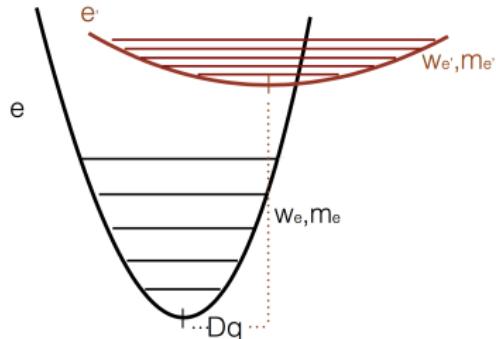
$$\chi_0^e(q_N) = \left( \frac{m_e \omega_e}{\pi} \right)^{1/4} \exp[-(m_e \omega_e) q_N^2 / 2]$$

$$\chi_1^e(q_N) = \sqrt{2} \left( \frac{m_e \omega_e}{\pi} \right)^{1/4} (\sqrt{m_e \omega_e} q_N) \exp[-(m_e \omega_e) q_N^2 / 2]$$

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- Example fc.f90

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- Given the same frequencies, displacement, reduced masses, how the FC factor changes with the **vibrational quantum number  $\nu$  (0, 1 or 2)** of the electronic excited state?

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- Given the same frequencies and reduced masses, how the FC factor changes with the **displacement  $\Delta q$**  (for a chosen  $\nu$ )?