Solution problem set 9

16.4

Suppose demand for labor is given by

$$l = -50w + 450$$

and supply is given by

$$l = 100w,$$

where l represents the number of people employed and w is the real wage rate per hour.

a. What will be the equilibrium levels for w and l in this market?
b. Suppose the government wishes to increase the equilibrium wage to \$4 per hour by offering a subsidy to employers for each person hired. How much will this subsidy have to be? What will the new equilibrium level of employment be? How much total subsidy will be paid?
c. Suppose instead that the government declared a minimum wage of \$4 per hour. How much labor would be demanded at this price? How much unemployment would there be?
d. Graph your results.

D: $L = -50w + 450$

S: $L = 100w$

$a)\ S = D$

$$100w = -50w + 450$$
$$w = 3, L = 300$$

b) D: $L = -50(w - s) + 450$ where s = subsidy

Government target is $w = 4$ → S: $L = 100w = 400$

$S = D$ implies that

$400 = -50(4 - s) + 450$ → $s = 3$

Total subsidy is 1200.

D: $L = -50w + 450$

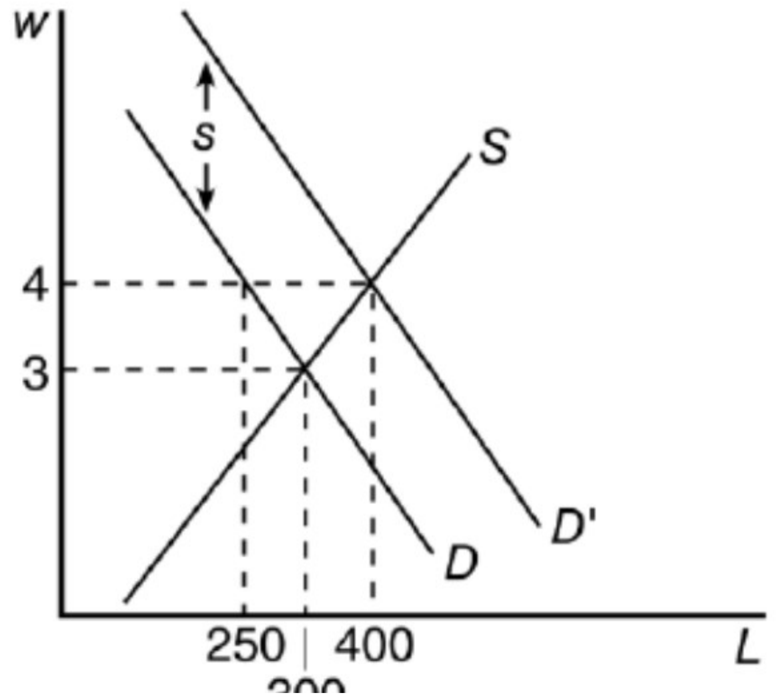
S: $L = 100w$

c) Minimum wage $w = 4$

D: $L_D = -50 \cdot 4 + 450 = 250$

S: $L_S = 400$

$u = L_S - L_D = 150$



16.5

Carl the clothier owns a large garment factory on an isolated island. Carl's factory is the only source of employment for most of the islanders, and thus Carl acts as a monopsonist. The supply curve for garment workers is given by

$$l = 80w,$$

where l is the number of workers hired and w is their hourly wage. Assume also that Carl's labor demand (marginal revenue product) curve is given by

$$l = 400 - 40MRP_l.$$

a. How many workers will Carl hire to maximize his profits, and what wage will he pay?
b. Assume now that the government implements a minimum wage law covering all garment workers. How many workers will Carl now hire, and how much unemployment will there be if the minimum wage is set at \$4 per hour?

Supply: $l = 80w$

Demand: $l = 400 - 40MRP$

$a)$

$$E = wl = \frac{l^2}{80} \rightarrow ME = \frac{l}{40}$$

$$MRP = 10 - \frac{l}{40}$$

$$ME = MRP \rightarrow \frac{l}{40} = 10 - \frac{l}{40} \rightarrow l = 200$$

$$w = \frac{l}{80} = 2.5$$

Supply: $l = 80w$

Demand: $l = 400 - 40MRP$

$b)$

$ME = 4$

$MRP = 10 - \frac{l}{40}$

$ME = MRP \rightarrow 4 = 10 - \frac{l}{40} \rightarrow l = 240$

At $w = 4$ Supply: $l = 80w = 320$

$$u = 320 - 240 = 80$$

17.3

As scotch whiskey ages, its value increases. One dollar of scotch at year 0 is worth $V(t) = \exp\{2\sqrt{t} - 0.15t\}$ dollars at time t. If the interest rate is 5 percent, after how many years should a person sell scotch in order to maximize the *PDV* of this sale?

$$v(t) = e^{2\sqrt{t} - 0.15t}$$

$$PDV = e^{2\sqrt{t} - 0.15t} e^{-0.05t} = e^{2\sqrt{t} - 0.20t}$$

The problem is

$$\max_t e^{2\sqrt{t} - 0.20t}$$

FOC is

$$\left(\frac{1}{\sqrt{t}} - 0.2\right) e^{2\sqrt{t} - 0.15t} = 0$$

$$\frac{1}{\sqrt{t}} - 0.2 = 0$$

$$t = 25$$

4) Paul is endowed by a capital of 10000 euro that can spend in the next 10 months. Every amount that is not spent increases by 10% in the following month, i.e. suppose that in month 1 he spends 1000 (and saves 9000), then in month 2 he has an endowment of 9900. Every amount that is not spent after the 10 months is lost. Assuming that his instantaneous utility function is $u(x) = \ln(x)$ and that his discounted utility function is $U = \sum_{t=1}^{10} 0.8^t u(c)$ compute the optimal consumption profile

Solution

$r = 0.1, U = \sum_{t=1}^{10} 0.8^t u(c), E = 10000, u(x) = \ln(x)$, δ=0.8

We start with a simpler exercise: with only 2 periods

In this case in period 1 consumption is c_1 and in period 2 consumption is $c_2 = (1+r)(10000 - c_1)$

The problem is to maximize

$$u(c_1) + \delta u(c_2) = u(c_1) + \delta u((1+r)(10000 - c_1))$$

FOC is $u'(c_1) - \delta(1+r)u'((1+r)(10000 - c_1)) = 0$

That can be written as

$$u'(c_1) = \delta(1+r)u'(c_2)$$

i.e. the marginal utility in period 1 is equal to the discounted and revaluated marginal utility in period 2

With 3 periods the problem is to maximize

$$u(c_1) + \delta u(c_2) + \delta^2 u(c_3)$$

Note that in each period the consumption can be expressed as the current endowment minus savings

$c_1 = 10000 - s_1$

then the endowment of period 2 is: $E_2 = (1 + r)s_1$.

then $c_2 = E_2 - s_2 = (1 + r)s_1 - s_2$ and finally $c_3 = (1 + r)s_2$

Then the function to maximize can be written as

$$U = u(10000 - s_1) + \delta u((1 + r)s_1 - s_2) + \delta^2 u((1 + r)s_2)$$

It can be maximized choosing the optimal values of s_1 and s_2.

FOCs are:

$$\frac{dU}{ds_1} = -u'(10000 - s_1) + \delta(1 + r)u'((1 + r)s_1 - s_2) = 0$$

$$\frac{dU}{ds_2} = -u'\big((1 + r)s_1 - s_2\big) + \delta(1 + r)u'((1 + r)s_2) = 0$$

That can be written as

$u'(c_1) = \delta(1 + r)u'(c_2)$ i.e. the marginal utility in period 1 is equal to the discounted revaluated marginal utility in period 2

$u'(c_2) = \delta(1 + r)u'(c_3)$ i.e. the marginal utility in period 2 is equal to the discounted revaluated marginal utility in period 3

This relation hold for all problems of this type, independently from the number of periods:

the marginal utility in period t is equal to the discounted revaluated marginal utility in period t+1

That can be written as

$$u'(c_1) = \delta(1+r)u'(c_2) \rightarrow \frac{1}{c_1} = \delta(1+r)\frac{1}{c_2} \rightarrow \quad c_2 = \delta(1+r)c_1$$

$$u'(c_2) = \delta(1+r)u'(c_3) \rightarrow \frac{1}{c_2} = \delta(1+r)\frac{1}{c_3} \rightarrow \quad c_3 = \delta(1+r)c_2 = \delta^2(1+r)^2 c_1$$

How much saving we need in period 1 to consume c_2? A: $\frac{c_2}{1+r} = \delta c_1$

How much saving we need in period 1 to consume c_3? A: $\frac{c_3}{(1+r)^2} = \delta^2 c_1$

Then

$$c_1 + \delta c_1 + \delta^2 c_1 = 10000$$

Solve can solve by c_1 and compute the optimal consumption in every period

With 10 periods

$$u'(c_t) = \delta(1+r)u'(c_{t+1})$$

$$c_{t+1} = \delta(1+r)c_t = 0.8 \cdot 1.1 \cdot c_t = 0.88 \cdot c_t$$

Then

$$c_2 = 0.88 \cdot c_1$$

$$c_3 = 0.88^2 c_1$$

$$c_t = 0.88^{t-1} c_1$$

$$\sum_{t=1}^{10} 0.88^{t-1} c_1 = 10000$$

$$\frac{1-0.88^{10}}{1-0.88} c_1 = 1000$$

$$c_1 = 166$$

$$c_2 = 0.88 \cdot 166 = 146$$

$$c_3 = 0.88^2 166 = 129$$

$$\ldots \ldots .$$

$$c_{10} = 0.88^9 166 = 52.5$$

5) Paul has to choose between two following payment streams:

Stream A: \$80 paid for ever starting from now.

Stream B: \$100 paid for ever starting 2 years from now

Assume that Paul discount rate is 10% and his instantaneous utility function is $u(c) = \ln c$

Which payment stream would Paolo prefer?

Find the discount rate that makes Paul indifferent between the two streams?

Suppose the discount rate is above the value you found in the previous point. What Paul prefer? And what if the discount rate is below?

Which payment stream would Paul prefer?

$$\delta = \frac{1}{1+0.1} = 0.909$$

Stream A: $80 paid for ever starting from now.

$$U = \ln 80 \frac{1}{1-0.909} = 48.2$$

Stream B: $100 paid for ever starting 2 years from now

$$U = \ln 100 \frac{0.909^2}{1-0.909} = 41.9$$

Paul prefers stream A

Which payment stream would Paul prefer?

$$\delta = \frac{1}{1+0.1} = 0.909$$

Stream A: $80 paid for ever starting from now.

$$U = \ln 80 \frac{1}{1-0.909} = 48.2$$

Stream B: $100 paid for ever starting 2 years from now

$$U = \ln 100 \frac{0.909^2}{1-0.909} = 41.9$$

Paul prefers stream A

Find the discount rate that makes Paul indifferent between the two streams?

$$\ln 80 \frac{1}{1-\delta} = \ln 100 \frac{\delta^2}{1-\delta} \rightarrow$$

$$\ln 80 = \ln 100\, \delta^2 \rightarrow \delta = \sqrt{\frac{ln80}{ln100}} = 0.9755$$

$$\delta = \frac{1}{1+r} = 0.9755 \rightarrow r = \frac{1}{0.9755} - 1 = 0.025$$

$\delta = \frac{1}{1+r} = 0.9755 \rightarrow r = \frac{1}{0.9755} - 1 = 0.025$

If the discount rate is above, Paul is impatient and prefers stream A

If the discount rate is lower, Paul discount less the future and prefers to wat to get more.

6) Suppose a machine is producing 1.000 units of a good every year. Each year this machine can go destroyed by a probability of 0.1. In case it is destroyed, it is not recovered and a new machine must be bought. Assume:

1. Assuming that this good is sold at a price of 10 per units and price remains constant in the coming years and that the interest rate is 5%, compute the present discounted value of the machine.

2. based on the result of previous point for which price you are willing to buy this machine.

3. Compute the point 1 assuming that in each year the probability to be destroyed is 5%, but after 10 years the machine must be replaced.