ASYMMETRIC INFORMATION

Transactions can involve a considerable amount of uncertainty. Uncertainty need not lead to inefficiency when both sides of a transaction have the same limited knowledge concerning the future, it can lead to inefficiency when one side has better information.

The side with better information is said to have private information or, equivalently, asymmetric information.

Several sources of asymmetric information:

- Parties will often have ‘‘inside information’’ concerning themselves that the other side does not have
- Other sources of asymmetric information arise when what is being bought is an agent’s service.

Asymmetric information can lead to inefficiencies:

- Insurance companies may offer less insurance and charge higher premiums
- With appliance repair, the repairer may replace parts that still function and may take longer than needed

Principal-Agent Model

One important way in which asymmetric information may affect the allocation of resources is when one person hires another person to make decisions

- patients hiring physicians
- investors hiring financial advisors
- car owners hiring mechanics
- stockholders hiring managers

there is only one party on each side of the market.

The party who proposes the contract is called the principal.

The party who decides whether or not to accept the contract and then performs under the terms of the contract (if accepted) is called the agent.

The agent is typically the party with the asymmetric information.

Two leading models

1) hidden-action model (or moral hazard)

   In a first model, the agent’s actions taken during the term of the contract affect the principal, but the principal does not observe these actions directly.

2) hidden-type model (or adverse selection model)

   the agent has private information about the state of the world before signing the contract with the principal. The agent’s private information is called his type

The hidden-type and hidden-action models cover a wide variety of applications.

Hidden-action models

The principal would like the agent to take an action that maximizes their joint surplus

The agent’s actions may be unobservable to the principal.

then the agent will prefer to shirk, choosing an action to suit himself rather than the principal.

two specific applications:

1) employment contracts signed between a firm’s owners and a manager who runs the firm on behalf of the owners.

2) Contracts offered by an insurance company to insure an individual against accident risk.

1. The Owner-Manager Relationship

Firm has one representative owner and one manager.

The owner, who plays the role of the principal, offers a contract to the manager, who plays the role of the agent.

The manager decides whether to accept the employment contract and, if so, how much effort $e \geq 0$ to exert.

Owner cannot observe e

An increase in e increases the firm's gross profit (π_g) but is costly to the manager:

$$\pi_g = e + \varepsilon$$

where ε is a random variable with 0 means and variance σ^2 that represents all economic factors outside of the control of the manager

Manager disutility from effort is $c(e)$ where $c'(e) > 0$ and $c''(e) > 0$ (convex)

Suppose that s is the salary offered to the manager.

Salary depends on some variable owner can observe.

Net profits π_n are:

$$\pi_n = \pi_g - s = e + \varepsilon - s$$

A risk neutral owner want to maximize the expected net profits:

$$E(\pi_n) = E(e + \varepsilon - s) = e - E(s)$$

The manager is risk averse and his expected utility is:

$$E(U) = E(s) - \frac{A}{2} Var(s) - c(e)$$

where $A > 0$ is a parameter capturing risk aversion

Now we study:

1. Full information case, owner observes e, first best
2. Asymmetric information case, second best

1. Full information case, owner observes e, first best

The owner pay the manager a fixed salary s^* if he exerts the first-best level of effort e^* (which we will compute shortly) and nothing otherwise.

Then in this case $Var(s^*) = 0$ and $E(s^*) = s^*$. Then the manager's expected utility is:

$$E(U) = s^* - \mathrm{c}(e^*)$$

Assuming that manager accepts the contract only if $E(U) \geq 0$, the owner will pay the minimum acceptable salary, i.e.

$$s^* = \mathrm{c}(e^*)$$

The expected net profits:

$$E(\pi_n) = e^* - s^* = e^* - \mathrm{c}(e^*)$$

Owner problem is: $\max\limits_{e^*} e^* - \mathrm{c}(e^*)$

FOCs is $1 - \mathrm{c}'(e^*) = 0$

i.e. marginal cost of effort is equal to marginal benefits

2. <u>Asymmetric information case, second best</u>

the owner cannot observe effort, the first best contract cannot be implemented.

However, the owner can induce the manager to exert some effort if the manager's salary depends on the firm's gross profit.

Suppose the owner offers a salary s:

$s(\pi_g) = a + b\pi_g$ where b is the *power*

We can represent this situation by a three-stage game:

Stage 1: the owner offers a salary scheme (he sets parameter a and b)

Stage 2: manager decides to accept or not

Stage 3: the manager decides the effort to exert (if he has accepted).

Solution: Subgame Perfect Nash equilibrium identified by backward induction.

Solution: we start from stage 3

The manager is maximizing his expected utility:

$E(U) = E(s) - \frac{A}{2}Var(s) - c(e)$ where

$E(s) = E(a + b\pi_g) = E(a + be + b\varepsilon) = a + be$

$Var(s) = Var(a + b\pi_g) = Var(a + be + b\varepsilon) = b^2\sigma^2$

replacing

$E(U) = a + be - \frac{A}{2}b^2\sigma^2 - c(e)$

The manager problem is:

$$\max_e E(U)$$

FOC is $b = c'(e)$

Given that $c(e)$ is increasing, higher power induces higher effort

Stage 2

The manager accepts the contract if and only if:

$$E(U) = a + be - \frac{A}{2}b^2\sigma^2 - c(e) \geq 0$$

$$a \geq c(e) + \frac{A}{2}b^2\sigma^2 - be$$

Stage 1

The owner must choose parameters a and b to offer a salary scheme to the manager.

The owner maximizes his net profits $e - E(s)$, i.e. his problem is:

$$\max_{a,b} e(1-b) - a$$

subject to

$a \geq c(e) + \frac{A}{2}b^2\sigma^2 - be$ (participation constraint)

$c'(e) = b$ (incentive compatibility constraint)

$$\max_{a,b} e(1-b) - a$$

subject to

$a \geq c(e) + \frac{A}{2} b^2 \sigma^2 - be$ (participation constraint)

$c'(e) = b$ (incentive compatibility constraint)

Given that parameter a appears by a negative sign into the profit function, the owner will set the lower possible value. Then the first constraint holds by equality, i.e. $a = c(e) + \frac{A}{2} b^2 \sigma^2 - be$

Replacing the two constraints into the net profit function the owner problem is an unconstrained maximization problem where the ownwr chooses the best effort to induce

$$\max_{e} e - c(e) - \frac{A\left(c'(e)\right)^2 \sigma^2}{2}$$

FOC is $1 - c'(e) - A\sigma^2 c''(e) = 0$

In the second best

$$c'(e) = 1 - A\sigma^2 c''(e) < 1$$

(note that also $c'(e) = b$)

In the first best we have

$$c'(e) = 1$$

given that $c(e)$ is increasing, we can conclude that the equilibrium effort in the second best is at a lower level respect to the first best

The salary paid will be

$$s = a + be = c(e) + \frac{A}{2} b^2 \sigma^2$$

Then the manager receives a salary that is higher that his effort cost

Exercise

Gross profits depend on the effort of the manager and on external factors, according the following table

	Bad Luck	Good Luck
Low effort $e = 0$	$10,000	$20,000
High effort $e = 1$	$20,000	$40,000

The probability of Bad Luck is 0.5
The cost of high effort is $10.000
and that the expected utility of the manager is

$$E(U) = E(w) - 0.1(w_{max} - w_{min}) - c(e)$$

The manager accepts the contract if $E(U) \geq 0$

Owner maximize net profits, i.e. gross profits minus the salary of the manager
<u>Full information case</u>
The owner implements the effort that maximizes his profits. Then he pays a salary if the manager exerts this effort, otherwise pays 0

Full information case

The owner implements the effort that maximizes his profits. Then he pays a fixed salary if the manager exerts this effort, otherwise pays 0

Then the manager's expected utility is $E(U) = w - c(e) \rightarrow w = c(e)$

with effort 0 he can set $w = 0$

His net expected profits are: $\pi_n = \frac{1}{2}10000 + \frac{1}{2}20000 - 0 = 15000$

With effort 1 he can set $w = 10000$

His net expected profits are: $\pi_n = \frac{1}{2}40000 + \frac{1}{2}20000 - 10000 = 20000$

Then the first best contract implements $e = 1$ with a salary $w = 10000$

Asymmetric information case

Suppose, for example, that the owner offers the manager a payment scheme:

$$w = a + b\pi_g$$

Manager prefers $e = 1$ if the utility from $e = 1$ is not lower than utility from $e = 0$, i.e.

$$E(w|e = 1) - 0.1(w_{max} - w_{min}) - c(1) \geq E(w|e = 0) - 0.1(w_{max} - w_{min})$$

Asymmetric information case

Suppose, for example, that the owner offers the manager a payment scheme:

$$w = a + b\pi_g$$

$$E(w|e=0) = a + \frac{1}{2}b10000 + b\frac{1}{2}20000 = a + b15000$$

$$w_{max} = a + b20000 \quad w_{min} = a + b10000$$

$$E(w|e=1) = a + \frac{1}{2}b20000 + b\frac{1}{2}40000 = a + b30000$$

$$w_{max} = a + b40000 \quad w_{min} = a + b20000$$

Manager prefers $e = 1$ if the utility from $e = 1$ is not lower than utility from $e = 0$:

$$E(w|e=1) - 0.1(w_{max} - w_{min}) - c(1) \geq E(w|e=0) - 0.1(w_{max} - w_{min})$$

$$a + b30000 - b2000 - 10000 \geq a + b15000 - b1000$$

$$b28000 - 10000 \geq b14000$$

$$b \geq 5/7 \rightarrow b = 5/7$$

Manager accepts if his utility is at least 0, i.e.

$$E(w|e=1) - 0.1(w_{max} - w_{min}) - c(1) \geq 0$$

$$a \geq -10000$$

Manager accepts if his utility is at least 0, i.e.

$$E(w|e=1) - 0.1(w_{max} - w_{min}) - c(1) \geq 0$$

$$a \geq -10000$$

Then $\mathrm{E}(w) = -10000 + \frac{5}{7}E(\pi_g) = 11428$

Owner profits are

$$\pi_n = \frac{1}{2}40000 + \frac{1}{2}20000 - w = 30000 + 10000 - \frac{5}{7}30000 = 18571$$

Now we check if owner prefers to implement $e = 0$

He can set $b = 0$, then the salary can be $w = 0$

Owner profits are

$$\pi_n = \frac{1}{2}20000 + \frac{1}{2}10000 = 15000$$

Then in this case in the second best high effort is implemented but at an higher expected salary w.r.t. the full information case

Now assume that The cost of high effort is 14.000

Full information case

with effort 0 he can set $w = 0$ and net expected profits are: $\pi_n = 15000$

With effort 1 he can set $w = 14000$

His net expected profits are: $\pi_n = \frac{1}{2}40000 + \frac{1}{2}20000 - 14000 = 16000$

Then the first best contract implements $e = 1$ with a salary $w = 16000$

Asymmetric information case $\rightarrow$ $w = a + b\pi_g$

$$E(w|e = 0) = a + b15000 \qquad w_{max} = a + b20000 \quad w_{min} = a + b10000$$

$$E(w|e = 1) = a + b30000 \qquad w_{max} = a + b40000 \quad w_{min} = a + b20000$$

Manager prefers $e = 1$ if the utility from $e = 1$ is not lower than utility from $e = 0$:

$$b28000 - 14000 \geq b14000$$

$$b \geq 1 \rightarrow b = 1$$

Manager accepts if his utility is at least 0, i.e.

$$a + 28000 - 14000 \geq 0$$

$$a \geq -14000$$

Then $\mathrm{E}(w) = -14000 + E(\pi_g) = 16000$

Owner profits are

$$\pi_n = \frac{1}{2}40000 + \frac{1}{2}20000 - w = 30000 + 14000 - 30000 = 14000$$

Now we check if owner prefers to implement $e = 0$

He can set $b = 0$, then the salary can be $w = 0$

Owner profits are

$$\pi_n = \frac{1}{2}20000 + \frac{1}{2}10000 = 15000$$

Then in this case in the second best low effort is implemented (by $e = 0$ the owner gets higher profits w.r.t $e = 1$)

Respect to the first best, there is a a loss of efficiency:

For this society as a whole, implementing $e = 1$ produces a net surplus of 16000 and implementing $e = 0$, the net surplus is 15000

2. Moral hazard in insurance

Definition of Moral hazard: The effect of insurance coverage on an individual's precautions, which may change the likelihood or size of losses

A risk-averse individual faces the possibility of incurring a loss (l) that will reduce his initial wealth (W_0).

The probability of loss is π.

An individual can reduce π by spending more on preventive measures (e).

Let $U(W)$ be the individual's utility from W.

An insurance company offers an insurance contract involving a payment x to the individual if a loss occurs. The premium for this coverage is p.

If the individual takes the coverage, then his wealth in state 1 (no loss) and state 2 (loss) are

$W_1 = W_0 - e - p$ and

$W_2 = W_0 - e - p - l + x$

and his expected utility is:

$$(1-\pi)U(W_1) + \pi U(W_2)$$

The risk-neutral insurance company's objective is to maximize expected profit:

$$p - \pi x$$

First best insurance contract

In this case the insurance can observe e. Then the insurance contract can determine the values of p, x and e.

The individual accepts the contract if

$$(1-\pi)U(W_1) + \pi U(W_2) \geq \bar{U}$$

Where $\bar{U}$ is its utility in absence of insurance

Then the insurance problem is

$$\max_{p,x,e} p - \pi x$$

subject to $(1-\pi)U(W_1) + \pi U(W_2) = \bar{U}$

The lagrangian is

$$L = p - \pi x + \lambda\big((1-\pi)U(W_1) + \pi U(W_2) - \bar{U}\big)$$

The lagrangian is

$$L = p - \pi x + \lambda\big((1-\pi)U(W_1) + \pi U(W_2) - \overline{U}\big)$$

FOCs are

$$\frac{dL}{dp} = 1 - \lambda\big((1-\pi)U'(W_1) + \pi U'(W_2)\big) = 0$$

$$\frac{dL}{dx} = -\pi + \lambda\pi U'(W_2) = 0$$

$$\frac{dL}{de} = -\frac{d\pi}{de}x - \lambda\left((1-\pi)U'(W_1) + \pi U'(W_2) + \frac{d\pi}{de}(U(W_1) - U(W_2))\right) = 0$$

From the first two conditions

$$\frac{1}{\lambda} = (1-\pi)U'(W_1) + \pi U'(W_2) = U'(W_2)$$

Then $W_1 = W_2 \rightarrow x = l$ (Full insurance that covers all losses)

Replacing $x = l$ and λ in the third condition we get

$$-\frac{d\pi}{de}l = 1$$

Social efficient level of e (precautionary measures)

Second best insurance contract

In this case the insurance cannot observe e. Then the insurance contract can only determine the values of p, and x .

If the individual accepts the contract he is free to choose e to maximize his expected utility

$$(1-\pi)U(W_1)+\pi U(W_2)$$

Usually the second best contract will offer only partial insurance

Under full insurance $W_1 = W_2$ then the individual expected utility is

$$U(W_1) = U(W_0 - e - p)$$

which is maximized for $e = 0$

With partial insurance the individual has incentive to choose some level of $e > 0$

Example

Driver wealth is $w = 100000$, value of the car is 20000.

Car is stolen by probability 0.25

Installing a car alarm costs 1750 and reduce the probability of a theft to 0.15

The utility function is $u(x) = \ln(x)$

<u>No insurance:</u>

No alarm $u = 0.75 \ln(100000) + 0.25 \ln(80000) = 11.457$

with alarm $u = 0.85 \ln(100000 - 1750) + 0.15 \ln(80000 - 1750) = 11.461$

Individual prefers to install the alarm

First best

In this case, the insurance requires the alarm to be installed and cover all loss, i.e. $x = l = 20000$

The insurance will charge a premium that leaves the individual indifferent between accept or not

$$\ln(100000 - 1750 - p) =$$
$$= 0.85\ln(100000 - 1750) + 0.25\ln(80000 - 1750) =$$
$$= 11.461$$

Solving we get $p = 3298$

The company profits are equal to $p - 0.15 \cdot x = 3298 - 0.15 \cdot 20000 = 298$

Second best

In this case, the insurance requires cannot require the alarm to be installed

If the insurance offers full insurance, the individual will not install the alarm

The insurance will charge a premium that leaves the individual between accept or not

$$\ln(100000 - p) = 0.85 \ln(100000 - 1750) + 0.25 \ln(80000 - 1750) =$$
$$= 11.461$$

Solving we get $p = 5048$

The company profits are equal to $p - 0.25 \cdot x = 5048 - 0.25 \cdot 20000 = 48$

Now assume that insurance company offer a contract with $x = 3374$ (partial insurance) and $p = 602$

The individual is indifferent between:

- to buy the insurance and install the alarm
- buy the insurance and not install the alarm
- not buy the insurance.

Then he weakly prefers the first option and insurance profits are 96

Suppose $p = 1000$ and $x = 5000$

What is the behavior of the individual?

Buy/not buy insurance, install/not install alarm.

What are the profits of the insurance?

Hidden-types models

The agent's type may be unobservable to the principal before signing the contract.

Differently from the hidden action models, the asymmetry of information is before signing the contract

two specific applications:

1) Nonlinear pricing

2) Private information in insurance

1. Nonlinear pricing

Consumer surplus is

$$u = \theta v(q) - T$$

from consuming a bundle of q units of a good for which he pays a total tariff of T.

$\theta v(q)$ reflects the consumer's benefit, $v'(q) > 0$ and $v''(q) < 0$.

The consumer's type is given by $\theta = \begin{cases} \theta_H \ by\ probability\ \ \beta \\ \theta_L \ \ by\ probability\ 1-\beta' \end{cases}$

The high type enjoys consuming the good more than the low type

there is a single consumer in the market.

The monopolist has a constant marginal and average cost c

Monopolist's profit from selling a bundle of q units for a total tariff of T is

$$\pi = T - q \cdot c$$

First best nonlinear pricing

Before offering a contract, the monopolist observes the type θ

Consumer will accept the contract if

$$u = \theta v(q) - T \geq 0$$

(participation constraint) . For the monopolist the best strategy is:

$$T = \theta v(q)$$

Monopolist's profits are:

$$\pi = \theta v(q) - c \cdot q$$

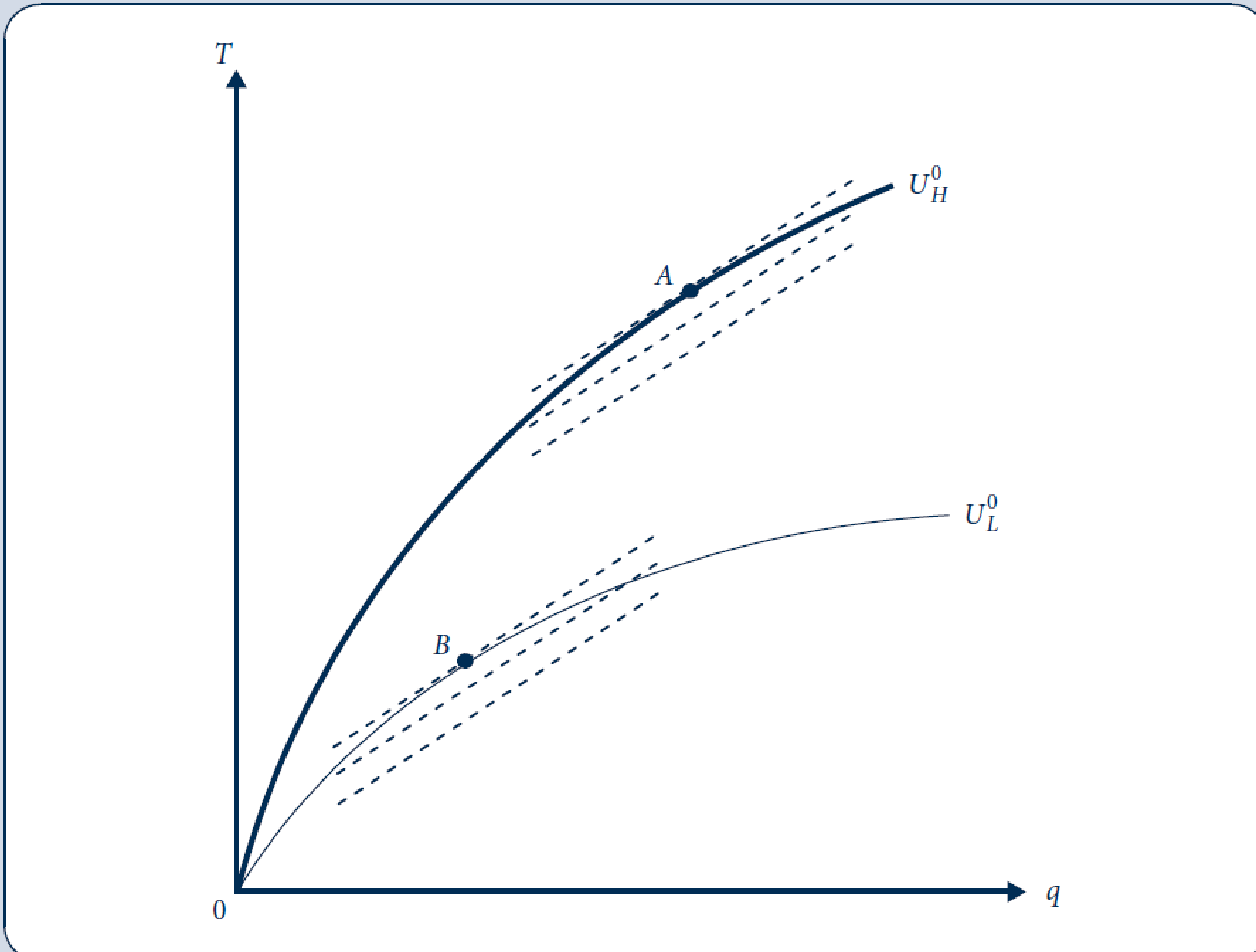
Then monopolist chooses q to maximize profits, FOC is

$$\theta v'(q) = c$$

Then the monopolist offers larger quantity to high type w.r.t low type.

Tariffs are set to extract all consumer surplus

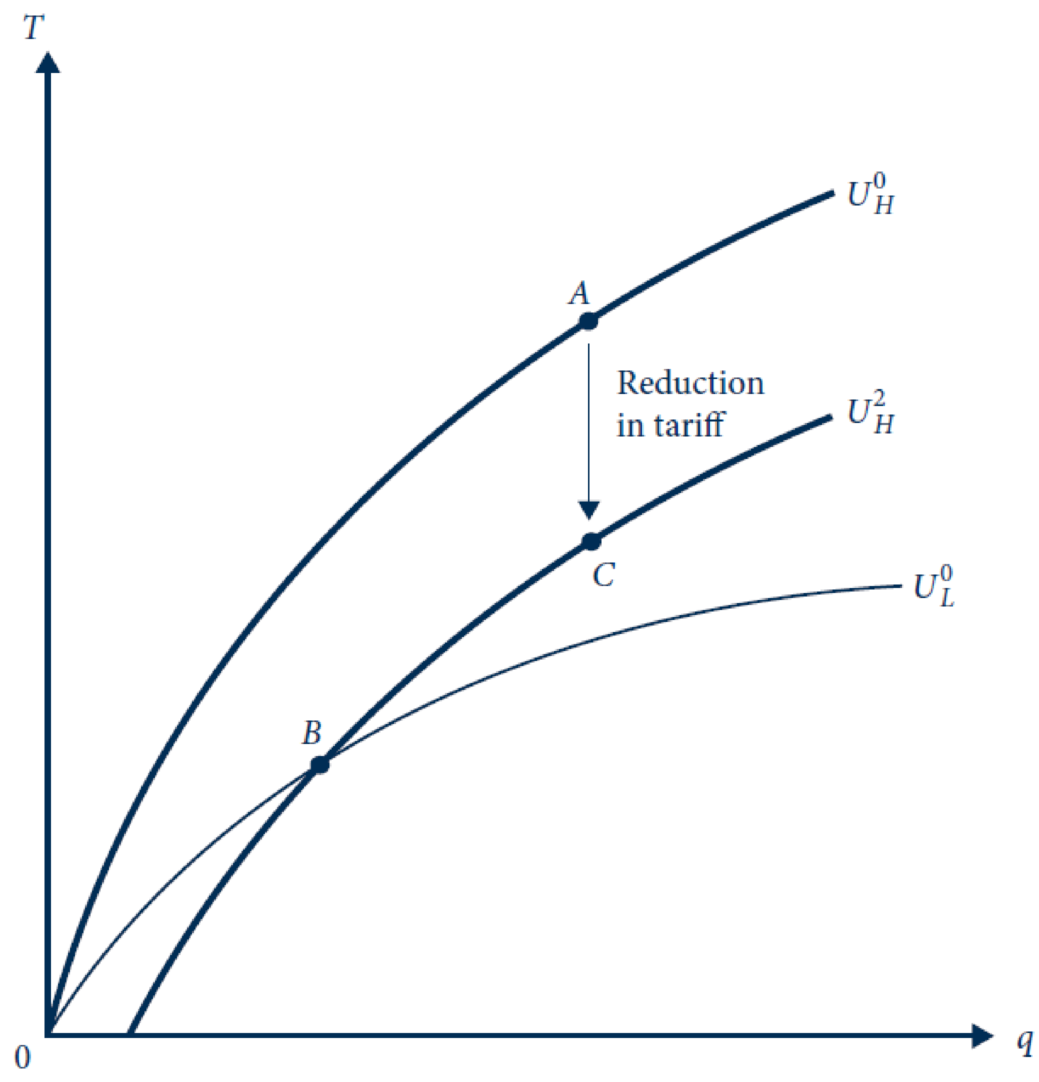
The consumer's indifference curves over the bundle of contractual terms are drawn as solid lines (the thicker one for the high type and thinner for the low type); the monopolist's isoprofits are drawn as dashed lines. Point A is the first-best contract option offered to the high type, and point B is that offered to the low type.



second best nonlinear pricing

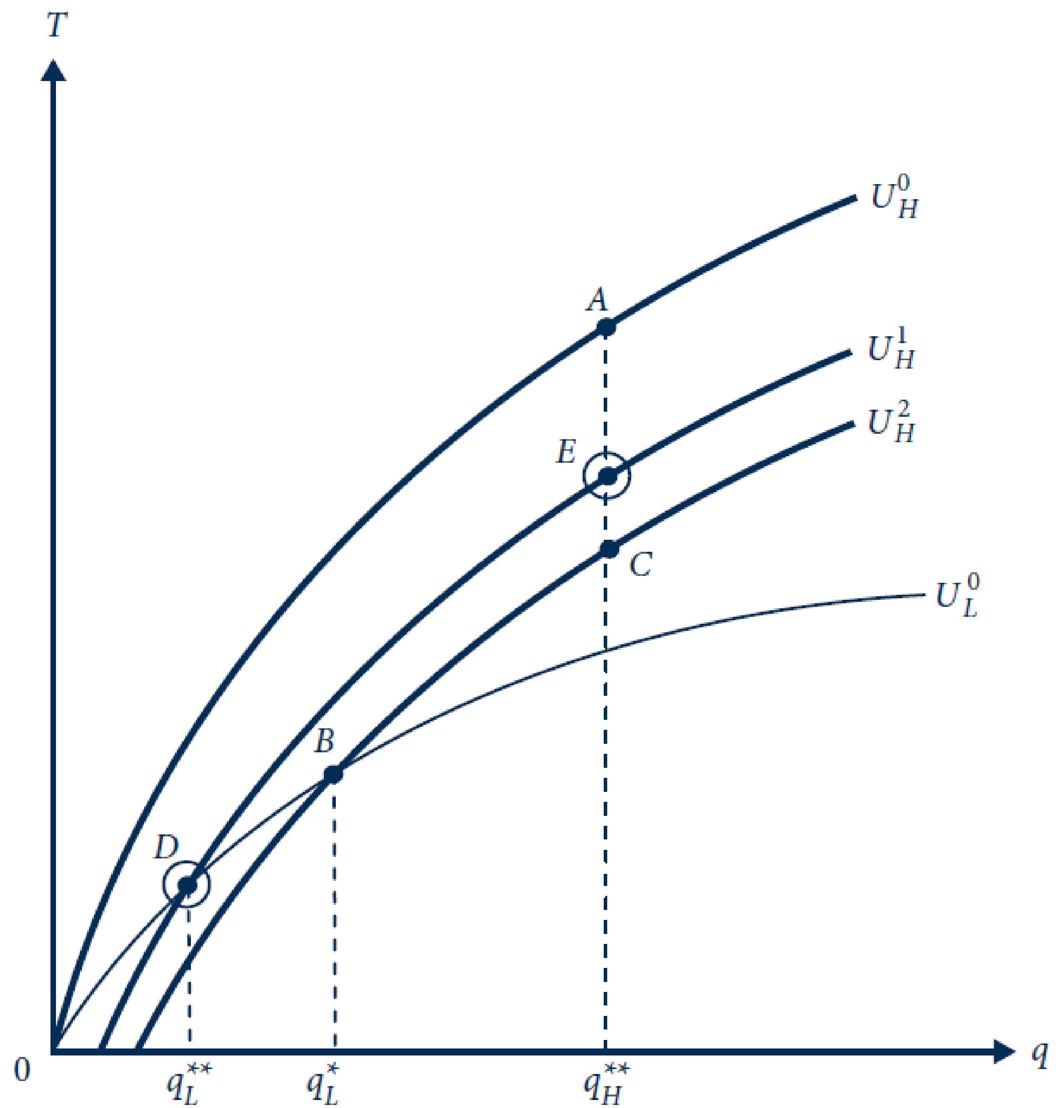
Before offering a contract, the monopolist cannot observes the type θ

First best contract is not more working



There is a better solution than the pair of contracts B and C ,

For example D and E



Second best contract is a pair of contracts (q_H, T_H) and (q_L, T_L) that maximizes monopolist expected profits:

$$\beta(T_H - cq_H) + (1-\beta)(T_L - cq_L)$$

Subject to

$$\theta_L v(q_L) - T_L \geq 0$$
$$\theta_H v(q_H) - T_H \geq 0$$
$$\theta_L v(q_L) - T_L \geq \theta_L v(q_H) - T_H$$
$$\theta_H v(q_H) - T_H \geq \theta_H v(q_L) - T_L$$

The first two are participation constraints for the low and high type of consumer, ensuring that they accept the contract

The last two are incentive compatibility constraints, ensuring that each type chooses the bundle targeted to him rather than the other type's bundle.

The two relevant constraints are the first and the last that will hold by equality

Then the monopolist maximization problem is

$$\max_{q_H,T_H,q_L,T_L} \beta(T_H - cq_H) + (1-\beta)(T_L - cq_L)$$

Subject to

$$\theta_L v(q_L) - T_L = 0$$

$$\theta_H v(q_H) - T_H = \theta_H v(q_L) - T_L$$

Replacing tariffs into the objective function we get a simple unconstrained maximization problem

$$\max_{q_H,q_L} \beta(\theta_H(v(q_H) - v(q_L)) \ - cq_H) + \theta_L\, v(q_L) - (1-\beta)cq_L$$

The FOCs are

$$-\beta\big(\theta_H v'(q_L)\big) + \theta_L\, v'(q_L) - (1-\beta)c = 0$$

$$\beta(\theta_H v'(q_H) - c) = 0$$

Rewritten are

$$\theta_L v'(q_L) = (1-\beta)c + \beta\big(\theta_H v'(q_L)\big)$$

$$\theta_H v'(q_H) = c$$

Rewritten are

$$\theta_L v'(q_L) = (1-\beta)c + \beta\big(\theta_H v'(q_L)\big)$$

$$\theta_H v'(q_H) = c$$

From the second condition the monopolist offers to the high type a quantity that satisfies the relation marginal benefits equal to marginal cost

From the first condition we can see that the quantity offered to the low type is lower respect to the first best contract .

To see this note that $\theta_L v'(q_L) < \theta_H v'(q_L)\ \forall q_L$

Then to satisfy the first condition we need $\theta_L v'(q_L) > c$

Using the constraints we can compute the optimal tariffs

$$T_L = \theta_L v(q_L)$$

$$T_H = \theta_H v(q_H) - (\theta_H v(q_L) - T_L)$$

So the tariff for the low type is equal to his surplus

And the tariff for the high type is his surplus minus the surplus he could get accepting the contract for the low type.

Example: coffe shop

$c = 5\,,\;\; v(q) = 2\sqrt{q}.$

Consumer types are $\theta_H = 20$ or $\theta_L = 15$ by equal probability

First best:

$$\theta v'(q) = c \;\; \rightarrow \frac{\theta}{\sqrt{q}} = 5 \rightarrow q = \frac{\theta^2}{25}$$

$q_L = \frac{15^2}{25} = 9 \quad q_H = \frac{20^2}{25} = 16$ and $T_L = 30\sqrt{9} = 90 \quad T_H = 40\sqrt{16} = 160$

Expected profits $\pi = 0.5(160 - 16 \cdot 5) + 0.5(90 - 9 \cdot 5) = 62.5$

Second best:

For the high type the quantity is equal to the first best, $q_H = 16$

For the low type the quantity is given by the FOC (monopolist problem)

$$\theta_L v'(q_L) = (1-\beta)c + \beta\big(\theta_H v'(q_L)\big)$$

$$\frac{15}{\sqrt{q_L}} = 0.5 \cdot 5 + 0.5\frac{20}{\sqrt{q_L}}$$

$$q_L = 4$$

$q_H = 16 \qquad q_L = 4$

Tariff for the low type is equal to his surplus

$$T_L = 30\sqrt{4} = 60$$

Tariff for the high type is equal to his surplus minus the surplus he could get from taking the bundle for the low type

$$T_H = 40\sqrt{16} - \left(40\sqrt{4} - 60\right) = 140$$

Expected profits $\pi = 0.5(140 - 16 \cdot 5) + 0.5(60 - 4 \cdot 5) = 50$

2. Adverse selection in insurance

an individual with state-independent preferences and initial income W_0 faces the prospect of loss l.

the individual can be one of two types:

- a high-risk type with probability of loss π_H

- a low-risk type with probability π_L,

where $\pi_H > \pi_L$.

The presence of hidden risk types in an insurance market is said to lead to adverse selection.

Insurance tends to attract more risky than safe consumers

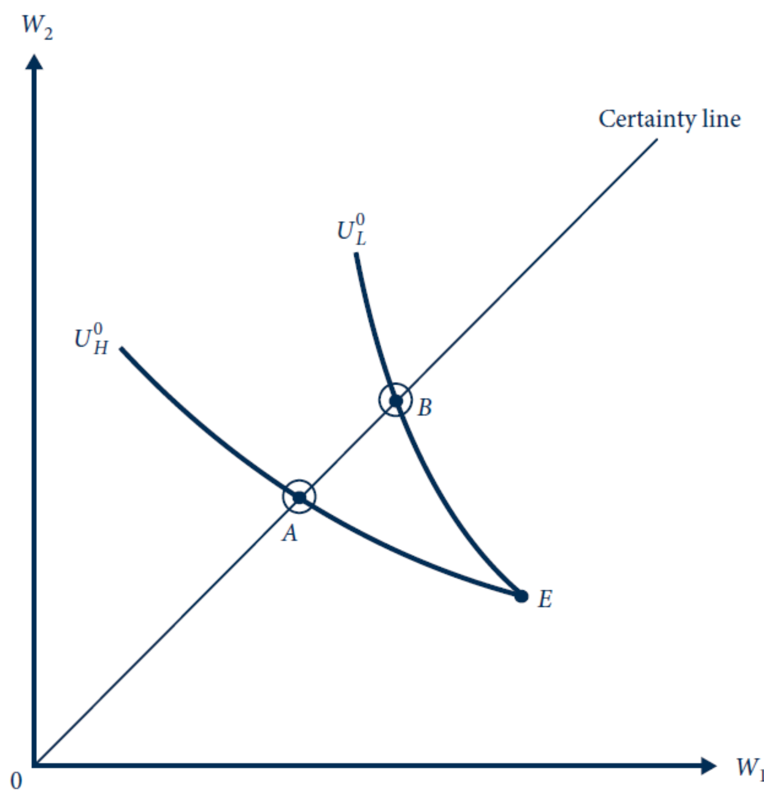
Definition of Adverse selection. The problem facing insurers that risky types are both more likely to accept an insurance policy and more expensive to serve.

First best

the insurer observes the individual's risk type and offer a different policy to each.

Full insurance for each type, $x = l$

Different premiums are charged to each type and are set to extract all of the surplus that each type obtains from the insurance.



Second best contracts

If the insurer cannot observe the agent's type, then the first-best contracts will not be incentive compatible:

the high-risk type would claim to be low risk and take full insurance coverage at the lower premium.

As in the nonlinear pricing problem, the second best will involve a menu of contracts:

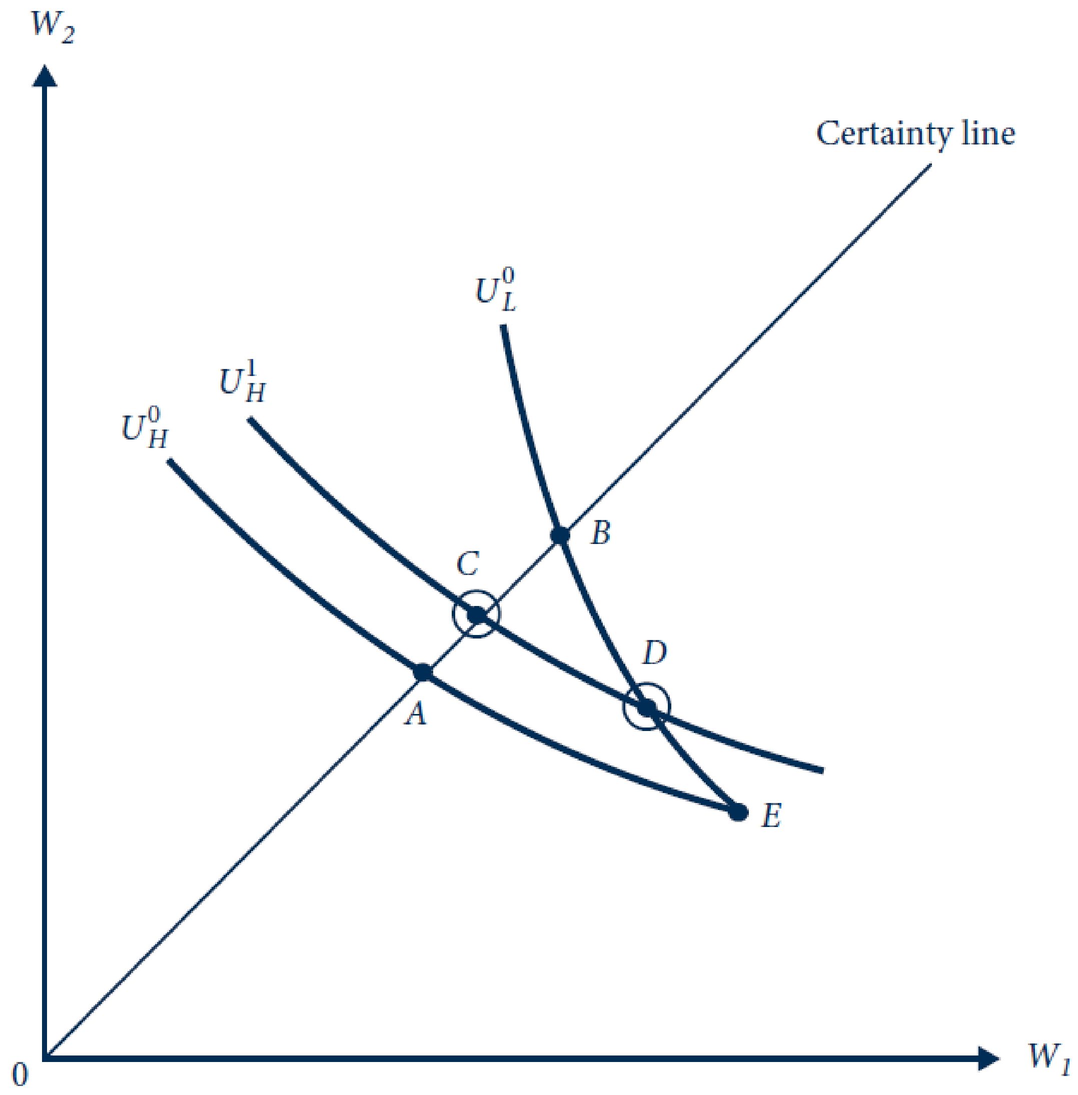
- Full insurance contract
- Partial insurance contract

Premiums are set so that

- The high risk type chooses full insurance
- The low risk type chooses only partial insurance

Premium for low risk type is computed leaving him indifferent between insurance and not insurance

Premium for high risk type is computed leaving him indifferent between full insurance and partial insurance


W_2
Certainty line
U_L^0
U_H^1
U_H^0
B
C
D
A
E
0
W_1

Example

Driver wealth is $w = 100000$, value of the car is 20000.

Car is stolen by probability 0.25 if red and 0.15 if gray. Then the loss is $l = 20000$

The utility function is $u(x) = \ln(x)$

<u>No insurance:</u>

Red car $u = 0.75\ln(100000) + 0.25\ln(80000) = 11.4571$

Gray car $u = 0.85\ln(100000) + 0.15\ln(80000) = 11.4795$

<u>First best contracts</u> (Full insurance)

The driver of a red car buys the full insurance if

$\ln(100000 - p) \geq 11.4571$ → $p = 5426$

The driver of a gray car buys the full insurance if

$\ln(100000 - p) \geq 11.4795$ → $p = 3287$

Insurance profits are:

Red car → $\pi = 5426 - 0.25 \cdot 20000 = 426$

Gray car → $\pi = 3287 - 0.15 \cdot 20000 = 287$

Second best contracts

Suppose the insurer does not observe the color of the customer's car and knows that 10 % of all cars are red and 90% are gray

Two contracts:

1. (p_H, x_H) for the high risk type
2. (p_L, x_L) for the low risk type

The insurance maximize its expected profits

$$0.1(p_H - 0.25x_H) + 0.9(p_L - 0.25x_L)$$

subject to participation constraint for the low type

$$0.85\ln(100000 - p_L) + 0.15\ln(80000 - p_L + x_L) \geq 11.4795$$

Incentive compatible constraint for the high risk type

$$0.75\ln(100000 - p_H) + 0.25\ln(80000 - p_H + x_H) \geq$$
$$\geq 0.75\ln(100000 - p_L) + 0.25\ln(80000 - p_L + x_L)$$

In the solution both constraints hold by equalities

$$0.85\ln(100000 - p_L) + 0.15\ln(80000 - p_L + x_L) = 11.4795$$

$$0.75\ln(100000 - p_H) + 0.25\ln(80000 - p_H + x_H) =$$
$$= 0.75\ln(100000 - p_L) + 0.25\ln(80000 - p_L + x_L)$$

In this case the solution requires a numerical approaches

$x_H = 20000$, $p_H = 4154$, $x_L = 11556$, and $p_L = 1971$.